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IN FINANCIAL MARKETS**

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Abstract

The aim of the works presented in this dissertation, in line with behavioral finance and agent-based as well as computational approaches to finance, is to explain some well-documented anomalies (*i.e.*, which cannot be fully rationalized within the traditional finance approach) through heterogeneous agent models based on the bounded-rationality hypothesis. More precisely, this research, which contributes to the ongoing debate about the explanations of some financial anomalies, focuses on the effect of the interplay between heterogeneous, boundedly rational agents on price dynamics.

For this purpose, in line with previous models based on extensive survey data works, we explicitly make the distinction between fundamentalists and chartists. However, rather than assuming that the very figure of technical analysis is trend followers, we also account, due to well-documented behavioral motivations, for the presence of contrarian traders within the chartist population.

The first model developed in this dissertation is analytically tractable and aims at providing the baseline for the subsequent models. In fact, through a simple model, wherein fully informed arbitrageurs (or *fundamentalists*) and chartists - both trend followers and contrarian traders - coexist in the market, we are able to identify (*i.*) the stability conditions of the system; (*ii.*) the effect of each trading strategy; (*iii.*) the effect of the interplay between different trading strategies on price dynamics. Our simple model - based on linear trading strategies and a linear price adjustment rule - is actually able to reproduce a variety of price dynamics (from stable to unstable systems) and, more important, the occurrence of over(under)-valued assets which are often observed in real financial data. While the presence of arbitrageurs is a key determinant of mispricing correction, trend following as well as contrarian strategies can destabilize market prices. The implementation of a number of comparative statics experiments confirms the robustness of our results and enables us to carefully choose parameter values used in subsequent models

which require numerical simulations.

Second, in line with earlier works based on *synchronization risk*, we relax the assumption according to which arbitrageurs are fully informed about asset fundamentals. This new model enables us to explore whether slow diffusion of information and/or dispersion of opinions among arbitrageurs could prevent arbitrageurs from correcting asset mispricing. For this purpose, we focus on the price dynamics within the time window in which arbitrageurs discover the asset fundamental value. As a result, computational simulations are convenient in order to grasp short-run price dynamics. Our model is able to reproduce some stylized facts observed in financial time series, namely, positive serial correlations in returns over short horizons, which characterize mispricing persistence. Moreover, our model suggests that when information diffusion is slow and/or arbitrageurs have heterogeneous beliefs, some chartist strategies, namely, contrarian ones, can favor mispricing correction.

Third, we investigate the role of fundamentalists' memory on price dynamics. For this purpose, we develop a model wherein fundamentalists, rather than knowing asset fundamentals, forecast future prices cum dividend through an adaptive learning rule. Computer simulations are required since the learning rule introduces a high degree of nonlinearity into the solution, so that the model cannot be solved analytically. Our model is actually able to reproduce simultaneously some well-documented anomalies, namely, short-run and long-run dependencies in time series (*i.e.*, momentum and reversals in returns), as well as excess volatility of returns. Predictability of returns over differing horizons can be explained by long memory in the learning mechanism of fundamentalists. Excess volatility can rather be explained by fundamentalists' short memory.

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Introduction

This dissertation puts forward the view that heterogeneity in agents' characteristics and less-than-perfectly rational agents' behaviors play a crucial role in shaping the overall price dynamics observed in financial markets. While investors' heterogeneity and bounded rationality are painfully arguable, a representative, rational agent approach has been the cornerstone of asset pricing theory for more than thirty years. Manifest illustrations of this viewpoint are Milton Friedman and John F. Muth, some of the strongest backers of the rational agent approach, hypotheses. On the one hand, according to the *Friedman hypothesis*, since non-rational agents will not survive evolutionary competition, they will be driven out of the market (Friedman, 1953), so that less-than-rational behaviors cannot markedly affect market prices. On the other hand, according to the *Muth hypothesis*, “[expectations] are essentially the same as the predictions of the relevant economic theory” (Muth, 1961, p. 315). However, extensive empirical evidence on asset pricing “anomalies”, *i.e.*, financial phenomena which are hardly rationalized within the traditional approach, suggests that alternative explanations may be required. The most obvious example of such anomalies is the recurring occurrence of bubbles and crashes in real financial markets (*e.g.*, the international crash of October 1987 and the DotCom Bubble in the late 1990s). Since the 1990s, behavioral finance has

emerged as an alternative approach to explain such anomalies. One of the main novelties of this approach is that it is built on how agents actually behave, through empirical evidence found out by psychologists, rather than how they should behave. Since less-than-perfectly rational agents' behaviors are widespread amongst investors and agents' decisions may be affected by systematic judgmental biases, in a heterogeneous world, a representative, rational agent framework may no longer be appropriate. Furthermore, the behavioral approach requires new tools of analysis. While in the traditional approach, simple analytically tractable models could be developed, within the behavioral approach, agent-based simulations and computational models, due to the complexity of the dynamics that these models tend to generate, have been increasingly employed. Behavioral finance and existing heterogeneous agent models have already been successful in coping with some of the weaknesses of the traditional finance and have been able to provide an accurate description of asset prices' behaviors. In particular, some heterogeneous agent models have been powerful in replicating stylized facts observed in real time series (*e.g.*, De Grauwe, Dewachter, and Embrechts, 1993; Lux, 1995; Lux and Marchesi, 1999; Hommes, 2001; De Grauwe and Grimaldi, 2005; Alfarano, Lux, and Wagner, 2008).

Essays in this dissertation focus on the effect on price dynamics of the interplay between heterogeneous strategies followed by boundedly rational agents.¹ In line with behavioral finance approach, we propose simple frameworks, based on linear

¹It is worth mentioning that while our research mainly focuses on anomalies in stock markets, almost all markets (*e.g.*, foreign exchange markets, derivatives) share a set of common statistical regularities, so that many of the issues considered in this dissertation could also be applied to other asset markets.

behavioral rules and linear price formation rule, to reproduce, through numerical simulations, some of the most important remaining anomalies (such as over- and under-valued assets that may persist over some period of time, momentum and reversals in returns, and excess volatility of returns). One of the oppositions to existing computational models is that in order to be able to faithfully replicate stylized facts observed in real time series, such models often require sophisticated and complicated frameworks, including many parameters. The main drawbacks of such an approach consists in *(i.)* the trickiness in understanding the underlying, causal mechanisms; *(ii.)* the possibility of assessing the relevance of the range of parameter values; *(iii.)* the difficulty of anchoring them to behavioral motivations. With this in mind, in order to overcome these weaknesses, in the models proposed in this dissertation there are few free parameters. Moreover, a careful study of the range of the parameter values which ensure price dynamics stability is carried out, as a preliminary step.

In Chapter 1, we survey the current leading asset pricing theories, namely, standard finance theory and behavioral finance theory. The first approach, based on the representative, rational agent framework and the efficient market hypothesis, maintains that prices should coincide with fundamentals, so that movements in prices should only reflect new information. The empirical evidence discussed in Section 1.2.2 suggests that asset prices may happen to be misaligned, for some period of time, and asset returns may be, at least partially, predictable. Accordingly, the traditional finance approach appears to be powerless to provide an accurate description of asset prices' behavior.

Behavioral finance, based on extensive evidence provided by psychologists on the

biases that may arise when agents make their decisions, rejects the representative, rational agent framework. This alternative approach rather emphasizes the role played by boundedly rational agents in shaping asset prices' behaviors. In Section 1.3, we discuss the major strengths of behavioral finance, based on the bounded-rationality hypothesis and limits to arbitrage, and how this approach can offer a more accurate description of asset prices' behaviors.

In Section 1.4, we review the existing heterogeneous agent models which have been the most influential for our research. We believe that the starting point of any analysis of asset prices' behaviors comes from a framework based on the interactions between heterogeneous agents. Furthermore, this survey enables us to identify the key hypotheses which will constitute the backbone of the heterogeneous agent models proposed in the following chapters. In particular, first, the discussion in Section 1.3 emphasizes that in a heterogeneous world, a representative, rational agent framework is not appropriate (*e.g.*, Kirman, 1992). With this in mind, a very influential hypothesis we choose to adopt in our research relies on bounded rationality. Second, extensive empirical evidence, as discussed for instance in Section 1.4, suggests that investors in financial markets tend to use simple trading strategies, namely, fundamentalist and technical analysis. Regarding technical analysis, there is strong evidence suggesting that the representation of technical analysis through trend followers is incomplete. In fact, a vast amount of works has supported the relevance and the profitability of contrarian strategies (*e.g.*, De Bondt and Thaler, 1985; Odean, 1998), which is worth accounting for in a heterogeneous agent model. One of the novelties of our study is to consider that both trend followers as well as contrarian traders can be present in the market.

In Chapter 2, we propose a simple, analytically tractable heterogeneous agent model which aims at assessing whether asset prices coincide with fundamentals when agents are boundedly rational. This model constitutes the foundation of the models developed in the subsequent chapters. Central for the outcomes of the model is the hypothesis according to which arbitrageurs (or *fundamentalists*) are well-funded and fully informed about asset fundamentals, so that they could ensure market efficiency. The study of the analytical solution of the foregoing model enables us to identify the effect of each trading strategy as well as the interplay between arbitrageurs and chartists - both trend followers and contrarian traders - on price dynamics. This simple model is also able to generate a variety of price dynamics. More important, our model can reproduce the occurrence of over- and under-valued assets which happen to emerge in real financial markets. The findings from this model clearly indicate that the composition of the population plays a crucial role in explaining asset prices' behaviors. Furthermore, while earlier works based on the distinction between fundamentalists and trend followers suggest that trend following strategies tend to destabilize market prices, we show that these findings can be extended to contrarian strategies as well. We eventually implement a number of comparative statics experiments which confirm the robustness of the results for differing parameter values of the model.

In Chapter 3, we extend the aforementioned model by relaxing the assumption according to which arbitrageurs are fully informed about asset fundamentals. In this new model, we propose a model wherein arbitrageurs are partially informed about the asset fundamentals. In line with earlier works based on *synchronization risk*, this model aims at investigating whether price underreaction and mispricing

persistence can be explained by slow diffusion of information, when chartists are present in the market.

Simulation results indicate that our model is able to reproduce the momentum effect that is observed, for instance, in individual stocks as well as industry portfolios. We find that slow diffusion of information and/or dispersion of opinions among boundedly rational arbitrageurs plays a crucial role in explaining positive serial correlations in returns over short horizons. Moreover, while earlier works suggest that trend following strategies destabilize markets, especially when trend followers dominate the market (*e.g.*, Lux, 1995; Lux and Marchesi, 2000), we find that misalignments in asset prices, which persist over time, can be explained by trend following strategies, even when arbitrageurs dominate the market. Lastly, while arbitrage strategies play a key role, contrarian strategies may also favor market efficiency. As a result, slow diffusion of information may not be sufficient to explain price underreaction.

Lastly, in Chapter 4, we explore the role of fundamentalists' memory on price dynamics. For this purpose, we depart from the assumption according to which fundamentalists have or end up having complete knowledge of the fundamentals. We rather develop a model wherein fundamentalists forecast future prices cum dividend through an adaptive learning rule. This model aims at investigating return predictability (*i.e.*, momentum and reversals in returns over differing horizons) as well as excess volatility of returns.

Simulation results indicate that our model is able to simultaneously reproduce some of the stylized facts observed in real time series *i.e.*, short- and long-run dependencies (which characterize momentum and contrarian effects) as well as excess

volatility of returns. Furthermore, fundamentalists' memory plays a crucial role in explaining asset prices' behaviors.

In fact, first, we find that, in contrast with earlier works, return predictability over short horizons can be reduced when fundamentalists have short memory. Second, while existing works have been able to reproduce either momentum in returns (*e.g.*, Lo and MacKinlay, 1988; Jegadeesh and Titman, 1993; Chan, Jegadeesh, and Lakonishok, 1996) or reversals in returns (*e.g.*, Fama and French, 1988; Jegadeesh and Titman, 1995a), our model is able to reproduce simultaneously positive and negative serial correlations over differing horizons. More precisely, we find that longer memory values rather induce both trends in prices over short horizons and oscillations in prices over long horizons. Third, our model is also able to reproduce excess volatility of returns (*e.g.*, Shiller, 1981; Cutler, Poterba, and Summers, 1989). While chartist strategies can reduce excess volatility of returns, such an anomaly can be explained by short memory in the learning mechanism of fundamentalists.

Summarizing, the dissertation is organized as follows. In Chapter 1, we review the existing literature on asset pricing theory. We discuss the major flaws associated with the traditional finance and the main strengths of behavioral finance, as a more accurate, better description of asset prices' behaviors. We survey some of the existing heterogeneous agent models which have been successful in explaining some of the remaining asset pricing anomalies, thereby setting the stage for the next chapters. In Chapter 2, we build a simple heterogeneous agent model, which is analytically tractable, and we discuss whether it can reproduce the occurrence of over- and under-valued assets. In Chapter 3, we present an extension of the

previous model and we evaluate whether it can generate positive serial correlations in returns over short horizons. In Chapter 4, we propose a heterogeneous agent model, which requires computational simulations, and we discuss how it is able to reproduce simultaneously both positive and negative serial correlations in returns over differing horizons, as well as excess volatility of asset returns. Lastly, we present the concluding remarks of our research and suggest possible extensions of this work.

Chapter 1

Asset Mispricing and Heterogeneous Agents in Financial Markets - A Survey

1.1 Introduction

Evidence in financial markets, about asset price' behaviors as well as investors' behaviors do not seem to be easily understood in the classical asset pricing theory, based on the efficient market hypothesis.

First, financial markets may exhibit, in some occasions, misalignments in asset prices *i.e.*, when asset prices depart from the asset fundamentals. Even if assessing the fundamental values of financial stocks is often tricky, some features of stock prices are very likely to be evidence of persistent departures from asset fundamentals.¹ An extreme example of misalignments in asset prices is financial bubbles followed by crashes (*e.g.*, the Internet Bubble of the late 1990s). Such an empirical evidence, for instance, is not fully explained by the classical asset pricing theory *i.e.*, “anomalies”. Indeed, the persistence of stock price misalignment (or *mispricing*) often results in predictability of stock returns, from which agents could take advantage and systematically make profits. However, this would contradict some predictions of the efficient market hypothesis, according to which prices are consistent with fundamentals and no investor can systematically beat the market and earn expected profits in excess of equilibrium levels. However, since financial markets are concerned with allocating capital to the most promising investment opportunities, “prices are right” becomes a crucial issue.

Second, it is worth accounting for the fact that, in real markets, agents (*i.e.*,

¹Further discussion and detailed references on such an empirical evidence are provided in Section 1.2.2.

investors or traders) are heterogeneous. In financial markets, agent heterogeneity uncovers a wide variety of situations, such as institutional versus individual investors, informed versus uninformed traders, differences in endowments and in trading strategies. Furthermore, there is a number of works which have suggested that agents may be subject to psychological biases.² From such an evidence on agents' behaviors, one may ask whether it is still reasonable to deal with the representative agent, assumed fully rational. Indeed, the term *rationality* embeds two key ideas, namely, (i.) when they receive new information, agents update their beliefs correctly, in the manner described by Bayes law; (ii.) given their preferences, agents make choices that are normatively acceptable *i.e.*, consistent with Savage's notion of subjective expected utility. However, accounting for evidence on individual behavioral complexities raises some further issues. How psychological biases affect agents and their trading strategies? How interactions between heterogeneous agents may affect asset prices' behaviors?

Facing these open questions, which we try to explore in this chapter, we propose to shed light on: (i.) what may be "wrong" in the traditional framework? (ii.) what could prevent the classical asset pricing theory from fully explaining asset pricing anomalies and why an alternative approach may be required?

We believe that one way to go is to ask whether misalignments in asset prices and other anomalies - including bubbles as an extreme case - are due to a coordination problem between heterogeneous agents. However, this viewpoint requires a better understanding of agents' behavior, in order to be able to investigate how agents interact and what is the outcome of their interactions.

²Further discussion and references are provided in Section 1.3.

With this purpose in mind, behavioral finance may be enlightening. First, while the classical asset pricing theory tries to understand financial markets using models in which agents are *rational* and exhibit *consistent beliefs*,³ behavioral finance proposes some crucial elements to understand how agents may deviate from rationality and how an alternative framework can be successful in explaining some remaining anomalies. Behavioral economists rather account for experimental evidence provided by cognitive psychologists on the biases that arise when people form their beliefs and on people preferences when they make their decisions. As an alternative to the traditional framework, behavioral approach may help to explain anomalies in financial markets, using models in which agents are boundedly rational. Accordingly, agents may be subjects to systematic biases which may affect their behavior. In financial markets, investor's demand for financial assets may be influenced by their *beliefs* or *sentiments*, so that they do not trade only based on market information. These considerations on agents' behavior are instructive and useful to understand asset price formation and may help to understand how agents use and interpret market information.

Behavioral finance rests on a second building block, namely, "*limits to arbitrage*".⁴ Contrary to the classical asset pricing theory, market forces which bring asset prices back towards fundamental values may be limited (*e.g.*, further discussion on limits to arbitrage is provided in DeLong, Shleifer, Summers, and Waldmann, 1990; Shleifer and Vishny, 1997; Wurgler and Zhuravskaya, 2002). Even in an economy wherein rational investors interact, agents' "irrationality" can have a substantial influence on prices, so that changes in investor's sentiments may affect financial

³ *Consistent beliefs* expresses the idea according to which agents' beliefs are correct.

⁴ The idea that behavioral finance is built on the two pillars of limits of arbitrage and investors' psychology is originally due to Shleifer and Summers (1990).

asset returns.

The purpose of this survey is therefore to investigate how behavioral finance, providing a more realistic analysis of agents' behaviors and asset price formation, may help us to understand *(i.)* how do, not fully rational, agents behave; *(ii.)* how do they interact in financial markets; *(iii.)* what is the outcome of their interactions.

In section 1.2, we review the literature on the efficient market hypothesis, highlighting the main successes and shortcomings of this approach. In section 1.3, we review earlier works on behavioral finance which propose a more accurate explanation of agents' behavior. In section 1.4, we present some heterogeneous agent models which, we believe, are useful starting points in order to explain some remaining asset pricing anomalies.

1.2 Efficient Market Hypothesis: Successes and shortcomings

The efficient market hypothesis is concerned with describing the behavior of asset prices in financial markets. The efficient market hypothesis actually proposes some important elements in order to define an efficient market:

(i.) “a market that adjusts rapidly to new information” (Fama, Fisher, Jensen, and Roll, 1969, p. 1).

This implies that, as mentioned, for instance, by Singal (2006), if new information about a stock (*e.g.*, changes in earnings), an industry (*e.g.*, changes in demand)

or the economy (*e.g.*, changes in expected growth), is revealed, an efficient market will quickly reflect that information through price changes.

(*ii.*) “[a] market in which prices “fully reflect” all available information” (Fama, 1970, p. 383).

Fama (1970) actually proposes different forms of market efficiency based on the amount of information that is assumed to be available in the market. The market is “weak-form” efficient, if current prices reflect only information regarding past prices. The market is “semi-strong form” efficient, if current prices reflect all publicly available information. Lastly, the market is “strong-form” efficient, if current prices reflect all private information.

As a result, from the definition of an efficient market, some predictions can be derived (*i.*) investors - individual or institutional - cannot beat the market *i.e.*, earn expected profits or returns in excess of equilibrium expected profits or returns; (*ii.*) the price adjustment should be accurate on average *i.e.*, prices should neither underreact nor overreact to particular information; (*iii.*) financial asset prices should always be consistent with “fundamentals” and prices should not move without any new information.

Following these predictions, a large amount of empirical studies have been devoted to testing whether financial markets are actually efficient. From an empirical viewpoint, earlier studies have supported the existence of financial puzzles *i.e.*, financial phenomena which are hardly rationalized with standard consumption-based models. Such an evidence has challenged the efficient market hypothesis. While these facts have been widely agreed on, they are not completely uncontroversial. A

broad branch of the literature has in fact questioned the statistical validity of empirical tests and the fundamental value specifications of some models. The latter issue highlights the fact that it is hard to assess fundamental values (or normal returns). While many asset prices' behaviors can be interpreted as deviations from fundamental values, it is only in few cases that asset mispricing can be established beyond any doubt.⁵ Fama (1970) emphasizes that in order to claim that an asset price differs from its properly discounted future cash flow, one needs a model of "proper" discounting. Any test of asset mispricing is therefore inevitably a *joint* test of mispricing and discount rates. Under these considerations, it becomes difficult to provide definitive evidence of market inefficiency. While this concern has cast doubts on some empirical findings, it is still possible to draw important conclusions about the informational efficiency of financial markets.

In what follows, we present some of the predictions of the efficient market hypothesis that have been undoubtedly validated by empirical evidence. These findings suggest that, to some extent, the efficient market hypothesis is a valid and useful theory to describe prices' behaviors in financial markets (for further details about the ongoing debate regarding the successes and threats to the efficient market hypothesis, see, for instance, Fama (1991, 1998); Beechey, Gruen, and Vickery (2000); Malkiel (2003); Schwert (2003)).

⁵We discuss empirical evidence on remaining anomalies and provide references in Section 1.2.2.

1.2.1 Some successes

New information is quickly incorporated into asset prices

Fama, Fisher, Jensen, and Roll (1969) have shown that stock prices respond rapidly to new information and subsequently display no apparent trends. Later on, many event studies⁶ generally confirmed this price adjustment pattern following major events (such as, for instance, stock splits, mergers, changes in firms' dividend policy). This finding contributed to demonstrate that financial markets usually incorporate new information very quickly (so that investors cannot beat the market) and most of the time correctly (so that price adjustment is accurate on average).

Asset prices move as random walks over time

In an efficient market, asset prices should fluctuate randomly over time, since they should reflect economic fundamentals (Samuelson, 1965; Fama, Fisher, Jensen, and Roll, 1969). As a result, asset price fluctuations should not be predictable. Some empirical works have suggested that asset returns could, in some circumstances, be partially predictable.⁷ However, the prediction according to which financial asset prices move as random walks over time remains approximately true. In fact, no empirical work has been really successful in providing a better description of asset price movements *i.e.*, as consistent over time as the random walk approach.

⁶The basic idea behind any event study is that it measures stock price changes in response to events *i.e.*, the average stock price reaction to the same type of event experienced by many firms. The event under consideration is examined in “event-time”, so that many similar events can be considered simultaneously, even though the event date differs across firms or stocks.

⁷We discuss in detail these empirical works and provide references in Section 1.2.2. The following chapters are in part devoted to providing explanations of such an evidence.

Fund managers cannot systematically outperform the market

The above description of the efficient market hypothesis mainly relies on the strong version of the hypothesis, according to which asset prices fully reflect all available information. However, this version could be true only if “all available information” was costless to obtain, which is a quite strong assumption.⁸ Jensen and Field (1978), in contrast, provide a weaker version of the hypothesis, according to which prices reflect information up to the point where the marginal benefits of using the available information do not exceed the marginal costs of collecting it.

Since managed funds employ managers who actively try to uncover information, careful examination of their performance can be instructive. Fund managers’ performance can actually be compared with alternative passive strategies (*e.g.*, buying-and-holding the market). The weak version of the efficient market hypothesis would actually predict that actively managed fund should equal passive returns *after* deducting management expenses. Across all the empirical research that has been devoted in testing such a prediction, the performance of actively managed funds appears to be broadly and robustly supportive of the efficient market hypothesis. The earliest studies on mutual fund performance have been undertaken by Sharpe (1966); Jensen (1968). Jensen (1968), for instance, finds that during the (1945-64) period, active mutual fund managers use to underperform the market by about the amount of their expenses. Malkiel (1995) repeated Jensen’s study for a subsequent period (1971-1991) and his study confirmed earlier results. Subsequent research, which has confirmed earlier findings as well, includes works by

⁸Further discussion and references on this issue are provided in the next subsection.

Lakonishok, Shleifer, Vishny, Hart, and Perry (1992) on US pension funds, and Grinblatt and Titman (1989); Lee and Rahman (1990) on US mutual funds. Even after deducting management expenses, actively managed funds do not usually outperform passively managed funds. Fund managers' performance tend to support the idea that financial asset prices move as random walks over time.

Some anomalies are questionable

Vast empirical research, devoted to test the predictions of the efficient market hypothesis, has frequently pointed out that anomalies (*i.e.*, empirical findings inconsistent with existing theories of asset pricing behavior) exist. However, most of the well-known anomalies in the finance literature have been largely contested and gave rise to lively debates about their robustness and validity. The main criticism relies on the idea that many empirical evidence of anomalies (*i.*) does not hold up in different sample periods (*i.e.*, empirical findings are highly dependent on the time period for which they have been identified); (*ii.*) vanish as soon as the finance literature publicizes their existence, casting doubt on whether they have initially existed. Such a criticism has, for instance, been raised against the *size effect* (*i.e.*, the finding that small capitalization firms earned higher average returns than predicted by the classical asset pricing theory) documented, for instance, by Banz (1981); Reinganum (1981); or the *weekend effect* (*i.e.*, the finding that returns are relatively larger on Fridays than those on Mondays) documented, for example, by French (1980); Schwert (1990); Keim and Stambaugh (1984).

Singal (2006) reviews further limitations and biases that may arise in the process of discovering asset mispricing. Singal (2006), as an illustration, suggests that spurious asset mispricings can emerge for several reasons: data mining, survivorship

bias, selection bias, misestimation of risk.

Lastly, fervent advocates of the efficient market hypothesis have also put considerable efforts on demonstrating that whatever anomalous stock prices' behaviors may be detected, such an evidence does not necessarily imply that profit opportunities are available (see, for instance, Malkiel, 2003). Accordingly, market efficiency still cannot be rejected.

These oppositions suggest that, as suggested by Schwert (2003), many “anomalies are [likely to be] more apparent than real” (p. 939). Overall, the above-mentioned considerations convey the idea that market efficiency remains the rule rather than an exception.

1.2.2 Some shortcomings

Market efficiency is of great importance because markets set prices. Since prices determine how available resources are allocated among different uses, “prices are right” is a central issue. However, while market efficiency is desirable, theoretical foundations of the efficient market hypothesis as well as empirical evidence have been challenged.

Theoretical challenges to efficient market hypothesis

The classical asset pricing theory, which rests on the law of one price and rational (and homogeneous) agent hypothesis, predicts that markets are efficient and prices quickly reflect available information about asset fundamentals (as put forward by Fama, Fisher, Jensen, and Roll, 1969; Fama, 1970). However, there are some limitations in achieving such an ideal.⁹

⁹Singal (2006), for instance, review some of the limitations examined in this subsection.

First, information is *costly*. Indeed, Grossman and Stiglitz (1980) stress that if markets are fully efficient *i.e.*, instantaneously reflect new information in prices, no market participant has an incentive to report new information. But if no one has any incentive to reveal new information, then it is impossible for prices to reflect new information and markets cannot be fully efficient. Market participants must have an incentive to make markets efficient *i.e.*, they must get some reward for this activity. This is usually achieved by the existence of a time lag in information adjustment, which allows market participants to earn some return on their cost of obtaining and processing information (Merton, 1987). However, if the return is abnormally high, this will attract more information processors, leading to a reduction in the time lag. Stock prices hence take time to adjust and to reflect new information because obtaining and processing that information is costly. The adjustment speed will determine the extent of market efficiency.

Second, traders face *other costs* when they trade, as for instance, in terms of time, brokerage costs, etc. When these transaction costs are high, financial assets are likely to remain misaligned over longer periods than when these costs are low. These costs are likely to limit investors trading activity. Like with the cost of information, traders must get an adequate return after accounting for costs to engage in an activity that makes markets more efficient. Otherwise, prices will not reflect all available information (Jensen and Field, 1978).¹⁰

Third, *arbitrage may be limited*. The efficient market hypothesis suggests that

¹⁰This issue is likely to have motivated the mentioned-above weaker version of the efficient market hypothesis provided by Jensen and Field (1978).

some investors *i.e.*, arbitrageurs, would trade if prices do not coincide with their information and would continue to do so until asset prices reflect available information. This activity ensures that prices will coincide with fundamentals. However, existing literature has emphasized that arbitrage may be limited.¹¹

First, the effectiveness of arbitrage crucially relies on the *availability of close substitutes*.¹² Arbitrageurs who sell or sell short overpriced assets must be able to buy the same (or essentially similar) assets which are not overpriced. While for some derivative assets, close substitutes are usually available, in many cases, assets do not have close substitutes. If, in addition, arbitrageurs are risk-averse, arbitrage activity becomes less attractive. For this reason, arbitrage is limited and less effective. Asset mispricing may thus persist for some time period. Even when close substitutes are available, *fundamental risk* remains a significant deterrent to arbitrage (see, for instance, Figlewski, 1979; Shiller, 1984b; Wurgler and Zhuravskaya, 2002). An arbitrageur may suffer from losses if new information about the asset fundamentals is revealed. An arbitrageur buying an undervalued asset will face a further decrease in the asset price, if bad news about its fundamental value occur, discouraging her to invest in such a risky activity.

Second, *noise trader risk*¹³ implies that it is not clear when, if ever, asset mispricing will disappear. Indeed, the presence of noise traders *i.e.*, uninformed traders or liquidity traders, can continue to influence prices over time. Asset mispricing may thus worsen even more, before eventually vanishing. If the departure

¹¹Shleifer and Vishny (1997) and Shleifer (2000), for instance, provide a more extensive and detailed discussion on the limits to arbitrage, examined in this subsection.

¹²A close substitute is an asset (or portfolio) with similar cash flows in all states of the world *i.e.*, with similar risk characteristics to those of the considered asset.

¹³Noise trader risk has been introduced by DeLong, Shleifer, Summers, and Waldmann (1990) and studied, for instance, by Shleifer and Summers (1990).

increase is significant, arbitrageurs may be forced to prematurely close their positions as they cannot maintain their position through increased losses (Shleifer and Vishny, 1997). Daniel, Hirshleifer, and Subrahmanyam (2001) suggest that due to risk aversion, arbitrageurs may not be able to remove all mispricing.

Lastly, most of the arbitrageurs act as agents when they manage other people's funds. Consequently, they must bear the constraints imposed on them by the owners of capital *i.e.*, the principals. The latter may be unwilling to leave the agents pursuing extra returns, which often implies taking higher risk. As a result, a contract signed with an agent often specifies permitted strategies, the amount of capital at risk and the maximum possible losses which can be made. While these constraints protect the owners of capital, they limit arbitrage activities in the market. Furthermore, since past performances are often used by principals to judge the abilities of the agents, arbitrageurs may choose to take less aggressive strategies in order to minimize losses and attract more capital. Shleifer and Vishny (1997), for instance, argue that agency problems associated with professional money managers and transaction costs can cause asset mispricing to persist in the market.

Overall, the key forces through which markets are supposed to attain efficiency, such as arbitrage, are likely to be much weaker and more limited than suggested by efficient market theorists (Shleifer, 2000; Barberis and Thaler, 2003; Singal, 2006).

Empirical challenges to efficient market hypothesis: Some remaining anomalies

Evidence, discussed above, that *(i.)* asset prices respond quickly and correctly to new information; *(ii.)* financial asset prices move as random walks; *(iii.)* fund managers do not tend to outperform the market, suggests that financial asset prices are mostly at levels consistent with fundamentals. However, such an evidence does not prevent asset prices from staying misaligned for a substantial time period. Beechey, Gruen, and Vickery (2000), for instance, suggest that even asset prices that are misaligned *i.e.*, not consistent with fundamentals, may have a behavior that is very close to the one described by the efficient market hypothesis (see also Summers, 1986). Researchers in finance have actually uncovered some financial market phenomena that are undoubtedly evidence of misalignments in financial markets, that may have persisted for a substantial time period. In what follows, we do not pretend to review the broad empirical works on anomalies and asset price misalignments, we rather focus on the ones which we believe are the most relevant for the purpose of the research developed in this dissertation.¹⁴

- **Financial bubbles and crashes: extreme cases of misalignments in asset prices**

The most striking and doubtless evidence of misalignment in asset prices comes from the existence of stock market bubbles and crashes.¹⁵ Financial markets have

¹⁴Singal (2006), for instance, provides an extensive review of financial anomalies.

¹⁵Financial bubbles are often captured as a sharp increase in asset prices followed by a huge drop (Kindleberger, Aliber, and Solow, 2005). An alternative definition of a bubble links asset prices to fundamentals (*i.e.*, economic factors that determine the price of an asset, such as cash flows and discount rates). Fundamental values are often measured as the discounted sum of

witnessed several episodes of financial bubbles and crashes. Earlier famous bubbles include the Dutch Tulip Mania of the 1630s, the Mississippi Bubble and the closely-connected South Sea Bubble of the 1720s (extensive review of earlier financial crises is provided in Garber, 1990; Kindleberger, Aliber, and Solow, 2005)¹⁶. More recent examples of financial bubbles followed by crashes include the international crash of October 1987 and the DotCom Bubble in the late 1990s. Roll (1988) reports that the strong market decline during October 1987 (most markets fell by more than 20%) followed an unprecedented market increase during the first nine months of the year (*e.g.*, 31% increase in the US market). Roll (1988) stresses that one of the symptom of a speculative bubble is the inverse relation between the price increase and the extent of the subsequent crash. Another striking feature of this episode is the unusual uniformity among markets when the bubble burst. While in the previous months of the crash, low correlations between world markets were identified, in October 1987, all major markets substantially dropped. More recently, the DotCom Bubble has been another illustration of financial bubble. Ofek and Richardson (2003) reveal that from early 1998 to February 2000, “the Internet sector earned over 1000 percent returns on its public equity” (p. 1113). However, by the end of 2000, these returns had completely vanished. Abreu and Brunnermeier (2003) ascertain that after having reached extremely high values in March 2000, in subsequent months, the Internet Index lost more than 75% of its value.¹⁷

expected future outcome. Accordingly, a bubble emerges when asset prices become significantly different from the fundamentals (Garber, 1990). Since this measure may not be a good estimate, alternative operational definitions have also been proposed (*e.g.*, further discussion on bubble definition is provided by Siegel, 2003).

¹⁶The first version of Kindleberger, Aliber, and Solow (2005) was published in 1978.

¹⁷See Cochrane (2002) and Lamont and Thaler (2003) for further discussion on the internet boom.

A common feature of these incidents is that the observed remarkable market values have been hard to reconcile with a plausible model of fundamental value. Regarding the October crash, this view is suggested, for instance, by Fama and French (1988); Roll (1988). On the DotCom Bubble, Ofek and Richardson (2003); Abreu and Brunnermeier (2003); Malkiel (2003); Shiller (2006) agree on the idea that the observed market values were inconsistent with rational valuations. While some academics have argued that these episodes were only a sign of market overreaction and will soon revert, in the following months, such a reversal has never been documented. On the contrary, price levels established in the aftermath of the crashes seem to be unbiased estimates of fundamentals.

These episodes have been clear evidence of persistent asset prices misalignments hard to reconcile with the predictions of standard neoclassical economic theory.¹⁸ A large number of works have proposed alternative explanations of financial bubbles based on bounded rationality (examples include Kirman (1991); Lux (1995); Shiller (2002); Abreu and Brunnermeier (2002, 2003); Scheinkman and Xiong (2003); De Grauwe and Grimaldi (2004); Baker and Wurgler (2007)).

- **Momentum and reversal in returns**

Empirical evidence suggests that stocks exhibit momentum and reversals in returns.¹⁹ Momentum usually emerges in short-term and medium-term returns (of about one month to one year), while reversals occur over longer time periods (of

¹⁸A more extensive review of the literature on bubbles can be found, for instance, in Brunnermeier (2001).

¹⁹For further details about this argument, the author suggests to refer, for instance, to Singal (2006) who provides an extensive discussion, among others, of this anomaly.

about three to five years).²⁰ Various academic researchers have studied serial correlations in common stock returns. These studies have actually uncovered both positive and negative serial correlations in returns depending on holding periods.

On the one hand, there is extensive empirical evidence which suggests that individual stocks as well as industry portfolios exhibit momentum. The idea behind the “momentum” effect is that stocks or industry portfolios which have done well in one period tend to do well also in the following period. Trading strategies based on buying and holding winner stocks as well as industry-momentum-based trading strategies appear to outperform index funds. These strategies appear to work for short (1 month) and medium (less than 1 year) periods. Such positive serial correlations in returns have mainly been documented for US common stock returns for holding periods in the 3- to 12-month range. Jegadeesh and Titman (1993), for instance, provide evidence of the cross-sectional predictability of returns over 6-12 month horizons. Rouwenhorst (1998) further supports Jegadeesh and Titman’s (1993) finding by providing out-of-sample evidence of a momentum effect in many European countries. Chan, Jegadeesh, and Lakonishok (1996) suggest that momentum in stock returns can be explained, at least partially, by the slow adjustment of the market to past profit surprises. Subsequent works have extensively addressed the momentum anomaly including, for instance, Lo and MacKinlay (1988); Moskowitz and Grinblatt (1999); O’Neal (2000); Lewellen (2002).

²⁰It is however worth mentioning that evidence also suggests that greater momentum appears in industries with no related futures markets (Singal, 2006).

The momentum in individual stock returns as well as industry returns unveils underreaction to new information, which may be inconsistent with market efficiency. Two broad types of explanations have been provided by the literature. One is based on irrational investor behavior. The other is rather based on efficient markets.

On the one hand, behavioral finance theories tend to explain underreaction and the momentum effect based on investor irrationality. Investors tend to be reluctant to change their beliefs, even when they face convincing new information (Rabin, 1998; Rabin and Schrag, 1999). Consequently, they use to underreact, which is likely to give rise to momentum. Nevertheless, “smart” investors, whose actions are often limited by the amount of capital available and uncertainty, may not be able to exploit such opportunities. The Internet bubble (1999-2000), from this viewpoint, could not be prevented from arising, despite strong beliefs among “smart” investors that this phenomenon was a clear evidence of asset overvaluation (*e.g.*, Abreu and Brunnermeier, 2003). An alternative explanation of momentum is rather based on slow diffusion of information (Chan, Jegadeesh, and Lakonishok, 1996; Hong and Stein, 1999; Hong, Lim, and Stein, 2000). Hong and Stein (1999) suggest, for instance, that gradual diffusion of news cause momentum. In such circumstances, feedback traders, who buy assets based on past returns, cause overreaction, by buying too much, because they attribute the actions of past momentum traders to news. When positions are reversed, this causes momentum. This finding is supported by subsequent empirical evidence (Hong, Lim, and Stein, 2000). According to Hong, Lim, and Stein (2000), stocks followed by fewer analysts, so that diffusion of information is slower, exhibit greater momentum. This finding is further supported by Doukas and McKnight (2005). Lastly, Singal (2006), for instance,

suggests that the presence of institutional investors can also explain the emergence of momentum. When a few managers start buying a specific stock, other managers are likely to buy the same stock, giving rise to momentum.

However, one of the main objections to this explanation in terms of underreaction to new information is whether such effects occur systematically in the stock market. Fama (1998), for instance, finds that apparent underreaction to information is about as common as overreaction.

On the other hand, momentum has also a rational explanation that comes from the relationship between financial and real assets. Since financial markets reflect expectations related to markets for real goods, whose prices move slowly due to slow changes in the demand for such products, financial asset prices also move slowly (*e.g.*, Moskowitz and Grinblatt, 1999).²¹ Singal (2006) also suggests that a possible reason for the persistence of industry and individual stock momentum is the absence of clear explanations, so that investors may be unwilling or unable to exploit such an opportunity. Lewellen (2002) presents evidence that portfolios of stocks sorted on size and book-to-market characteristics have similar momentum effects as those in Jegadeesh and Titman (1993, 2001) and in Fama and French (1996). However, Lewellen (2002) argues that the presence of momentum effect in large diversified portfolios invalidate the explanations based on behavioral biases in information processing.

On the other hand, there is extensive evidence of long-term reversals *i.e.*, long-run negative serial correlations in stock returns. These negative serial correlations

²¹This explanation helps to explain momentum only in industries for which no related futures markets exist. For further details on this distinction, see Singal (2006).

have mainly been documented for US common stock returns for holding periods in the 3- to 5-year range. De Bondt and Thaler (1985, 1987) find that past losers *i.e.*, stocks with low returns in the past three to five years, have higher average returns than past winners *i.e.*, stocks with high returns in the past three to five years (“contrarian” effect). Fama and French (1988) find that 25% to 40% of the variation in long holding period returns can be predicted in terms of a negative correlation with past patterns. Similarly, Poterba and Summers (1988) find substantial mean reversion in stock market returns at longer horizons. Subsequent works documenting price reversals include, for instance, Chopra, Lakonishok, and Ritter (1992); Jegadeesh and Titman (1995a); Forner and Marhuenda (2003).

Reversals in returns also find some explanations in behavioral finance. Indeed, some studies have attributed such a predictability to the tendency of stock market prices to “overreact”. De Bondt and Thaler (1985), for instance, suggest that investors are subject to waves of optimism and pessimism, which cause prices to deviate systematically from asset fundamentals and later exhibit mean reversion.²² Overall, both anomalies still lack comprehensive and consistent explanations as well as unified theory.

- **Volatility puzzle**

A well-established puzzle is that stock returns and price-dividend ratios are both highly variable. Empirical evidence suggests that the volatility of stock returns

²²Overreaction to past events is consistent with the behavioral decision theory of Kahneman, Slovic, and Tversky (1982), according to which investors are systematically overconfident in their ability to forecast future stock prices. We defer further discussion about the biases highlighted by behavioral finance to Section 1.3 of this chapter.

is much higher than the volatility of the short-term real interest rate. Campbell (2003a), for instance, on a study of US and international financial data,²³ reveals that stock markets are volatile in every country considered, while consumption is smooth and aggregate dividends exhibit an intermediate volatility. However, such an evidence contradicts one of the predictions of the efficient market hypothesis according to which price movements should reflect fundamentals ones. This anomaly refers to the *Volatility Puzzle* (Campbell and Cochrane, 1999).²⁴ Furthermore, the work by Shiller (1981) shows that stock market prices are far more volatile than what could be justified by a simple model in which stock prices equal the expected net present value of future dividends. Indeed, within a classical asset pricing model, both discount rates and expected dividend growth are constant over time, generating a constant price-dividend ratio. But under such assumptions, it is hard to explain historical volatility of stock returns with any model in which investors are rational and discount rates are constant (Shiller, 1981).

Now it is well-understood that also rational variations in discount rates, through changes in risk aversion, could explain the equity volatility puzzle. Constantinides (1990) and Campbell and Cochrane (1999) propose an alternative explanation based on a habit formation framework with time-varying risk aversion. Indeed, changes in consumption relative to habits entail changes in risk aversion, generating variations of the price-dividend ratio. Since return volatility is higher than

²³Campbell (2003a) study is based, for instance, on stock market data from Morgan Stanley Capital International and macroeconomic data from the International Financial Statistics of the International Monetary Fund for eleven developed countries.

²⁴Campbell (2003b), for instance, provides an extensive discussion of the “*volatility puzzle*” as well as alternative structural models. This work reveals that both rational and behavioral approaches have made some progress in understanding this puzzle.

the volatility of dividend growth, the price-dividend ratio cannot anymore remain constant. However, on the one hand, Campbell and Cochrane (1999) model does not resolve the equity premium puzzle,²⁵ relying on high average risk aversion, it does explain the stock market volatility puzzle. On the other hand, Constantinides (1990) model fits the equity premium puzzle, but it is still not completely satisfactory in explaining the equity volatility puzzle. These approaches are not able to provide a unified theory for the equity premium and volatility puzzle.

- **The price of closed-end funds**

While the efficient market hypothesis implies that asset prices should be consistent with fundamentals, which is intrinsically hard to measure for a wide range of assets, in some cases, this issue is overcome *e.g.*, in the case of closed-end funds.

A closed-end fund is an actively managed portfolio of stocks, which are all individually traded on a stock exchange. A fixed number of shares is then issued in the closed-end fund, which is itself listed and traded like a stock on a stock exchange. However, unlike a typical mutual fund or open-ended fund, shares in a closed-end fund cannot be returned to the fund company or liquidated, they must be traded in a secondary market. The fund pays dividends equal to the weighted sum of the dividends paid by the stocks in its portfolio. The price of each share in a closed-end fund should thus reflect the value of the underlying asset. Since tradable securities of other firms constitute most of the assets of closed-end funds, it is relatively easy to observe both the value of the stock of the closed-end fund and the value of its

²⁵The equity premium puzzle is another well-established anomaly evidence that suggests that stock markets have historically exhibited a high excess rate of return (Mehra and Prescott, 1985; Constantinides, 1990; Campbell and Cochrane, 1999; Campbell, 2003a). Further details and discussion on the equity premium puzzle is, for instance, provided by Campbell (2003b).

assets. However, these values usually do not coincide. Indeed, closed-end funds usually trade at a premium or at a discount with respect to the net asset value of the assets held by the fund (see, for instance, Dimson and Minio-Kozerski, 1999; Garay and Russel, 1999, for extensive surveys and discussions on this anomaly).²⁶

This puzzle is usually described through four elements. First, closed-end funds tend to begin trading at a premium of almost 10%. Second, closed-end funds move quickly to an average discount. In an earlier study, Lee, Shleifer, and Thaler (1991) found that major US closed-end funds traded at an average discount of 10% between 1965 and 1985. In the 1990s, discounts of 10% to 20% have actually been the norm (Shleifer, 2000). Third, discounts then tend to vary substantially over time and appear to be mean-reverting. Eventually, on the termination of the closed-end fund (either through a liquidation or an open ending), the price converges to the net value of the assets.

Some attempts to explain the *closed-end fund discount puzzle* have been proposed based on the costs involved in trying to exploit such an opportunity (see, for instance, Pontiff, 1996; Gemmill and Thomas, 2002). However, Shleifer (2000) suggests that explanations in terms of agency costs, tax liabilities and illiquidity of assets fail to account for much of the existing evidence. Lee, Shleifer, and Thaler (1991), in contrast, propose an alternative explanation in terms of investors' expectations. This explanation is actually supported by the fact that closed-end funds tend to be mainly held by individual investors. However, to the best of our knowledge, none of these theories seem to provide a sufficient explanation for the pricing of closed-end funds.

²⁶Shleifer (2000) and Singal (2006) also provide some discussion on this anomaly.

The above discussion makes clear that both theoretical and empirical predictions of the efficient market hypothesis have been challenged. These observations raise doubts on the validity of the efficient market hypothesis as an accurate description of financial asset prices' behavior.

While introducing market imperfections can significantly improve the predictions of the traditional theory, some deeper issues about the explanatory capacity of this theory have to be raised. Indeed, if one believes in efficient markets, one would believe that market prices provide the information that agents need to trade in an optimal way. But one may then ask whether this assumption is reasonable, whether agents are effectively able to read properly prices and use in an optimal way the information provided by the market.

The previous discussion on some remaining financial anomalies provides some elements to believe that this may not be the case. Investor sentiment may have a significant effect on asset prices. Looking more closely at agents' behavior may be enlightening to understand market prices' behavior and some of the remaining anomalies.

1.3 Behavioral Finance: An alternative approach to explain market anomalies

While the traditional finance is based on the efficient market hypothesis and investor rationality, behavioral finance, based on non-rational behavior amongst investors, attempts to offer an alternative, and better, approach to explaining

remaining anomalies. Behavioral finance rests on the belief, derived from psychological principles of decision making, that investors do not always behave in a rational, predictable and unbiased manner. Consequently, contrary to a “normative” theory, which discusses how agents should behave,²⁷ behavioral approaches focus on how agents interpret and act on information to make their investment decisions.

An earlier way to consider that agents in financial markets do not trade only based on information and might deviate from rationality, pioneered by Kyle (1985) and Black (1986), was to deal with *noise* as opposed to information. While people sometimes trade on information in the usual way *i.e.*, shifts in investor demand for financial assets are rational and reflect reactions to public announcements affecting asset fundamentals (such as future growth rate of dividends, risk or risk aversion, tax trading or trading done for institutional reasons), people may trade on noise as if it were information. In this case, changes in asset demand are no more caused by news about economic fundamentals but by non-fundamental considerations such as changes in expectations or market sentiment. The presence of noise entails that what is observed by agents is imperfect. Subsequent works on the effect of the presence of noise include, for instance, DeLong, Shleifer, Summers, and Waldmann (1990) and Dow and Gorton (1997).

Although this approach is a step forward in accounting for the fact that agents

²⁷The main objection to “normative” theories is that if people end up not behaving in this way, it is likely that the theory will be powerless to explain asset price’ behavior.

may be unable to perfectly observe their environment, this theory is not satisfactory enough. In fact, the noise approach does not provide a complete explanation of *(i.)* what drives *investors' sentiment*; *(ii.)* what are the sources of the biases that agents are subject to; *(iii.)* how agents form their beliefs and their decisions. In contrast, psychologists and behavioral economists have provided rich conceptual tools for understanding and modeling agents' behavior (Camerer, 1995; Rabin, 1998; Hirshleifer, 2001). Psychology, exploring human judgment and behavior, can shed some light on how agents differ from the way they are traditionally described by economists. Behavioral models, based on experimental evidence provided by cognitive psychologists on the systematic biases that arise when people form beliefs, often assume a specific form of irrationality. Behavioral finance seems able to propose an alternative approach about people's preferences (Barberis and Thaler, 2003; Subrahmanyam, 2007).

In what follows, we review some works that propose alternative specifications of how agents form expectations and alternative assumptions about investors' preferences, based on evidence from psychology which are useful to finance academics. This section does not contain all experimental evidence suggesting deviations from rationality, since it constitutes a vast body of works by psychologists. We rather choose to focus on the most relevant and stimulating elements for our research. For extensive surveys on behavioral finance, see for instance, Barberis and Thaler (2003); Glaser, Noth, and Weber (2004); Subrahmanyam (2007).

1.3.1 Biases in Judgments

A key element of any model of financial markets is a specification of how agents form their expectations. In this subsection, we present what we have learned from psychologists and economists regarding how agents form their beliefs in practice.

Economists have usually assumed that, when faced with uncertainty, agents correctly form their probabilistic assessments and are able to optimally achieve their objectives. However, since time and cognitive resources are limited (*i.e.*, limited attention, processing power or memory), agents tend to implement rules-of-thumb *i.e.*, *heuristics* which reduce the complexity and the probability assessment process to simpler judgmental operations (see, for instance, Simon, 1955; Kahneman, Slovic, and Tversky, 1982). Simon (1957) suggests that agents are limited in their knowledge about their environment and their computing abilities. Besides, agents face search costs to obtain sophisticated information, when following optimal decision rules. According to Simon (1955), because of these limitations, bounded rationality with agents using simple, but reasonable, rules of thumb for their decisions under uncertainty appears to be a more realistic description of human behavior than perfect rationality with fully optimal decisions rules. This theory has highly influenced subsequent research and has been supported by evidence from psychology laboratory experiments of Kahneman and Tversky (1973); Tversky and Kahneman (1974).²⁸

Heuristics appear to be effective when applied to appropriate problems, but when used out of their domain of applicability, they may lead to systematic errors.

²⁸We provide further discussion on heuristics and biases in the following subsections.

Although Kahneman, Slovic, and Tversky (1982) emphasize the positive role of heuristics, it cannot be taken for granted that this mechanism always works, especially, given that financial problems are presented to agents in a manner that may not favor the most accurate decisions. Economists, however, often argue that errors are independent across agents and thus cancel out in equilibrium. A common objection to this viewpoint is that since agents tend to use heuristics that worked well in the past, they become widespread amongst agents. Agents thus tend to be subject to similar biases. Systematic biases have been confirmed in a vast literature in experimental psychology, some of these works include Camerer (1995, 1998); Bossaerts (2001); Hirshleifer (2001).

Consequently, it is worthwhile examining the biases in judgment which agents may face, in order to understand how agents depart from perfect rationality. In what follows, since there is a large number of evidence provided by psychologists and economists, we only focus on the most relevant biases for our purpose.

Overconfidence

The self-deception theory implies *overconfidence*, one of the most well-documented bias (*e.g.*, Odean, 1998; Hirshleifer, 2001).²⁹ Overconfidence entails that individuals tend to overestimate the quality of their knowledge, which is likely to explain why people may overreact.³⁰ However, economists often argue that, since agents fail more often than they expect to, rational learning over time would eliminate

²⁹Empirical as well as experimental tests and the methodology used in overconfidence models are not presented in this section, for further details see, for instance, Glaser, Noth, and Weber (2004).

³⁰See Hirshleifer (2001) for detailed discussion and further references on experimental evidence regarding overconfidence.

overconfidence. While learning may be an effective process to correct biases, in financial markets, feedbacks are often too low and noisy. The learning process is actually itself biased and prevents agents from learning (Gervais and Odean, 2001).³¹ Odean (1998) suggests that overconfident traders cause the market to underreact to information provided by rational traders.

Furthermore, overconfidence may be explained by two other biases. First, *self-attribution bias* *i.e.*, the tendency of agents to attribute any success to competences and any failure to bad luck. Doing this repeatedly, agents start to believe that they are very talented. (Hirshleifer, 2001), for instance, suggests that self-attribution causes agents to learn to be overconfident. Gervais and Odean (2001) formally model self-attribution bias in a dynamic setting with learning. They show that if this bias is severe, self-attribution may prevent a finitely-lived agent from ever learning about her ability. Doukas and Petmezas (2007) find support for the self-attribution hypothesis in the market for corporate control. This study suggests that managers earn smaller returns in each successive acquisition, suggesting that they become increasingly overconfident with each successful acquisition.

Second, *hindsight bias*, *i.e.*, the tendency of agents to believe, after the occurrence of an event, that they predicted it before it happened, can explain people overconfidence (Hirshleifer, 2001). In fact, if agents erroneously think that they have been able to predict the past, they may start to believe that they can predict the future more accurately than they actually can. Such bias in judgment can thus favor overconfidence.

³¹Experimental evidence supports the idea that agents may fail to learn from past errors (Camerer, 1995; Rabin, 1998; Hertwig and Ortmann, 2001).

Many behavioral models examine how overconfident investors affect market outcomes and this helps to predict high trading volume in markets with overconfident investors. Indeed, at the individual level, overconfident investors will trade more aggressively. The higher the degree of overconfidence, the higher his trading volume is (De Bondt and Thaler, 1995; Odean, 1998). Kyle and Wang (1997), for instance, find that overconfident investors might earn higher expected profits, since overconfidence causes aggressive trading.

Models of overconfidence make further predictions. Odean (1998), for instance, finds that overconfident investors have lower expected utility than rational investors and tend to hold undiversified portfolios. While Benos (1998) finds similar results, higher profits of overconfident investors are the result of a first mover advantage in his model. Benos (1998) and Odean (1998) both show that the presence of overconfident investors helps to explain excess volatility of asset prices. Some works suggest that the activity of overconfident investors may have a significant impact on market outcomes and overconfident investors may survive in the long run (DeLong, Shleifer, Summers, and Waldmann, 1991; Kyle and Wang, 1997; Hirshleifer and Luo, 2001; Wang, 2001).

Daniel, Hirshleifer, and Subrahmanyam (1998, 2001) propose to explain momentum and reversals in stock returns based on overconfidence and self-attribution bias. On the one hand, overconfidence about private information that often leads to overreaction, may help to explain phenomena like long-run reversals in returns. On the other hand, self-attribution tends to maintain overconfidence, which allows prices to continue to overreact, which is characteristic of the momentum effect.

Representativeness

Tversky and Kahneman (Kahneman and Tversky, 1973; Tversky and Kahneman, 1974) established that when agents try to assess whether some data was generated by a specific model, they often use the *representativeness heuristic*. This heuristic entails that people evaluate the likelihood of a model by the extent to which the observed evidence reflects the essential characteristics of the model (see also, for instance, Grether, 1992; Camerer, 1995, for further experimental evidence of the representativeness heuristic).³² While similarity can be a good indicator in order to infer information, Hirshleifer (2001) and Barberis and Thaler (2003) highlights that such a heuristic can be misleading. In fact, while according to Bayes Law, which states that:

$$p(\textit{statement}B|\textit{description}) = \frac{p(\textit{description}|\textit{statement}B)p(\textit{statement}B)}{p(\textit{description})}$$

agents rather tend to overweight how “representative” a piece of evidence is of the reference model (*i.e.*, $p(\textit{description}—\textit{statement}B)$) and to underweight the base rate (*i.e.*, $p(\textit{statement}B)$) (see Tversky and Kahneman, 1974). This bias in judgment refers to *base rate neglect*. Representativeness may, therefore, generate systematic departures from Bayesian judgment (*i.e.*, biases).

Furthermore, agents’ perception of representativeness may mislead the conditional probability of the event. In fact, when assessing whether some evidence was generated by a particular state of the world, without knowing the initial data-generating process, agents tend to infer information and draw conclusions on too few facts

³²Grether (1980), Hirshleifer (2001) and Barberis and Thaler (2003) provide detailed discussions and references on the representativeness heuristic.

i.e., sample size neglect. People therefore tend to exaggerate the extent to which a small sample coincides with the parent population from which the sample is drawn. This bias is known as the “law of small numbers” (Rabin, 2002).

The law of small numbers might provide an explanation of some anomalies that have been identified in financial markets. Many empirical evidence suggests that the use of representativeness can cause *trend chasing*, because agents tend to believe that trends have systematic causes (see, for instance, Shiller, 1988; De Bondt, 1993, for further evidence). One example of trading strategies, based on past returns which can earn statistically significant profits, is trend following (or *momentum*) strategy.³³ Jegadeesh and Titman (1993, 2001) show that, for the U.S. stocks, this strategy results in significant positive profits. Closely related to this argument, returns of “value” stocks (*i.e.*, with a high dividend yield, a low price-earnings ratio and/or a high book-to-market ratio) have on average exhibited higher returns than “growth” stocks (*i.e.*, with a low dividend yield, a high price-earnings ratio and/or a low book-to-market ratio) (see, for instance, De Bondt and Thaler, 1985; Fama and French, 1992; Lakonishok, Shleifer, and Vishny, 1994; Daniel, Hirshleifer, and Subrahmanyam, 1998).

DeBondt and Thaler (1990) and Barberis, Shleifer, and Vishny (1998) suggest that investors in financial markets seem to underreact in the short term to news about a firm’s financial prospects, but to overreact in the medium and longer term to such news. Belief in the law of small numbers may provide an alternative

³³In the previous section, the momentum effect has been discussed. “Winners” *i.e.*, stocks with high returns over the last 3 to 12 months, are bought and “losers” *i.e.*, stocks with low returns over the same period, are sold.

explanation of the underreaction/overreaction phenomenon. There are some advantages of this hypothesis. First, while the observed patterns do not inevitably follow from the belief in the law of small numbers, it seems to be a quite natural outcome. Second, this evidence tightly connects financial anomalies to a far more general psychological phenomenon. Lastly, this hypothesis seems to provide a unified model derived from a single psychological bias.

Conservatism

Another bias which has been identified in empirical research is *conservatism i.e.*, the underweighting of likelihood information.³⁴ While representativeness leads to underweighting of base rates, there are situations where base rates are over-emphasized relative to evidence. Agents may not change their beliefs as much as a rational Bayesian would do in the face of new evidence. Experiments surveyed, for instance, in Camerer (1995) suggest that subjects are far too conservative in drawing conclusions from samples in which an event is favored. Hirshleifer (2001) suggests that conservatism can be explained by the fact that processing new information and updating beliefs is costly. This is likely when information is abstract, so more difficult to process. As a result, agents are likely to overreact to information that is easily processed, through scenarios and concrete examples. If investors use this type of heuristics, this systematic bias may have not only an impact on the responsiveness of prices to new information, but also on the price reaction afterwards when this error becomes obvious, generating, for instance, overreaction.

Evidence of conservatism appears to be at odds with representativeness and

³⁴See Barberis and Thaler (2003); Hirshleifer (2001) for further references on conservatism.

this conflict seems to indicate that agents use Bayes rule on average, sometimes weighing base rate too little and sometimes too much. However, this does not seem to be a satisfactory justification for adhering to Bayes rule, as a descriptive principle in all circumstances.

There may, however, be some reasons that could explain why base rates are sometimes underweighted and sometimes overweighted.³⁵ Barberis and Thaler (2003), for instance, suggest that if an evidence is representative of an underlying and previously known model, agents tend to overweight the relevance of the data. However, when agents face some new information which is not highly representative of some well-known model, information will be underweighted and agents react too little, relying much on their priors *i.e.*, conservatism.

Barberis, Shleifer, and Vishny (1998) model investors who make systematic errors when evaluating public information. This model suggests that investors are prone to conservatism bias and tend to expect even a few realization of a random variable to reflect the properties of the parent population from which the realizations are drawn. Barberis, Shleifer, and Vishny (1998) suggest that extrapolation from random sequences, wherein agents expect patterns from small samples to continue, tend to cause overreaction as well as subsequent reversals. This finding is further supported by Frieder (2008). In contrast, Barberis, Shleifer, and Vishny (1998) also find that conservatism tends to cause momentum through underreaction.

Hirshleifer (2001) rather suggests that in a stable environment, *self-deception* may cause conservatism because an agent who has explicitly adopted a belief may

³⁵Hirshleifer (2001), also, provides further details and references on conservatism evidence.

be reluctant to admit that he previously made a mistake. However, when the environment is volatile and more complex, the use of different beliefs may not be an issue.

Belief perseverance

Experimental evidence puts forward that once they have formed strong judgment values, agents tend to be inattentive to new information contradicting their beliefs (Rabin, 1998). Barberis and Thaler (2003), for instance, suggest that this bias can be explained by the fact that, first, agents tend to avoid looking for evidence that contradicts their prior opinions. Second, when they find such evidence, agents tend to cast some doubt on the relevance and/or the validity of such an information.

A stronger effect of belief perseverance is *confirmation bias*, according to which agents tend to misread evidence as additional support for their initial opinion. A formal model of confirmatory bias is proposed by Rabin and Schrag (1999). The authors show that agents come to believe with near certainty in a false hypothesis, even after receiving an infinite amount of information. In financial markets, this bias is likely to cause some investors to stick to unsuccessful trading strategies, causing asset mispricing to persist.

While the relevance of these biases in judgment raised some objections about their real impact on agents behavior (*e.g.*, such biases could be overcome through learning), subsequent studies have clearly revealed that these biases cannot be eliminated through training subjects, repetitions or increasing rewards (Camerer, 1995; Rabin, 1998; Hertwig and Ortmann, 2001). Biases in judgments are therefore likely to be systematic and widespread amongst investors.

1.3.2 Preferences

An essential element of any model which tries to understand asset prices behavior or trading behavior is the assumption regarding *investors' preferences i.e.*, how investors evaluate risky gambles. In the following subsections, we present what we have learned from psychologists and economists about how agents appear to make their decisions in practice.

From expected utility theory to prospect theory

In its standard version, the theory of rationality rests on the idea that agents make their choices consistently with their preferences and accounting the constraints which are imposed on them *i.e.*, *rational "calculation"*. Accordingly, economic agents behave in the best way to achieve their goals. From this viewpoint, it would be pointless to examine the psychological aspects involved in decision making. The role of rational decision-making theory is essentially *normative i.e.*, study how agents *should* behave. Extending this theory to situations of uncertainty, Morgenstern and Von Neumann (1947) propose an axiomatic approach by formalizing the expected utility hypothesis (Camerer, 1995; Egidi, 2005) in order to describe agents' choices under uncertainty. If agents' preferences satisfy a number of plausible axioms,³⁶ then preferences can be represented by the expectation of a utility function. This approach, as a good approximation of how agents evaluate a risky gamble, like financial markets, became a reference for most of the models on risky situations. The main assumption being that agents evaluate gambles according to the expected utility framework. However, the expected utility approach has not

³⁶Completeness, transitivity, continuity and independence.

been supported by evidence.

Experimental works have, in fact, shown that agents systematically violate the expected utility theory, when choosing among risky gambles (Allais, 1953; Kahneman and Tversky, 1979; Rabin, 2000). Allais (1953) paradox and subsequent studies have revealed systematic violations of expected utility. According to the Allais' paradox, a *certain* outcome may be perceived as more desirable than any random outcome, even if the latter is very likely. This demonstration and subsequent ones (see, for instance, Camerer, 1995) suggest that the expected utility theory, as originally proposed, could not be an accurate description of agents' behaviors. These findings may be crucial in order to understand a number of financial phenomena. As a result, in order to be consistent with experimental evidence, more sophisticated versions of the utility theory under uncertainty have been proposed.³⁷ Nevertheless, these attempts have not been completely successful, mainly because they were quasi-normative, trying to capture some expected utility violations by slightly weakening the original expected utility axioms.

The most successful attempt at capturing experimental findings was prospect theory proposed by Kahneman and Tversky (1979). The main novelty of this alternative approach is that it has restructured the problem of agents' attitudes to risk gambles, by referring to the mental processes involved. According to Tversky and Khaneman (1986), normative approaches fail to explain agents' choices under uncertainty because agents act routinely, which cannot be justified on normative

³⁷These versions include, for instance, weighted-utility theory, implicit Expected Utility, disappointment aversion, regret theory, rank-utility theories (for further details, see, for instance Camerer, 1995; Barberis and Thaler, 2003).

grounds:

“[T]he logic of choice does not provide an adequate foundation for a descriptive theory of decision making. [...] deviations of actual behavior from the normative model are too widespread to be ignored, too systematic to be dismissed as random error, and too fundamental to be accommodated by relaxing the normative system.” (p. 252)

The original version of prospect theory was designed for gambles with at most two non-zero outcomes (Kahneman and Tversky, 1979). Kahneman and Tversky (1979) propose that when offered a gamble: “get outcome x with probability p , outcome y with probability q ”, where $x \leq 0 \leq y$ or $y \leq 0 \leq x$, people assign it a value of

$$\pi(p)v(x) + \pi(q)v(y)$$

where v is the value function and π is the probability weighting function. When choosing between different gambles, agents pick the one with the highest value.

This formulation has several features.³⁸ First, utility is defined over *gains and losses* rather than over final wealth positions.³⁹ Such a representation is more consistent with the way gambles are actually presented and experienced by individuals (Kahneman and Tversky, 1979).⁴⁰ Besides, this representation is also consistent with basic principles of agents’ perception and judgment (Rabin, 1998).

³⁸Barberis and Thaler (2003) offer an in-dept treatment.

³⁹An idea initially proposed by Markowitz (1952).

⁴⁰Kahneman and Tversky (1979) provide further evidence on the fact that people actually focus on gains and losses and that the description of the problem influence agents’ choices *i.e.*, *framing effect*. However, such behavior contradicts normative theories of choice, according to which choices should be independent of the problem description.

Second, the value function v is concave in the region of gains and convex in the region of losses, which helps to account for the fact that agents are *risk averse* over gains and *risk seeking* over losses. The value function is hence kinked at the origin (or *reference point*), indicating a greater sensitivity to losses than gains, a feature known as *loss aversion*.⁴¹ Investors tend to suffer greater disutility from a wealth loss than the utility from an equivalent wealth gain in absolute terms.

Camerer (1998) suggests that such a representation helps to describe financial regularities and a form of prospect theory better fits the data than either expected utility theory or any other proposed generalizations.

Third, the probability transformation is nonlinear. Kahneman and Tversky (1979) find that small probabilities are overweighted (*i.e.*, agents prefer a small loss than a small probability of a large loss) and agents place much more weight on outcomes that are certain relative to outcomes that are merely probable (*i.e.*, agents are more sensitive to differences in probabilities at higher probability levels).⁴²

Overall, prospect theory proposes that attitudes toward risk are jointly determined by the value function v and the probability weighting function π rather than solely by the utility function. Moreover, by describing how gambles are perceived and what are the judgmental principles that govern the evaluation and the weighting of outcomes, prospect theory provides a useful framework for the analysis of choice under uncertainty.

Benartzi and Thaler (1995) propose to explain the equity premium puzzle

⁴¹Loss aversion helps to capture agents' aversion to wealth bets over modest stakes, which is not captured by the traditional risk neutral hypothesis (Rabin, 2000).

⁴²This feature refers to the "*certainty effect*".

through prospect theory. Loss-averse investors getting utility from annual changes in financial wealth tend to charge a high equity premium because they fear further drops in financial wealth. Loss aversion is exacerbated by frequent evaluations of investors' financial wealth *i.e.*, *myopic loss aversion* (Benartzi and Thaler, 1995). Barberis and Huang (2001) and Barberis, Huang, and Santos (2001) propose a further test of an explanation of the equity premium puzzle based on prospect theory. More precisely, Barberis, Huang, and Santos (2001) make a first attempt at building a dynamic equilibrium model of stock returns wherein investors get utility from both consumption and changes in the value of their risky asset holdings. Barberis, Huang, and Santos (2001) are hence able to show that loss aversion can provide a partial explanation of the high reward-to-risk on the aggregate stock market. Subsequent works (*e.g.*, De Giorgi, Hens, and Mayer, 2007) confirm the relevance of an explanation of the equity premium puzzle based on prospect theory. De Giorgi, Hens, and Mayer (2007), through a static two-period optimization problem and including asymmetric risk aversion, are also able to explain the equity premium puzzle. Furthermore, while loss aversion can explain the equity premium puzzle, Barberis and Huang (2001) suggest that loss aversion in individual stocks causes excess volatility (Shiller, 1981). Agents tend to respond to past stock gains by increasing their desire to hold stocks.⁴³ The decrease in discount rate leads stock prices to further increase which feeds higher volatility of returns. Grinblatt and Han (2005) suggest that loss aversion can also explain the momentum effect. More precisely, past winners tend to have excess selling pressure, so that they become undervalued. In contrast, investors tend to continue to buy past losers, so

⁴³Further evidence on “house money effect” is provided in Thaler and Johnson (1990). The idea behind is that people tend to account for prior gains and losses when making decisions regarding risky gambles.

that they become quickly overvalued. This attitude causes underreaction to public information. However, as soon as the misvaluation reverses, momentum emerges. Subsequent works propose a more sophisticated version of prospect theory (*e.g.*, De Giorgi and Hens, 2006) which appears to be powerful in simultaneously explaining the equity premium, the value and the size puzzle.⁴⁴

Framing effect

In the previous subsection, we presented how prospect theory could explain why agents make different choices in situations with identical final wealth levels. This illustrates the fact that this theory can account for the effects of problem description *i.e.*, *framing effect* (Tversky and Kahneman, 1986). Framing refers to the way a problem is posed for the decision maker. The influence of framing on agents' choices is related to loss aversion and diminishing sensitivity. Agents are often more affected by losses than gains. As a result, a frame that emphasizes the losses associated with a choice will make that choice less attractive. In contrast, a frame that exploits diminishing sensitivity by making losses smaller than the scales involved makes that choice more attractive.⁴⁵

In many contexts, agents can think about a problem in different ways, feature that is known as *mental accounting* (Thaler, 1999).⁴⁶ Thaler (1999) reviewed three components of mental accounting that received the most attention. The first component captures how outcomes are perceived and experienced and how

⁴⁴De Giorgi and Hens (2006) suggest to replace the piecewise power value function of Tversky and Kahneman (1992) with a piecewise negative exponential value function.

⁴⁵Related arguments can explain the above-mentioned *house money effect* (Thaler and Johnson, 1990).

⁴⁶For more extensive treatments and further references on framing and mental accounting, see, for instance, Hirshleifer (2001).

decisions are made as well as evaluated. The second component involves the assignment of activities to specific accounts. The third one concerns the frequency with which accounts are evaluated. However, the rules that govern each of these components can affect the perceived attractiveness of choices, which explains that preferences are no more stable. Mental accounting therefore matters when one tries to understand agents' choices.

An important feature of mental accounting is *narrow framing* (Rabin, 1998), which is the tendency to treat gambles separately from other portions of wealth. When offered a gamble, agents often evaluate it as if it was the only one and lack to merge it with pre-existing bets. This prevents agents to compare subsequent outcomes and/or decisions. As a result, narrow framing entails that people tend to analyze problems in a too isolated way. Mental accounting can explain the *disposition effect* emphasized by Shefrin and Statman (1985) *i.e.*, the tendency of holding assets that have losted value and to sell assets that have increased in value.⁴⁷ Experienced gains and losses cause pleasant or unpleasant feelings, which may be a successful mental design when agents make their decisions. However, such a mechanism may be misleading when individuals avoid recognizing losses. Self-deception theory further feeds this tendency. A loss is often perceived as an indicator of failure, which people tend to discard. Furthermore, when they face two logically (but not *clearly*) equivalent statements of a problem, agents tend to choose different options, reversing their preferences (Tversky and Khaneman, 1986). Evidence of systematic *preference reversals* suggests that agents' choices

⁴⁷Grinblatt and Han (2002), for instance, suggest that investors subject to the disposition effect may explain the momentum effect (as discussed in Section 1.2.2).

may not be well-described by maximization of a utility function (Tversky and Thaler, 1990).

Framing effects are often viewed as heuristic errors. However, when time and resources are limited, this mental design may be relevant. However, since the way in which a problem is presented may draw attention to different aspects of a problem, agents are likely to make mistakes.

Ambiguity aversion

The expected utility theory focuses on how people act when the outcome of gambles have known objective probabilities. However, in reality, this is rarely the case. To account for such an evidence, subjective expected utility has been proposed (Savage, 1972; Efron, 1978). Accordingly, preferences are represented by the expectation of a utility function which is then weighted by the agent's subjective probability assessment. Preferences are, in this case, described as subjective properties of the agent. Since probabilities in subjective expected utility are derived from preferences, rather than assumed, as in expected utility, subjective expected utility theory enables to grasp a wider range of situations. However, in the last few decades, experimental evidence also recognizes clear deviations from the subjective expected utility theory.⁴⁸ Most of the empirical evidence on deviations with respect to the predictions of the subjective expected utility theory relies on the distinction between known and unknown probability. However, within the subjective expected utility theory, such a distinction is irrelevant and cannot be account

⁴⁸An earlier violation was established by the Ellsberg's (1961) paradox, making the distinction between unambiguous and ambiguous probability.

for since subjective probabilities are never unknown.

Empirical evidence in fact suggests that agent decisions may actually be affected by such a distinction. Agents tend to dislike situations where they are uncertain about the probability distribution of a gamble *i.e.*, situations of ambiguity (*e.g.*, Camerer, 1995, reviews some empirical tests). This tendency refers to *ambiguity aversion*.⁴⁹ Camerer (1995) defines *ambiguity* as the *known-to-be-missing information i.e.*, not knowing relevant information that could be known (see, also, Heath and Tversky, 1991). Ambiguity aversion implies that there may be a gap between subjects' beliefs about the likelihood of an event and their willingness to bet on the event. In subjective expected utility, such a gap cannot exist because beliefs are derived from betting preferences. Heath and Tversky (1991) argue that ambiguity aversion actually emerges because people may not feel enough competent when assessing the relevant probability distribution. According to this viewpoint, competence (*i.e.*, knowledge, skill or comprehension) explains the gap that may exist.

Many works have used ambiguity aversion to explain some financial phenomena. Maenhout (2004), for instance, suggests that if investors have some doubts about whether their model of stock returns is well-specified (*i.e.*, a situation of ambiguity), they are likely to require a higher equity premium in order to compensate for the perceived ambiguity in the probability distribution. However, Maenhout (2004) also recognizes that ambiguity may only be a partial explanation of the equity premium puzzle. In fact, in order to be fully explained, this explanation

⁴⁹Barberis and Thaler (2003) provides an in-depth treatment of ambiguity aversion.

would entail that agents have an extreme dislike for ambiguity, which may not be sustainable.

Ambiguity aversion may, however, help to explain the observed insufficient diversification made by investors, which refers to “home bias” (*e.g.*, French and Poterba, 1991). Such an evidence can actually be explained by ambiguity aversion. In fact, investors may find the national stock markets less ambiguous than foreign stock markets. As a result, they tend to hold more assets that they perceive as more familiar (*e.g.*, domestic ones), leading to undiversified portfolios.⁵⁰

Overall, although we have not presented the broad class of anomalies with respect to standard utility theories under risk and uncertainty (for further details see, for instance, Camerer, 1995), errors and violations of rationality are recognized as systematic. The limits of rational behaviors can no longer be defended with the idea of a social process of selecting the best. Many of the anomalies can be related to the idea that values are judged relative to a reference point, probabilities are not weighted linearly and decision weights are not the same as beliefs. Preferences also seem to depend on the way problems are described (creating framing effects) and on the procedure by which they are revealed (creating preference reversals). These phenomena are likely to influence portfolio choices and suggest that agents use simple procedures to make choices rather than maximizing over well-formed preferences.

⁵⁰Barberis and Thaler (2003) provide further discussion on the explanation of insufficient diversification through ambiguity aversion and detailed references.

1.4 Heterogeneous Agent Models

The discussion in the previous sections sheds light on the fact that, contrary to the predictions of traditional finance, *(i.)* financial markets should not always be presumed to be efficient *i.e.*, prices are not always right, rather, they happen to be misaligned and anomalies may arise in financial markets; *(ii.)* agents are subjects to psychological biases and can significantly affect asset prices. Since rational agents may be powerless to drive asset prices towards asset fundamental values, we suggest that it may be worthwhile focusing on how heterogeneous agents interact in the market and how they may affect market equilibrium.⁵¹

While the main tool of analysis of the traditional approach, based on a representative, rational agent hypothesis, rests on simple analytical tractable models, behavioral agent-based approach, based on boundedly rational, heterogeneous agents using rule of thumb strategies fits better with agent-based simulation models. The main tool of analysis of the behavioral approach has, in contrast, been numerical and computational methods.

In this section, we therefore review models of financial markets based on heterogeneous agents which have already proposed to explain mispricing phenomena, and bubbles as an extreme case. Since in the recent literature, there is already a quite large number of heterogeneous agent models, we do not pretend to discuss all of them (extensive review of heterogeneous agent models which have been developed in economics and in finance is, for instance, provided in Hommes (2006).

⁵¹The reader should bear in mind the discussion of the previous section on how agents form their beliefs, and on agents' preferences. In order to make our presentation as clear as possible, we will not repeat too heavily the elements discussed in the previous discussion.

LeBaron (2000) rather proposes extensive review and further references regarding works in agent-based finance.). Instead, we review, what we consider, a representative selection of works which has significantly influenced our research. The discussion presented in this section has several objectives.

First, we examine how heterogeneous agents are analytically captured? Which types of heterogeneity features have been considered? How heterogeneous agents interact? What may be the outcome of their interactions? What is the effect of these interactions on market equilibrium?

Second, can the assumption of heterogeneous agents improve our understanding of empirical evidence observed in financial markets? Has a model wherein agents are heterogeneous a greater explanatory ability than a model which does not account for this specificity?

Third, are the models proposed by the literature, and examined in this section, satisfactory enough? Did they give an acceptable description of agents' heterogeneity?

For this purpose, in Section 1.4.1, we focus on heterogeneous agent models based on the bounded-rationality hypothesis and present models which have the common feature of being built on the distinction between fundamentalists and chartists. In Section 1.4.2, we rather present heterogeneous agent models based on the limits to arbitrage approach, wherein rational traders coexist with boundedly rational agents.

1.4.1 Bounded Rationality: Fundamentalists versus chartists models

From the late 1980s, research has been devoted to obtain information about investors' expectations. Evidence from survey data works has unveiled that financial practitioners tend to use different trading and forecasting strategies (see, for instance, Shiller, 1987; Frankel and Froot, 1987a,b, 1990a,b; Shiller, 1990; Allen and Taylor, 1990; Ito, 1990; Liu, 1996; Menkhoff, 1997, 1998; Lui and Mole, 1999).⁵² In addition to fundamental analysis, technical analysis and chartist strategies appear to be extensively employed among investors.

The primary aim of technical analysis is to pinpoint recurring patterns in historical prices in order to forecast future price trends, by identifying the initiation to new trends. Hommes (2006), for instance, points out that technical analysis is based on the belief that price trends are to some extent predictable. Well-known examples of simple trading rules include, for example, *filter rules i.e.*, buy when the price has increased by a given proportion above a recent trough; *moving average intersection i.e.*, buy when a shorter moving average penetrates a longer moving average from below. These simple technical trading rules are quite inexpensive to implement but are not likely to generate excess profits in efficient markets. Technical analysts, though, use much more sophisticated trading rules that happen to be profitable. More recent evidence from survey data works confirm the widespread use of technical analysis (see, for instance, Cheung and Wong, 2000; Shiller, 2000;

⁵²It is worth mentioning that another powerful method in order to obtain information about investors' expectations is to design laboratory experiments (recent works include, for instance, Sonnemans, Hommes, Tuinstra, and Van De Velden, 2004; Bottazzi and Devetag, 2005; Hommes, Sonnemans, Tuinstra, and Van de Velden, 2005; Bottazzi, Devetag, and Pancotto, 2009).

Maditinos, Sevic, and Theriou, 2007).

Furthermore, empirical evidence suggests that chartist strategies can survive in the long run, when applied to the US exchange market (see, for instance, Levich Lee, Richard, et al., 1993; Chang and Osler, 1995; Lento and Gradojevic, 2007). Empirical tests on stock markets seem to confirm the foregoing results (see, for instance, Brock, Lakonishok, and LeBaron, 1992; Lo, Mamaysky, and Wang, 2000; Neely, Weller, and Dittmar, 2009). The widespread use of technical trading analysis is, however, an issue from the efficient market hypothesis viewpoint. In fact, in an efficient market, participants who look for patterns in historical data should only play a marginal role, which is inconsistent with the actual extent of technical trading.

The findings of these empirical works are likely to have influenced the development of heterogeneous agent models. Indeed, in many heterogeneous agent models, two important types of agents are distinguished, namely, fundamentalists and chartists. Fundamentalists are assumed to base their trading strategies on asset fundamentals and economic factors (such as dividends and earnings). They tend to buy (*sell*) assets that are undervalued (*overvalued*) *i.e.*, whose prices are below (*above*) asset fundamentals. Chartists (or technical analysts), in contrast, disregard asset fundamentals. They are therefore assumed to base their trading strategies on observed historical patterns in prices. Technical analysts try to extrapolate observed price patterns, such as trends, and exploit these patterns in their investment decisions.

Heterogeneous agent models with fundamentalists, chartists and market makers: Disequilibrium models

Beja and Goldman (1980) propose a dynamic heterogeneous agent model for asset markets. This kind of models can generate complex dynamics and asset price chaotic fluctuations. Besides, this work is one of the first dynamic heterogeneous agent models with a representation of the market institution by a *market maker*, who adjusts prices according to aggregate excess demand.⁵³ This model, through a simple, behavioral framework, under a market maker scenario, shows that speculative trading may destabilize asset prices.

Chiarella (1992) considers a nonlinear generalization of Beja and Goldman's model, where chartists' excess demand is captured by a nonlinear function. Like Beja and Goldman (1980), this work suggests that a large impact of speculative demand and/or high adaption speed tends to destabilize the market.

More recent disequilibrium models based on the distinction between fundamentalists and chartists include Farmer and Joshi (2002) and Westerhoff (2003). Farmer and Joshi (2002) propose a model asset market in order to examine the price dynamics induced by several trading strategies, mainly fundamentalist and chartist. However, while Beja and Goldman (1980) assume linear trading rules of each type of strategy, Farmer and Joshi (2002) examine linear as well as nonlinear

⁵³This is an important feature that is worth stressing as this assumption is also used in the models presented in the subsequent chapters. Further discussion and references is provide in what follows.

trading rules.⁵⁴ Farmer and Joshi (2002) provide a qualitative description of financial asset markets which is able to reproduce patterns observed in real financial data. The main finding of this study is that commonly used trading strategies affect asset prices. Since each strategy is activated differently, bursts in trading by either group can emerge in the market. The feedback effects studied in this work help to explain, for example, clustered volatility.

Westerhoff (2003) proposes a heterogeneous agent model for exchange rates based on a nonlinear price adjustment rule in order to describe the activity of market makers. In Westerhoff (2003), market makers have two source of information, namely, current order flows (*i.e.*, positive (*negative*) excess demand drives prices up (*down*)) and inventory levels (*i.e.*, a negative (*positive*) inventory reveals that exchange rates have been set too low (*high*) in the past). The main result of this work is that market makers tend to adjust prices more strongly when the signal from order flows and inventory levels have opposite signs. Westerhoff (2003) suggests that such a behavior causes markets to be less efficient due to higher volatility, distortion and trading volume.

Studies along somewhat different lines have been proposed by Lux (1995, 1998) and Lux and Marchesi (1999, 2000). The specificity of these heterogeneous agent models is that they account for interaction effects among fundamentalists and chartists. Most of these works have already been successful in explaining stylized facts observed in financial data, especially bubbles and clustered volatility. In

⁵⁴Farmer and Joshi (2002) study simple linear behavioral rules depending on agents' positions as well as orders. In Farmer and Joshi (2002), nonlinear behavioral rules are introduced to account for the existence of transaction costs, which often prevent agents from trading. Nonlinear rules are based on the existence of a threshold for the value of the mispricing which triggers agents' trades.

these models, such anomalies arise from the interaction and switching between fundamentalists and chartists. Lux (1998), for instance, offers a socio-economic model of the interaction of speculators, based on a twofold switching mechanism. On the one hand, chartists are assumed to be of two types, namely, *optimistic* (bullish) and *pessimistic* (bearish), and can switch among these subgroups. On the other hand, traders switch between chartist and fundamentalist trading strategies. The main result of this model is that such a framework is able to generate complex dynamics as well as reproduce some stylized facts observed in financial data (*e.g.*, leptokurtosis of the distributions of returns) which are mainly explained by the presence of chartists in the market.

Heterogeneous agent models with fundamentalists, chartists without market makers: Equilibrium models

Works emphasizing the importance of fundamentalists and chartists have actually been pioneered by Zeeman (1974). Zeeman (1974), through a highly stylized model for stock exchange markets, offers a qualitative description of the observed stylized facts of bull and bear markets. This model, though, contains a number of important, behavioral elements, also used in recent heterogeneous agent models.⁵⁵

Studies along somewhat different lines have been proposed by Frankel and Froot. These authors have initiated models for exchange rates, with time varying weights of forecasting strategies (Frankel and Froot, 1986, 1990a,b). In such models, there are three types of agents, namely, fundamentalists, chartists and portfolio

⁵⁵Rheinlaender and Steinkamp (2004) propose a stochastic version of Zeeman's model using random system theory.

managers. Portfolio managers form their expectations as a weighted average of the predictions of fundamentalists and chartists. Portfolio managers then update the respective weights over time in a rational way according to the respective performances of fundamentalists or chartists. The main finding of these models is that exchange rates may exhibit a temporary bubble when the fundamentalist weight is driven to zero. However, as the exchange rate moves too far away from its fundamental value, portfolio managers start giving more weight again to fundamentalists' forecasts, bringing back the exchange rate towards its fundamental value. This serie of papers has been of substantial influence for subsequent works (*e.g.*, De Grauwe, Dewachter, and Embrechts, 1993; De Grauwe and Grimaldi, 2005).

De Grauwe, Dewachter, and Embrechts (1993) has been one of the heterogeneous agent models wherein the weights of the two types of investors are endogenously determined and vary over time. The endogenous switching mechanism for the weights of chartists and fundamentalists acts as a stalizing force on exchange rates. The greater the exchange rate departs from its fundamental value, the higher the weight of fundamentalists will be, which subsequently pushes back the exchange rate towards its fundamental value. The main result of this model is that the fundamental steady state becomes unstable and chaotic exchange rate fluctuations around the fundamental value arise as the weight of chartists increases.

Kirman (1991) also provides a model for exchange rate with fundamentalists and chartists, though, it is also based on a model of opinion formation (Kirman,

1993).⁵⁶ This work belongs to the area of models wherein agents interact stochastically. The idea behind these models is that even weak (local) interactions among individuals yield strong dependencies and may generate large movements at the aggregate level. The main finding of this work is that when the market is dominated by fundamentalists, the exchange rate is stable and is pushed back towards its fundamental value. However, when the market is dominated by chartists, the system exhibits a greater variety of dynamics (including unstable system) and the volatility of the exchange rate fluctuations is high. This heterogeneous agent model is hence able to capture, at least qualitatively, the volatility clustering phenomenon, with exchange rates switching irregularly between phases of high and low volatility.⁵⁷

However, we believe that in order to propose a model based on switching strategies, such a mechanism should be based on extensive empirical evidence. If this is not the case, one may wonder whether requiring that agents know all the payoffs of a vast range of strategies is more realistic than imposing agents to know the beliefs of all other, non-rational agents.⁵⁸

⁵⁶Unlike Frankel and Froot (1986), for instance, Kirman (1991) provides a micro-foundation of asset demand.

⁵⁷Kirman and Teyssiere (2002) discuss stylized facts such as clustered volatility and long memory, generated by the model in more detail.

⁵⁸This is the main justification of the absence, in the works developed in this dissertation, of switching mechanism among different strategies.

1.4.2 Limits to Arbitrage: Rational versus behavioral agent models

As examined in the first part of our discussion, a crucial issue of the efficient market hypothesis is the stabilizing power of arbitrage. Arbitrageurs generally trade against noise trader demands and therefore stabilize prices, though sometimes imperfectly. However, as we have also seen, there are several reasons that may explain that this scenario does not always hold. Limits to arbitrage, as reviewed in Section 1.2.2, can be explained by market imperfections, such as cost of information and trade, capital constraints and imperfect substitutes. However, standard market imperfections cannot explain large and substantial mispricing, such as the overpricing of Internet stocks during the 1990s (see, for instance, the survey by Ofek and Richardson, 2003). As mentioned previously, another reason for powerless arbitrage is that it can be difficult and risky for rational arbitrageurs to correct asset mispricing caused by non-rational agents *i.e.*, noise trader risk.

In the heterogeneous agent models discussed in the previous subsection, none of the two types of agents were rational, since none of them took the presence of the others into account. We now turn to review some works, based on the limits to arbitrage approach, wherein rational traders coexist with boundedly rational traders. In what follows, we mainly present models based on synchronization risk and noise trader risk.

Synchronization risk

As presented in Section 1.2.2, the literature on the limits to arbitrage has pointed out that there are two broad types of risk that prevent arbitrage to work, namely, fundamental risk and noise trader risk.⁵⁹ Abreu and Brunnermeier (2002, 2003) rather propose a new limit to arbitrage, which can explain that asset mispricing may persist even when rational arbitrageurs are present in the market, namely, *synchronization risk*.

Abreu and Brunnermeier (2002, 2003) offer models for asset markets in which there is a single risky asset and two types of agents *i.e.*, rational arbitrageurs and behavioral traders.⁶⁰ In these works, arbitrage is limited because rational traders face uncertainty about when their peers will exploit a common arbitrage opportunity. *Synchronization risk* arises because arbitrageurs become sequentially aware of the existence of the mispricing in the market and incur holding costs. The main result of Abreu and Brunnermeier (2002) is that the combination of synchronization risk with holding costs causes arbitrageurs to delay acting on their information. Under such circumstances, rather than correcting the mispricing, rational arbitrageurs tend to “time the market”. While arbitrage eventually works, it can be markedly delayed. Abreu and Brunnermeier (2002) show that behavioral influences on asset prices can be resistant to arbitrage in the short as well as in the intermediate run.

Abreu and Brunnermeier (2002, 2003) rationalize the occurrence of bubbles (*i.e.*,

⁵⁹We defer discussion on some models based on the noise trader approach to later subsection.

⁶⁰Abreu and Brunnermeier (2003), in a similar framework, extends Abreu and Brunnermeier (2002) work to the study of stochastic bubbles. Abreu and Brunnermeier (2003) suggest that a stochastic bubble persists due to delayed arbitrage.

extreme forms of misalignments in aggregate stock prices).⁶¹ Before a bubble bursts, the stock price usually grows at a higher rate, even if it is mutually known that assets are overpriced.

Abreu and Brunnermeier (2002, 2003) works have influenced subsequent research. More recently, Sakawa and Watanabel (2006), in an attempt to reexamine Abreu and Brunnermeier (2003) model, show that the synchronization risk from earlier works actually applies to a discrete-time setting. Bhojraj, Bloomfield, and Tayler (2009) provide experimental evidence that, when *synchronization risk* is severe, relaxing margin restrictions allows for more aggressive short selling, which further exacerbates observed overpricing.

Noise trader risk

In this subsection, we briefly present early models due to DeLong, Shleifer, and Summers (1990) and DeLong, Shleifer, Summers, and Waldmann (1990) which are based on the *noise trader approach* (see, for instance, Shleifer and Summers, 1990, for more detailed presentation).⁶²

The work by DeLong, Shleifer, Summers, and Waldmann (1990) aims at assessing the validity of the *Friedman hypothesis*, according to which non-rational agents do not survive evolutionary selection, so that they are driven out of the market (Friedman, 1953). DeLong, Shleifer, Summers, and Waldmann (1990) propose a model for asset markets in which there are two types of agents, namely, noise

⁶¹Some of the important contributions on bubbles are summarized in Brunnermeier (2001).

⁶²Another early model along this line include Shiller (1984a).

traders and rational traders. Noise traders select their portfolios based on the incorrect belief that they have special information about the fundamental price of the risky asset.⁶³ Rational traders rather exploit noise traders misperception, buying (*selling*) when noise traders depress (*push up*) prices. Rational trader strategies end up bringing back the risky asset price towards the asset fundamentals, but not completely.

As a result, contrary to the *Friedman hypothesis*, DeLong, Shleifer, Summers, and Waldmann (1990) show that noise traders can earn higher expected returns and are hence able to survive in the long run. More precisely, in this model, short horizons make arbitrageurs averse to noise trader risk. Noise traders trading activity could, at least temporarily, deepen the mispricing.⁶⁴

The purpose of the work by DeLong, Shleifer, and Summers (1990) is rather to show that, contrary to the *Friedman hypothesis*, in the presence of positive feedback traders,⁶⁵ rational speculation can be destabilizing. Furthermore, the model is able to explain overreaction to news about economic fundamentals, caused by rational and informed traders accounting for the presence of feedback traders.

⁶³In DeLong, Shleifer, Summers, and Waldmann (1990), there are two assets, a safe asset paying a fixed dividend in each period and a risky asset paying an uncertain dividend.

⁶⁴DeLong, Shleifer, Summers, and Waldmann (1990) also discuss a dynamic version of the model with time varying fractions of each type of traders, based on the relative performance of each strategy, so that the strategy that has performed better attracts more traders. The results of this version support the above-mentioned findings.

⁶⁵In this work, noise traders are actually replaced by *positive feedback traders*

1.5 Concluding Remarks

The purpose of this analysis was to provide a guide for further research on whether asset mispricing phenomena, including bubbles as an extreme case, may be due to a coordination problem between heterogeneous agents. The above discussion provides some elements in this direction.

First of all, the classical asset pricing theory may fail to explain some anomalies in financial markets which require alternative explanations.⁶⁶ Indeed, according to the efficient market hypothesis, the marketplace of ideas should somehow work optimally and hence, by inference, the prominent theories that appear to move investors' decisions should be based on the best possible information too. However, given the previous discussion on some remaining financial anomalies, one may question the accuracy of this description of asset prices' behavior. Furthermore, according to the efficient market hypothesis, investors should disregard the current market and have no concern for the long run viewpoint, since broad diversification is the ideal. Such a strong belief may lead investors to make considerable mistakes when they trade and choose their portfolios. The remaining anomalies discussed in Section 1.2.2, in fact, suggest that investors' expectations and investors' awareness may be noteworthy in order to understand asset prices' behaviors.

Second, human patterns of less-than-perfectly rational behavior, examined in section 1.3, suggest that it is difficult to assert that agents are able to read the information contained in market prices. First, agents are subject to judgmental

⁶⁶We believe that it is important to keep in mind the above-discussed weaknesses of the efficient market theory.

biases which prevent them from forming correct beliefs and properly using available information. Second, agents may make choices which are not normatively acceptable and may not be able to achieve their goals in the best way. Hence, while there is no evidence supporting clear “foolishness” (or irrationality) among investors, human patterns of less-than-perfectly rational behavior, widely investigated by behavioral finance, are central to explain asset prices’ behaviors.

Lastly, the discussion in section 1.4 supports the relevance of agents’ heterogeneity in explaining asset mispricing in financial markets. Heterogeneity based on information - differential information and differential interpretation - appears to be a crucial source of heterogeneity among agents in financial markets and a key assumption for asset pricing models that aims at explaining asset mispricing.

The above discussion suggests that we have to move away from the assumption that financial markets always work well, and that price changes always reflect relevant information. Evidence from behavioral finance may help us to understand financial anomalies. Our discussion makes clear that when one wants to address the implications of financial market outcomes, psychological results are needed to explain interactions between investors.

Overall, the discussion provided in this chapter has been instructive for the main questions which have guided our analysis and convincing enough to believe that mispricing phenomena may be better understood by focusing on the interaction between heterogeneous agents.

In the following chapters, based on some crucial aforementioned assumptions *i.e.*, bounded rationality and agents heterogeneity, we discuss the models which we have developed, in order to propose alternative explanations of some remaining anomalies.

Chapter 2

A Model of Price Formation with Fully Informed Traders

2.1 Introduction

In the previous chapter, we reviewed the existing literature on the backbones and main underlying issues behind our research *i.e.*, *(i.)* what is known about stock prices' behavior and why the efficient market hypothesis may be unsatisfactory to explain important stylized facts observed in financial data; *(ii.)* what is known about agents' behavior and how useful has been behavioral finance to account for bounded rationality; *(iii.)* which kind of heterogeneous agent models have been proposed by the literature to offer an alternative to representative, rational agents models. This discussion led us to believe that heterogeneous agent models, based on the bounded-rationality hypothesis, are the relevant framework to understand asset prices' behavior and may help to provide an alternative asset pricing theory.

In this chapter, we investigate whether, as ascertained by the efficient market hypothesis, *(i.)* financial asset prices are consistent with “fundamentals”; *(ii.)* prices move only due to new information, when agents are boundedly rational. For this purpose, in line with the heterogeneous agent models described in subsection 1.4.1 (Beja and Goldman, 1980; Lux, 1995, 1998; Farmer and Joshi, 2002; Westerhoff, 2003), we model an asset market wherein there are two types of boundedly rational agents, namely, arbitrageurs (or *fundamentalists*) and chartists. This model enables us to identify under which circumstances, arbitrage strategies enable asset prices to reflect asset fundamentals, although chartists are present in the market. Furthermore, in this work, we pursue the same goal as, for instance, Farmer and Joshi (2002), that is investigate the effect of commonly used trading strategies as well as the interplay between different strategies on price dynamics,

through a market maker based method of price formation. However, rather than exploring a variety of fundamentalist and chartist strategies, we restrict our study to three of them, namely, arbitrage strategies, trend following as well as contrarian strategies, showing how they affect asset prices.

More important, the work developed in this chapter constitutes the foundation for the models discussed in the following chapters. Indeed, in the model discussed in this chapter, arbitrageurs are assumed to know asset fundamentals. In the next chapter, we rather relax this assumption by allowing arbitrageurs to be partially informed about asset fundamentals and investigate whether this new model is able to explain price underreaction as well as mispricing persistence.

This chapter is organized as follows. In Section 2.2, we present a simple model of linear price formation with fully informed arbitrageurs. In Section 2.3, we solve the model. This work enables us to identify the stability conditions of the system under consideration. In Section 2.4, we discuss the results regarding the price dynamics that emerge in the market according to the composition of the population as well as the type of convergence that our model is able to generate. This study enables us to fully understand the impact of each trading strategy as well as the effect of the interplay between arbitrageurs and chartists on price dynamics. In Section 2.5, we present a number of comparative statics experiments. This work enables us to assess the robustness of the main results of our model. Furthermore, this investigation is crucial in order to select the parameter values that will be used in the numerical simulations implemented in the following chapters. Section 2.6 summarizes the main findings of this work and Section 2.7 presents some concluding remarks.

2.2 The Model

As mentioned above, we develop a heterogeneous agent model, based on the assumption of bounded rationality, in order to investigate whether asset prices reflect asset fundamentals, when chartists are present in the market.

In the model discussed in this chapter, we consider a market in which there is a single risky asset with price P_t and fundamental value V_t . We consider a simple situation in which the intrinsic value of the asset is assumed to be constant over time.¹ Consequently, future cash flows of the asset as well as its fundamental value are held constant. In the market, there are N agents, who are assumed to be of two types, namely, arbitrageurs (or *fundamentalists*) and chartists.² The number of arbitrageurs and chartists is n_a and n_c , respectively, with $n_a + n_c = N$. The portion of arbitrageurs in the market is denoted $\eta \equiv n_a/N$. The chartist group is however composed of two subgroups: trend followers and contrarian traders. Their respective numbers are denoted n_{TF} and n_{CT} with $n_{TF} + n_{CT} = n_c$. The portion of trend followers among the chartist group is denoted $z \equiv n_{TF}/n_c$.

In each period, agents can place buy or sell orders in the market. In order to make their buy/sell decisions, first arbitrageurs observe the asset price which prevails in the market and know the asset fundamental value. The fundamental value of the asset is based on the asset future payoffs *i.e.*, on the prospect of future cash flows only. Beliefs about future prices independent of cash flows are precluded from this work. Indeed, in the work presented in this chapter, arbitrageurs are

¹Frequent and repeated new information that could marginally affect the intrinsic value of the asset over time is not considered in this work.

²For more detailed discussion and further references on heterogeneous agent models based on the distinction between fundamentalists and chartists, see Section 1.4 and 1.4.1.

assumed to have homogeneous as well as realistic beliefs about future payoffs. This may be justified by the fact that information about the asset fundamental value diffuses quickly and correctly among arbitrageurs. As a result, arbitrageurs' perceived asset fundamental value coincides with true asset fundamentals. Besides, this type of agents bases their trading strategy upon any differential between the observed asset price in each period and its fundamental value. Arbitrageur's orders are thus function of the difference between the asset fundamental value and its current price:

$$X_{t+1}^A = \beta(V_t - P_t) \quad (2.1)$$

where the term β is a positive reaction coefficient.

In Farmer and Joshi (2002), such a behavioral rule defines *order-based strategies*, which implies that agents continue to buy (*sell*) as long as there is asset mispricing *i.e.*, $P_t \neq V_t$. More precisely, as long as there is no asset mispricing (*i.e.*, $V_t = P_t$), arbitrageurs remain inactive. However, any differential between the asset price and the asset fundamental value represents a potential gain for arbitrageurs. They thus sell (*buy*) the asset as long as its price is above (*below*) the asset fundamental value *i.e.*, when the asset is overvalued (*undervalued*). This behavioral rule actually justifies the use, along this work, of the term “arbitrageurs” rather than “fundamentalists”. Indeed, while fundamentalists tend to buy (*sell*) overvalued (*undervalued*) stocks and to hold them in the long run, arbitrageurs exploit more aggressively such potential gains, buying or selling the asset as long as its price does not reflect the asset fundamentals.³

³Besides, this hypothesis holds because in this work, arbitrageurs are assumed to be well-funded *i.e.*, they do not face any financial constraints.

Second, chartists observe asset prices which prevail in the market but do not account for the asset fundamental value. This type of agents bases their trading strategies upon observed historical patterns in past prices in order to forecast future movements in prices. As mentioned above, in what follows, we consider two types of chartists, namely, trend followers and contrarian traders. On the one hand, trend followers believe that past price movements tend to repeat in the future. Their orders are expressed as:

$$X_{t+1}^{TF} = \varphi(P_t - P_{t-1}) \quad (2.2)$$

where the term φ is a positive reaction coefficient. Trend followers thus buy (*sell*) the asset when its price has increased (*decreased*) in previous periods.

On the other hand, contrarian traders rather believe that past price movements tend to revert in the future. Their orders are thus expressed as:

$$X_{t+1}^{CT} = -\varphi(P_t - P_{t-1}) \quad (2.3)$$

where the term φ is the same positive reaction coefficient as in eq. (2.2). Such a simplification is justified by the fact that we assume that contrarian strategies and trend following strategies have the same effect on price dynamics, being both based on technical analysis.⁴ Contrarian traders buy (*sell*) the asset when its price has decreased (*increased*) in previous periods.

However, if observed prices were constant in previous periods, chartists - both trend followers and contrarian traders - would expect that the asset price stay

⁴This simplification also avoids introducing an additional parameter into the model, especially given that the parameter value is the same.

unchanged and they would thus remain inactive (*i.e.*, neither buying, nor selling the asset).⁵

At this point, it is worth stressing that, while most of the heterogeneous agent models based on the distinction between fundamentalists and chartists (as discussed in Section 1.4 and 1.4.1) essentially consider that the very figure of technical analysis is a trend follower, we depart from this hypothesis. In line with earlier works (see, for instance, Odean, 1998, 1999; Lux, 1995, 1998; Lux and Marchesi, 1999, 2000), we rather assume that chartists may not only chase trends. Instead, some investors could try to exploit such trends in prices or simply decide to go against the crowd (or the general tendency of the market) *i.e.*, contrarian traders. In the existing literature, an alternative terminology for contrarian traders is also used, namely, “pessimistic” (or *bearish*) chartists (as in Lux, 1995; Lux and Marchesi, 1999, for instance). The behavioral motivation for the presence of contrarian traders comes from the “disposition effect”, according to which investors are reluctant to sell assets at a loss relative to their purchase price.⁶ Subsequent empirical evidence has supported the existence of this psychological bias. Odean (1998, 1999), for instance, find that investors are more likely to sell stocks which have raised in value relative to their purchase price, rather than stocks which have fallen. De Bondt and Thaler (1985), as mentioned previously, suggest that investors are subject to waves of optimism and pessimism, which cause prices to deviate systematically from asset fundamentals and later exhibit mean reversion. Genovese and Mayer (2001) find a similar behavior in the housing market. Overall, these

⁵It is worth mentioning that chartists do not face any financial constraints as well.

⁶As suggested in Barberis and Thaler (2003), the explanation of this bias can also rely on prospect theory and narrow framing.

findings give further support to investment techniques that rest on a “contrarian” strategy *i.e.*, buying stocks that have been out of favor long time periods and avoiding stocks that have large run-ups over the last several years. Furthermore, there is extensive empirical evidence on the profitability of contrarian strategies which suggests that contrarian traders can survive in the long run (*e.g.*, Lehmann, 1990; Jegadeesh, 1990; Jegadeesh and Titman, 1995b,a; Dechow and Sloan, 1997; Galariotis, 2004).

Lastly, following, for instance, Day and Huang (1990) and more recently Farmer and Joshi (2002), we assume that in each period a market maker mediates transactions by matching agents’ demand and supply.⁷ In each period, the market maker sets the asset price according to aggregate excess demand in the market as follows:

$$P_{t+1} = P_t + \mu(n_a X_{t+1}^A + n_{TF} X_{t+1}^{TF} + n_{CT} X_{t+1}^{CT}) \quad (2.4)$$

where the term μ is a positive price adjustment parameter. The terms n_a , n_{TF} and n_{CT} represent the number of arbitrageurs, trend followers and contrarian traders, respectively.⁸ If agents’ net orders are negative (*positive*), the market maker must buy (*sell*) some shares of the asset and decrease (*increase*) the asset price in the next period. Transactions are then executed at this new price.

More precisely, substituting eq. (2.1), eq. (2.2) and eq. (2.3) into eq. (2.4) yields

⁷A market maker based method of price formation enables to study the price dynamics induced by each trading strategy as well as by the interplay between arbitrageurs and chartists.

⁸It worth stressing at this point that the portion of arbitrageurs and trend followers is rather η and z , respectively.

the following equation:

$$P_{t+1} = \mu n_a \beta V_t + (1 + n_a \beta + \mu \varphi(n_{TF} - n_{CT}))P_t + \mu \varphi(n_{CT} - n_{TF})P_{t-1} \quad (2.5)$$

which constitutes our deterministic model driving the price dynamics.

2.3 Analytical Solution

We now consider the model presented in the previous section and seek to examine the price dynamics that are likely to emerge in a market in which arbitrageurs and chartists coexist. Our model of price formation - with linear behavioral rules and linear price formation rule - based on the assumption that arbitrageurs have homogeneous as well as realistic beliefs about asset fundamentals can be solved analytically. We now turn to *(i.)* identify the long-run equilibrium of the system; *(ii.)* study the conditions under which the $\{P_t\}$ sequence would converge towards the long-run equilibrium. Our model actually attempts to test the efficient market hypothesis according to which asset prices should reflect the asset fundamentals. Non-convergent price dynamics in the long run would contradict such hypothesis. As a result, in what follows, any dynamics of the $\{P_t\}$ sequence that lead to non-convergent price dynamics are precluded from this study.

To perform the foregoing analysis, we must identify the law governing the price evolution in our model. For this purpose, we need to find out the solution to the deterministic equation driving the price dynamics, as given in eq. (2.5). Rewriting

eq. (2.5) yields the following **second-order linear difference equation**:⁹

$$P_t = a_0 + a_1 P_{t-1} + a_2 P_{t-2} \quad (2.6)$$

$$\text{with } \begin{cases} a_0 = \mu N \eta \beta \bar{V} \\ a_1 = 1 - \mu N \eta \beta + \mu N \varphi (1 - \eta) (2z - 1) \\ a_2 = -\mu N \varphi (1 - \eta) (2z - 1) \end{cases}$$

Because frequent and repeated new information that could affect the intrinsic value of the asset over time is precluded from this work, in what follows, the fundamental value of the asset V_t is constant over time and can thus be denoted \bar{V} . Furthermore, it is worth mentioning that coefficient a_2 must be different from zero, since if $a_2 = 0$, eq. (2.6) becomes of the first order. Actually, careful examination of the possible values of the coefficient a_2 enables us to identify the specific cases for which eq. (2.6) is of the first order. With $\mu = 0.001$, $N = 1000$, $\varphi = 1$, a_2 equals zero if either $\eta = 1$ (*i.e.*, there are only arbitrageurs in the market) or $z = 0.5$ (*i.e.*, chartist population is equally divided between trend followers and contrarian traders).

In order to understand the main features of the model and to derive the solution to eq. (2.6), in what follows, as a first step, we study the solution and the stability conditions in the simplest cases (*First-order case*) in which first, there are only arbitrageurs in the market ($\eta = 1$); second, the chartist population is split between

⁹Equation (2.6) is linear because all values of the dependent variable are raised to the first power and of the second order because P_t depends on P_{t-1} and P_{t-2} only. It is however worth noting that eq. (2.6) has no forcing process.

trend followers and contrarian traders ($z = 0.5$). Indeed, when either $\eta = 1$ or $z = 0.5$, $a_2 = 0$ and the linear difference equation (2.6) simplifies to a first-order linear difference equation. Then, as a second step, we consider the presence of both arbitrageurs and chartists in the market, by allowing $z \neq 0.5$, which leads us to study the second-order linear difference equation given by eq. (2.6) (*Second-order case*). In order to do so, we mainly follow the methodology presented in Gandolfo (1997) and Enders (2003).

2.3.1 The First-order Case

As a first step, we study the solution and the stability conditions in the simplest case in which there are only arbitrageurs in the market. In this case, eq. (2.6) simplifies to

$$P_t = a_0 + a_1 P_{t-1} \quad (2.6a)$$

$$\text{with } \begin{cases} a_0 = \mu N \beta \bar{V} \\ a_1 = 1 - \mu N \beta \end{cases}$$

The term a_1 must be different from zero, since if a_1 is zero the equation is no longer a difference equation.¹⁰

At this point, it is worth mentioning that eq. (2.6) also becomes of the first order when the chartist population is split between trend followers and contrarian

¹⁰At this point, examination of the possible values of the coefficient a_1 enables us to identify cases in which eq. (2.6a) is no longer a difference equation. Indeed, with $\mu = 1/N$, $N = 1000$ and $\beta = 1$, a_1 equals zero. However, it is necessary to study the behavior over time of the function P_t for any value of μ , N and β in order to identify the conditions on the parameter values so that the system is stable.

traders (*i.e.*, $z = 0.5$). In this case, eq. (2.6) simplifies to:¹¹

$$P_t = a_0 + a_1 P_{t-1} \tag{2.6b}$$

$$\text{with } \begin{cases} a_0 = \mu N \eta \beta \bar{V} \\ a_1 = 1 - \mu N \eta \beta \end{cases}$$

Equation (2.6a) and eq. (2.6b) are both **first-order linear difference equations** since P_t depends on P_{t-1} only. However, since the procedure to derive the solution to eq. (2.6a) and eq. (2.6b) is the same, in what follows, unless differently specified, we focus on solving eq. (2.6a). We eventually make specific comments on each case.

The **general solution** to a difference equation is defined to be a particular solution plus all homogeneous solutions. To split this procedure into its component parts, consider first the **homogeneous equation**:

$$P_t - a_1 P_{t-1} = 0 \tag{2.7a}$$

The solution to eq. (2.7a) is called the **homogeneous solution**. The trivial solution $P_t = P_{t-1} = \dots = 0$ obviously satisfies eq. (2.7a), but this solution is not unique. In order to find the complete solution to eq. (2.7a), let us suppose that, in the initial period ($t = 0$), the function P takes an arbitrary value A . From eq.

¹¹In this case, with $N = 1000$, $\mu = 1/N$ and $\beta = 1$, $a_1 = 0$, only when $\eta = 1$ which is contradictory to $z = 0.5$, so that, when the chartist population is split between trend followers and contrarian traders, $a_1 \neq 0$.

(2.7a), the following sequence can be computed:

$$P_1 = a_1 P_0 = a_1 A$$

$$P_2 = a_1 P_1 = a_1(a_1 A) = a_1^2 A$$

$$P_3 = a_1 P_2 = a_1(a_1^2 A) = a_1^3 A$$

$$P_4 = a_1 P_3 = a_1(a_1^3 A) = a_1^4 A$$

...

And so the **solution** appears to be

$$P_t = (a_1)^t A \tag{2.8a}$$

As a check, substitute this function in eq. (2.7a):

$$(a_1)^t A - a_1(a_1)^{t-1} A = 0 \tag{2.9a}$$

If eq. (2.8a) is a solution, eq. (2.9a) must hold identically. Now, since

$$a_1(a_1)^{t-1} A = (a_1)^t A$$

eq. (2.9a) can be written as

$$(a_1)^t A - (a_1)^t A = 0$$

and is indeed satisfied for all t . Since the function found satisfies the difference equation and contains one arbitrary constant, eq. (2.8a) is the **complete solution**

of the homogeneous equation (2.7a).

Now, since eq. (2.8a) gives only the form of the function P_t but not its position in the Cartesian plane (t, P) , an additional condition is needed. Indeed as soon as the function is constrained to pass through a given point, say (t^*, P^*) , its position, which depends on one arbitrary constant only, is determined and the arbitrariness of the constant A disappears. The additional condition thus says that $P_t = P^*$ for $t = t^*$ where t^* and P^* are known values. Substituting these values in eq. (2.8a) yields:

$$P^* = (a_1)^{t^*} A$$

and so

$$A = \frac{P^*}{(a_1)^{t^*}} \quad (2.10a)$$

The value of P in the initial period ($t = 0$) is actually assumed to be known and equals the asset fundamental value, so that:

$$P_t = P_0 = \bar{V}$$

for $t = 0$, which gives that $A = P_0 = \bar{V}$.

The arbitrary constant A can thus be eliminated, once the particular solution is obtained, by imposing an initial condition for P_0 . So, in order to complete the study of the first-order linear equation (2.6a), one needs to find the particular solution to eq. (2.6a). To this end, consider now the **non-homogeneous equation** by writing (2.6a) as:

$$P_t - a_1 P_{t-1} = a_0 \quad (2.11a)$$

$$\text{with } \begin{cases} a_0 = \mu N \beta \bar{V} \\ a_1 = 1 - \mu N \beta \end{cases}$$

As a **particular solution**, substitution of λ , an undetermined constant, into eq. (2.11a) yields $(1 - a_1)\lambda = a_0$ from which:

$$\lambda = \frac{a_0}{(1 - a_1)} \quad (2.12a)$$

and so $P^* = \frac{a_0}{(1 - a_1)}$ is a particular solution.¹² This particular solution represents the long-run equilibrium value of the asset price. Notice that with $a_0 = \mu N \beta \bar{V}$ and $a_1 = 1 - \mu N \beta$, the particular solution simplifies to $P^* = \bar{V}$ *i.e.*, to the asset fundamental value which is assumed to be constant over time.

Since the complete solution of the homogeneous equation $P_t - a_1 P_{t-1} = 0$ is $P_t = (a_1)^t A$, where A is an arbitrary constant, the **general solution** of (2.6a) is:

$$P_t = (a_1)^t A + \bar{V} \quad (2.13a)$$

The arbitrary constant A can now be eliminated given that we know the price in the initial period, namely, P_0 . Since the solution must hold for every period, including the initial period, it must be the case that:

$$P_0 = (a_1)^0 A + \bar{V}$$

¹² $P^* = \frac{a_0}{(1 - a_1)}$ would not be satisfactory if $(1 - a_1) = 0$. However, here, $(1 - a_1) \neq 0$ since $a_1 = 1 - \mu N \beta$ with $\mu > 0$, $N > 0$ and $\beta > 0$.

Since $(a_1)^0 = 1$, the value of A is given by:

$$A = P_0 - \bar{V}$$

Substituting this solution for A back into eq. (2.13a) yields:

$$P_t = (a_1)^t(P_0 - \bar{V}) + \bar{V} \quad (2.14a)$$

which constitutes the **general solution** to eq. (2.6a).

From eq. (2.14a), we can notice that if the initial price happens to be \bar{V} , then $P_t = \bar{V}$ *i.e.*, the asset price stays constant at \bar{V} . Instead, if at some point in time (*e.g.*, because of an exogenous disturbance), the system is no more in equilibrium (*i.e.*, $P_t \neq \bar{V}$), eq. (2.14a) tells us whether the $\{P_t\}$ sequence will converge to the long-run equilibrium price. This mainly depends on the absolute value of the parameter a_1 . If $|a_1| < 1$ ($|a_1| > 1$), the movement will be convergent (*divergent*). Since $a_1 = 1 - \mu N \beta$, the condition which ensures stability of the system (*i.e.*, $|a_1| < 1$), implies that: $0 < \mu < \frac{2}{\beta N}$ or $0 < \beta < \frac{2}{\mu N}$.

Substituting $N = 1000$, $\beta = 0.9$ and $\mu = 0.001$, into eq. (2.14a), yields the thin line in Fig. 2.1. However, it is worth mentioning that such values of β and μ are chosen so that the above-mentioned stability conditions of the model are satisfied. In fact, substituting these values for N , β or μ into these conditions yields $0 < \mu < 0.002$ or $0 < \beta < 2$. Besides, when $\eta = 1$, we avoid using $\beta = 1$, since as mentioned above, in this case, eq. (2.6a) is no more a linear difference equation and $P_t = V_t = \bar{V}$.

Fig. 2.1 shows the time path of the general solution given by eq. (2.14a) in which $P_0 = 90$, $\bar{V} = 100$ and t runs from 1 to 20.

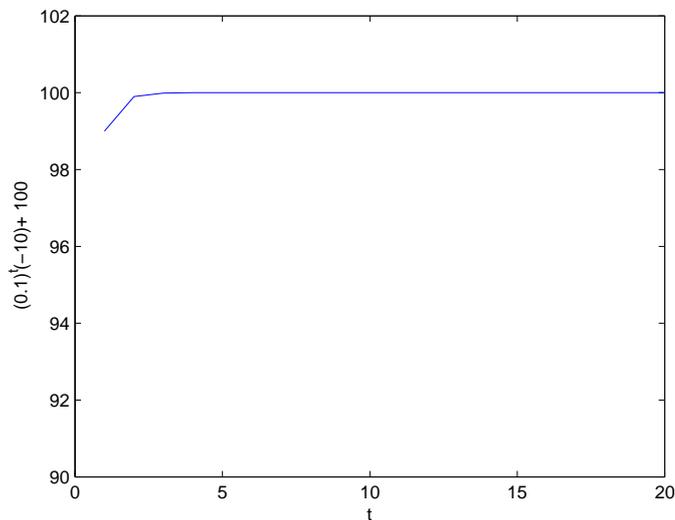


Figure 2.1: The behavior of the general solution when $\eta = 1$.

Price correction is almost immediate and asset mispricing (*i.e.*, $P_t \neq V_t$) does not persist in a market where there are only arbitrageurs. As a result, arbitrageurs are immediately able to correct the mispricing *i.e.*, arbitrage works.

Our setting enables us to reproduce one of the predictions suggested by the efficient market hypothesis, according to which asset prices should reflect any new information.

As explained above, eq. (2.6) becomes of the first order also when the chartist population is split between trend followers and contrarian traders (*i.e.*, $z = 0.5$). At this point, it is worth stressing that when $z = 0.5$, from eq. (2.2) and eq. (2.3),

trend followers' orders and contrarian traders' ones exactly compensate, so that chartist strategies do not further affect price dynamics. The price dynamics are, in this case, driven by eq. (2.6b). Given the findings from the case in which there are only arbitrageurs in the market, the general solution to eq. (2.6b) is given by:

$$P_t = (a_1)^t(P_0 - \bar{V}) + \bar{V} \quad (2.14b)$$

with $a_1 = 1 - \mu N \eta \beta$.

When $z = 0.5$, the condition which ensures stability of the system ($|a_1| < 1$) implies that:

$0 < \mu < \frac{2}{\beta N \eta}$ or $0 < \beta < \frac{2}{\mu N \eta}$ which, with $\mu > 0$, $N > 0$ and $\beta > 0$, holds if $\eta \neq 0$.¹³

With $N = 1000$, $\beta = 1$ and $\mu = 0.001$, the study of the general equation (2.14b) reveals that when the chartist population is split between trend followers and contrarian traders, the asset price goes back towards its fundamental value \bar{V} . However, mispricing duration is decreasing in the portion of arbitrageurs in the market. At this point, it is worth mentioning that these values for β and μ are chosen so that the above-mentioned stability conditions of the model are satisfied. In fact, substituting these values for N , β or μ into these conditions yields $0 < \mu < 2/(1000\eta)$ or $0 < \beta < 2/\eta$. With $0 < \eta < 1$, $0 < \mu < 2/(1000\eta)$ or $0 < \beta < 2/\eta$, so that the values of N , μ and β chosen above satisfy these conditions.

¹³It is worth mentioning that if there is no arbitrageur in the market ($\eta = 0$), then whatever the composition of the chartist population, the asset price remains constant at its initial value. This is merely due to the fact that chartist's orders are only triggered by price fluctuations. When $\eta = 0$, such fluctuations rather never arise, since the absence of arbitrageurs in the market preclude any intrinsic price changes from occurring. Because when $\eta = 0$, the asset price never converges towards the asset fundamentals, in what follows, this setting is not considered.

Fig. 2.2 shows the time path of the general solution given by eq. (2.14b), for the case in which $P_0 = 90$, $\bar{V} = 100$, for differing values of η and t runs from 1 to 40.

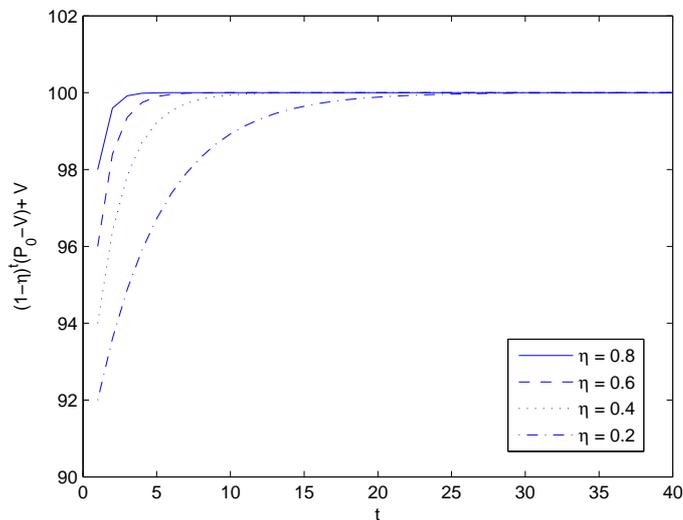


Figure 2.2: Behavior of the general solution when $z = 0.5$, for differing values of η

Indeed, while the $\{P_t\}$ sequence eventually converges to the asset fundamentals, price correction takes some time. Besides, the larger the portion of arbitrageurs in the market, the shorter the mispricing lasts. Furthermore, price correction is markedly delayed only if chartists widely dominate the market.

Arbitrageurs appear to play a key role in mispricing correction. More precisely, while chartist strategies do not further affect the mispricing (since $z = 0.5$), mispricing duration may be markedly increased if the portion of arbitrageurs in the market is low.

2.3.2 The Second-order Case

In the previous subsection, the study of the model presented in Section 2.2 was focused on finding out the solution to the first-order linear difference equation (2.6a) and (2.6b). Indeed, a preliminary study of the model has enabled us to detect that some parameter values cause eq. (2.6) to become of the first-order, namely, *(i.)* when there are only arbitrageurs in the market; *(ii.)* when the chartist population is split between trend followers and contrarian traders. The general solutions to eq. (2.6a) and eq. (2.6b) have eventually been formed, so that the behavior of the $\{P_t\}$ sequence, in these two specific cases, has been uncovered. However, the foregoing compositions of the population do not cover the whole range of possible situations in a market in which arbitrageurs and chartists coexist. This work has rather been a preliminary step in order to investigate the effect of arbitrage strategies and chartist strategies on price dynamics. To this end, we now turn to focus on the price dynamics that emerge when arbitrageurs as well as chartists are present in the market and the latter can further affect the evolution of the asset price (*i.e.*, $z \neq 0.5$). This is done by considering the whole range of possible values of z *i.e.*, $0 \leq z \leq 1$. In this case, price dynamics are described through the following **second-order linear difference equation** (2.6):

$$P_t = a_0 + a_1 P_{t-1} + a_2 P_{t-2} \quad (2.6)$$

$$\text{with } \begin{cases} a_0 = \mu N \eta \beta \bar{V} \\ a_1 = 1 - \mu N \eta \beta + \mu N \varphi (1 - \eta) (2z - 1) \\ a_2 = -\mu N \varphi (1 - \eta) (2z - 1) \end{cases}$$

As in the first-order case, consider first the **homogeneous equation**:

$$P_t - a_1 P_{t-1} - a_2 P_{t-2} = 0 \quad (2.7)$$

Given the findings from the first-order case, the complete solution of this homogeneous equation has the form $P_t^h = A\alpha^t$. Substitution of this solution into eq. (2.7) yields:

$$A\alpha^t - a_1 A\alpha^{t-1} - a_2 A\alpha^{t-2} = 0 \quad (2.8)$$

Clearly, any arbitrary value of A is satisfactory. By dividing eq. (2.8) by $A\alpha^{t-2}$, the problem is now to find the values of α that satisfy:

$$\alpha^2 - a_1\alpha - a_2 = 0 \quad (2.9)$$

Equation (2.9) is called the **characteristic equation**. Substituting the respective expressions of a_1 and a_2 into eq. (2.9) yields:

$$\alpha^2 - (1 - \mu N\eta\beta + \mu N\varphi(1 - \eta)(2z - 1))\alpha - (-\mu N\varphi(1 - \eta)(2z - 1)) = 0$$

Solving eq. (2.9) yields two values of α , called the **characteristic roots**. Using the quadratic formula, the two characteristic roots are:

$$\begin{aligned} \alpha_1, \alpha_2 &= \left(a_1 \pm \sqrt{a_1^2 + 4a_2} \right) / 2 \\ &= \left(a_1 \pm \sqrt{d} \right) / 2 \end{aligned} \quad (2.10)$$

where the term d is the discriminant *i.e.*, $[(a_1)^2 + 4a_2]$.

Each of these two characteristic roots yields a valid solution to eq. (2.7). These

solutions are however not unique. Indeed, for any two arbitrary constant A_1 and A_2 , the linear combination $A_1(\alpha_1)^t + A_2(\alpha_2)^t$ also solves (2.7). The form of the **complete homogeneous solution** being identified, the first step in order to solve the second-order linear difference equation (2.6) is now achieved. However, the study of the homogeneous solution is incomplete because the values of α_1 and α_2 , which depend on the portions of arbitrageurs (η) and trend followers (z) in the market, are not determined yet. The determination of the possible values of the characteristic roots is actually done in the following subsections.

However, given the findings from the first-order case, the **general solution** to eq. (2.6) can already be found. Indeed, the general solution to a difference equation is the sum of all homogeneous solutions and one particular solution. We now turn to derive the particular solution of the second-order linear difference equation (2.6). To this end, consider the non-homogeneous equation by writing eq. (2.6) as

$$P_t - a_1 P_{t-1} - a_2 P_{t-2} = a_0 \quad (2.11)$$

$$\text{with } \begin{cases} a_0 = \mu N \eta \beta \bar{V} \\ a_1 = 1 - \mu N \eta \beta + \mu N \varphi (1 - \eta) (2z - 1) \\ a_2 = -\mu N \varphi (1 - \eta) (2z - 1) \end{cases}$$

As a **particular solution**, substitution of λ , an undetermined constant, into eq. (2.11) yields $\lambda(1 - a_1 - a_2) = a_0$ from which:

$$\lambda = \frac{a_0}{(1 - a_1 - a_2)} \quad (2.12)$$

and so $P^* = \frac{a_0}{(1-a_1-a_2)}$ is a particular solution.

It is worth stressing that $P^* = \frac{a_0}{(1-a_1-a_2)}$ would not be satisfactory if $(1-a_1-a_2) = 0$. However, with $\mu > 0$, $N > 0$ and $\beta > 0$, $(1-a_1-a_2) = 0$ if and only if $\eta = 0$.¹⁴ This particular solution represents the long-run equilibrium value of the asset price. Notice that with $a_0 = \mu N \beta \bar{V}$, $a_1 = 1 - \mu N \eta \beta + \mu N \varphi (1 - \eta)(2z - 1)$ and $a_2 = -\mu N \varphi (1 - \eta)(2z - 1)$, the particular solution simplifies to $P^* = \bar{V}$ *i.e.*, the asset fundamental value, which is assumed to be constant over time. Since the complete solution of the homogeneous equation $P_t - a_1 P_{t-1} - a_2 P_{t-2} = 0$ is $P_t = (\alpha_1)^t A_1 + (\alpha_2)^t A_2$, where A_1 and A_2 are arbitrary constants, the **general solution** to eq. (2.6) is:

$$P_t = (\alpha_1)^t A_1 + (\alpha_2)^t A_2 + \bar{V} \quad (2.13)$$

In order to complete the study of the second-order linear difference equation (2.6), we now turn to determine the values of the characteristic roots α_1 and α_2 . Indeed, according to our setting, the specific values of α_1 and α_2 depend on the parameter values of the model. More precisely, as defined in eq. (2.10), the specific values of the characteristic roots depend on the values of a_1 and a_2 . More important, the terms a_1 and a_2 depend on two key variables of our model, namely, the portion of arbitrageurs (η) and the portion of trend followers (z) in the market. However, at this point, it is worth reminding that this work aims at understanding the price dynamics that may emerge in the market for differing composition of the population. As a result, the entire range of possible values of η and z is considered and for any combination of η and z will be associated distinct

¹⁴This is a further justification for not considering the case when $\eta = 0$.

values of a_1 and a_2 . With this in mind, in order to perform this study, we first derive an expression for the discriminant as a function of the model parameters. Substituting the respective expressions of a_1 and a_2 into the expression for the discriminant yields:

$$d = 1 + \mu N(-\eta\beta + \varphi(1 - \eta)(2z - 1))^2 - 4\mu N\varphi(1 - \eta)(2z - 1)$$

Clearly, the value of the discriminant depends on the portion of arbitrageurs (η) and trend followers (z) in the market. As presented in Section 2.2, the possible values for $\eta \equiv n_a/N$ and $z \equiv n_{TF}/n_c$ ranges from 0 to 1. Three cases are thus possible for the value of $d = (a_1)^2 + 4a_2$, namely, $d > 0$, $d = 0$ and $d < 0$. In what follows, we turn to first, identify the specific range of values for η and z that holds for every possible value of the discriminant. Second, we further study and find out the stability conditions for each of the three cases mentioned above. Lastly, in order to uncover and further understand the price dynamics that emerge in the market, we form the general solution of the second-order linear difference equation (2.6) in each case.

CASE 1: Real and distinct characteristic roots.

With reasonable parameter values, namely, $\varphi > 0$, $\beta > 0$, $\mu > 0$ and $N > 0$, $d > 0$ if and only if:¹⁵

$$\begin{aligned}
 & i. \quad 0 < \eta < 1 \\
 & ii. \quad 0 \leq z < \frac{-N\beta\eta\mu - N\varphi\mu + N\eta\varphi\mu - 1}{2N(\eta - 1)\mu\varphi} - \sqrt{\frac{\beta\eta}{N(\eta - 1)^2\mu\varphi^2}}
 \end{aligned}$$

For $N = 1000$, $\mu = 0.001$, $\beta = 1$ and $\varphi = 1$, condition *ii.* simplifies to:

$$0 \leq z < -\sqrt{\frac{\eta}{(\eta - 1)^2}} - \frac{1}{\eta - 1}$$

The conditions on the values of η and z that lead to $(a_1)^2 + 4a_2 > 0$ are summarized in Fig. 2.3.¹⁶

When $(a_1)^2 + 4a_2 > 0$, d is a real number and there will be two distinct and real characteristic roots. In this case, there are two separate solutions to the homogeneous equation (2.7) denoted by $(\alpha_1)^t$ and $(\alpha_2)^t$. In addition, as mentioned above, any linear combination of the two *i.e.*, $P_t^h = A_1(\alpha_1)^t + A_2(\alpha_2)^t$, is also a solution.

¹⁵Actually, for d to be positive, z would be allowed to be negative. However, within our model and given that z is the portion of trend followers in the market, z cannot be negative. This precisely justifies the lower bound for z . Furthermore, given the remark made previously regarding the necessary conditions for eq. (2.12a) to be satisfactory, while for d to be positive, η would be allowed to equal zero, $\eta = 0$ has to be precluded from this work. This precisely justifies the lower bound for η .

¹⁶It is worth reminding that eq. (2.6) is of the second order, if and only if $a_2 \neq 0$. This condition however does not hold if $z = 0.5$, which constitutes a further restriction to be imposed on the value of z .

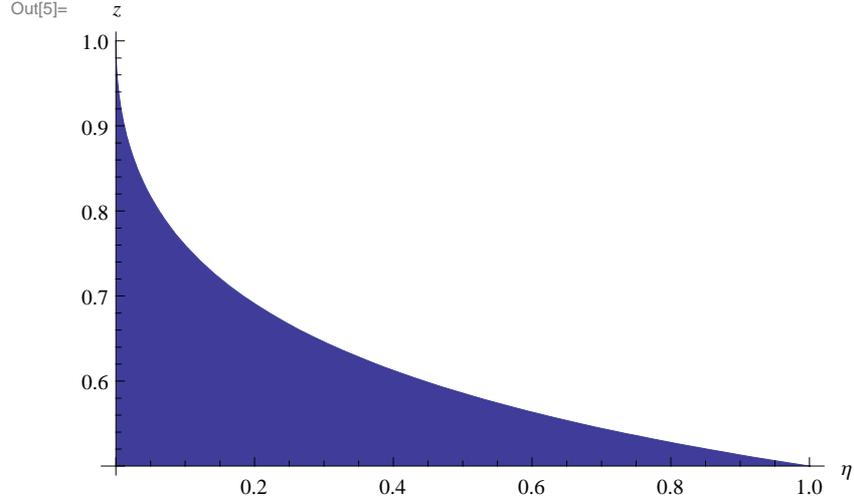


Figure 2.3: Possible combinations of η and z so that $d > 0$.

Given the findings in the first-order case, the behavior of the homogeneous solution depends on the absolute value of α_1 and α_2 . More precisely, the homogeneous solution is convergent if $|\alpha_i|$ are less than unity. More precisely, stability only requires that the largest root ($\alpha_1 = (a_1 + \sqrt{d})/2$) is less than unity and the smallest root ($\alpha_2 = (a_1 - \sqrt{d})/2$) is greater than -1.

First, the largest characteristic root (α_1) is less than unity if:

$$a_1 + (a_1^2 + 4a_2)^{1/2} < 2 \quad \text{or} \quad (a_1^2 + 4a_2)^{1/2} < 2 - a_1$$

Hence, $a_1^2 + 4a_2 < 4 - 4a_1 + a_1^2$, that is:

$$a_1 + a_2 < 1 \quad \text{or} \quad a_2 < 1 - a_1 \quad (2.14)$$

Substituting the respective expressions for a_1 and a_2 into (2.14) yields:

$$\eta > 0 \tag{2.15}$$

since $\beta > 0$, $\mu > 0$ and $N > 0$.

Second, the smallest root (α_2) will be greater than -1 if:

$$a_1 - (a_1^2 + 4a_2)^{1/2} > -2 \quad \text{or} \quad 2 + a_1 > (a_1^2 + 4a_2)^{1/2}$$

Hence, $4 + 4a_1 + a_1^2 > a_1^2 + 4a_2$, that is:

$$a_2 < 1 + a_1 \tag{2.16}$$

Substituting the respective expressions for a_1 and a_2 into (2.16) yields:

$$z > \frac{1}{-4\varphi(1-\eta)} \left(\frac{2}{\mu N} - \eta(\beta - 2\varphi) - 2\varphi \right) \tag{2.17}$$

Now, for $N = 1000$, $\mu = 0.001$, $\beta = 1$ and $\varphi = 1$, eq. (2.17) simplifies to:

$$z > \frac{\eta}{-4(1-\eta)} \tag{2.18}$$

which actually implies, as illustrated in Fig. 2.4, that for $0 < \eta < 1$, $z \geq 0$.

To sum up, given the values of N , μ , β and φ , combining the conditions on the values for η and z which first, ensure that the discriminant is positive (*i.e.*, conditions *i.* and *ii.*); second, which guarantee that the homogeneous solution is

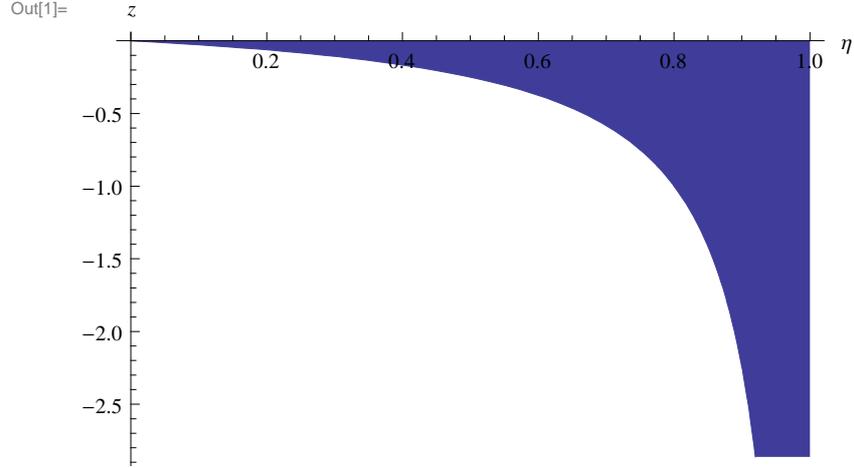


Figure 2.4: Possible combinations of η and z so that $d > 0$.

convergent (*i.e.*, eq. (2.15) and eq. (2.18)) yields the following conditions on η and z when $d > 0$:¹⁷

$$0 < \eta < 1$$

$$0 \leq z < -\sqrt{\frac{\eta}{(\eta-1)^2}} - \frac{1}{\eta-1}$$

These conditions which characterize the stability region when $d > 0$ are summarized in Fig. 2.5.

At this point, it is worth reminding that when the discriminant is positive, as explained previously, the general solution to eq. (2.6) is given by:

$$P_t = (\alpha_1)^t A_1 + (\alpha_2)^t A_2 + \bar{V} \quad (2.13)$$

¹⁷The reader has to keep in mind that a further restriction has been previously imposed on the value of z *i.e.*, $z \neq 1/2$.

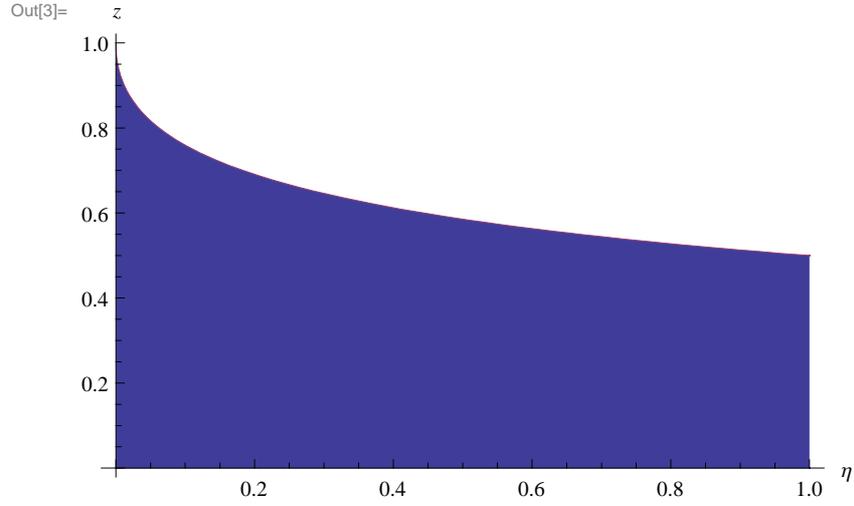


Figure 2.5: Possible combinations of η and z within the stability region when $d > 0$.

Given the foregoing findings, for all η and z within the stability region depicted in Fig. 2.5, the $\{P_t\}$ sequence given by eq. (2.13) is convergent and the asset price goes back towards the asset fundamental value.

Knowing the possible values of η and z so that the discriminant is positive, it is possible to first, derive the values of a_1 and a_2 ; second, to determine the possible values of the characteristic roots α_1 and α_2 . Then, given two initial conditions for the $\{P_t\}$ sequence, namely, P_0 and P_1 , the arbitrary constants A_1 and A_2 from eq. (2.13) can be eliminated. Eventually, substituting $N = 1000$, $\mu = 0.001$, $\beta = \varphi = 1$ into eq. (2.13) and simulating the $\{P_t\}$ sequence yields the line in Fig. 2.6. This figure shows the time path of the general solution to eq. (2.6) when $d > 0$, for the case in which $\eta = 0.3$, $z = 0.6$ and t runs from 1 to 20.

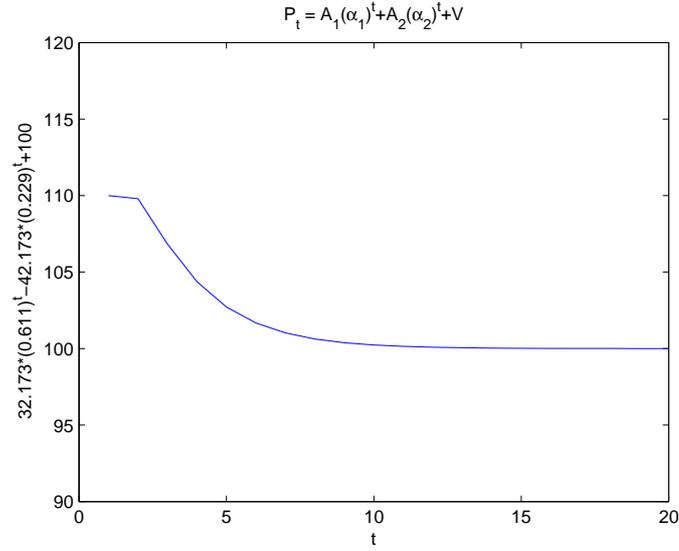


Figure 2.6: Behavior of the general solution when $d > 0$, $\eta = 0.3$ and $z = 0.6$.

The $\{P_t\}$ sequence in fact converges towards the asset fundamentals.¹⁸

CASE 2: Real and repeated characteristic roots.

With reasonable parameter values, namely, $\varphi > 0$, $\beta > 0$, $\mu > 0$ and $N > 0$, $d = 0$ if and only if:¹⁹

- i. $0 < \eta < 1$
- ii. $z = \frac{-N\beta\eta\mu - N\varphi\mu + N\eta\varphi\mu - 1}{2N(\eta - 1)\mu\varphi} - \sqrt{\frac{\beta\eta}{N(\eta - 1)^2\mu\varphi^2}}$

¹⁸While in this work, we mainly study the conditions which ensure that the system is stable, it is worth stressing that our model is able to generate unstable dynamics as well. In fact, if $\eta = 0$, then for all $0 \leq z < 1$, the $\{P_t\}$ sequence will never converge. As an illustration, if $\eta = 0$ and $z = 0$ (*i.e.*, the chartist population is only composed of contrarian traders), the $\{P_t\}$ sequence exhibits constant oscillations. Contrarian strategies prevent the asset price from converging towards the fundamentals and tend to destabilize the market.

¹⁹The reader has to keep in mind that the further restriction that has been previously imposed on the value of z *i.e.*, $z \neq 1/2$, still holds when $d = 0$.

For $N = 1000$, $\mu = 0.001$, $\beta = 1$ and $\varphi = 1$, condition *ii.* simplifies to:

$$z = -\sqrt{\frac{\eta}{(\eta - 1)^2}} - \frac{1}{\eta - 1}$$

The conditions on the values of η and z that lead to $(a_1)^2 + 4a_2 = 0$ are summarized in Fig. 2.7.

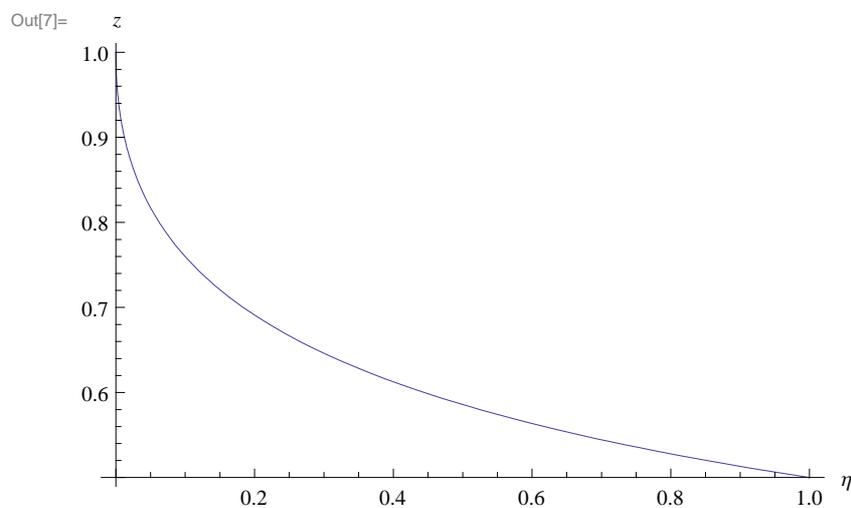


Figure 2.7: Possible combinations of η and z so that $d = 0$.

When $(a_1)^2 + 4a_2 = 0$, d equals zero and there are two real and repeated roots, namely, $\alpha_1 = \alpha_2 = a_1/2$ is a homogeneous solution. However, when $d = 0$, there is a second homogeneous solution given by $t(a_1/2)^t$. As a proof, substituting it into the homogeneous equation (2.7) yields:

$$t(a_1/2)^t - a_1[(t - 1)(a_1/2)^{t-1}] - a_2[(t - 2)(a_1/2)^{t-2}] = 0$$

Dividing it by $(a_1/2)^{t-2}$ and rearranging this expression yields:

$$-[(a_1^2/4) + a_2]t + [(a_1^2/2) + 2a_2] = 0$$

Since in this case, $(a_1)^2 + 4a_2 = 0$, each bracketed expression is zero; hence $t(a_1/2)^t$ solves (2.7). For any arbitrary constants A_1 and A_2 , the **complete homogeneous solution** is thus

$$P_t^h = A_1(a_1/2)^t + A_2t(a_1/2)^t$$

The behavior of the homogeneous solution is always convergent as long as $|a_1| < 2$, since $\lim[t(a_1/2)^t]$ is necessarily zero as $t \rightarrow \infty$. Here from $a_1 = 1 - \mu N \eta \beta + \mu N \varphi (1 - \eta)(2z - 1)$, it is possible to derive the values of z so that the homogeneous solution converges. The upper and lower bounds for the value of z are derived by studying $|a_1| < 2$ which yields:

$$\frac{1}{2\varphi(1-\eta)} \left(\frac{-3}{\mu N} + \eta(\beta - \varphi) + \varphi \right) < z < \frac{1}{2\varphi(1-\eta)} \left(\frac{1}{\mu N} - \eta(\varphi - \beta) + \varphi \right) \quad (2.19)$$

Now, with $N = 1000$, $\mu = 0.001$, $\beta = 1$, $\varphi = 1$, eq. (2.19) yields:

$$-\frac{1}{(1-\eta)} < z < \frac{1}{(1-\eta)} \quad (2.20)$$

With $0 < \eta < 1$ *i.e.*, η values which ensure that $d = 0$, the homogeneous solution converges if:

$$0 \leq z \leq 1 \quad (2.21)$$

It is worth stressing that this condition on z is less restrictive than the one which ensures that $d = 0$.

To sum up, given the values of N , μ , β and φ , the conditions on the values of η and z so that first, d equals zero; second, the homogeneous solution is convergent, are the following:²⁰

$$0 < \eta < 1$$

$$z = -\sqrt{\frac{\eta}{(\eta - 1)^2}} - \frac{1}{\eta - 1}$$

These conditions characterize the stability region when $d = 0$. In this case, the stability region is represented by the convex curve depicted in Fig. 2.7.

When $d = 0$, the general solution to eq. (2.6) is given by:

$$P_t = A_1(a_1/2)^t + A_2t(a_1/2)^t + \bar{V} \quad (2.22)$$

Given the foregoing findings, for any combination of η and z within the stability region, as depicted in Fig. 2.7, the $\{P_t\}$ sequence, given by eq. (2.22), is convergent and the asset price goes back towards the asset fundamental value.

Knowing the possible values η and z so that the discriminant equals zero, it is possible to derive the value of a_1 and more important, the possible values for the characteristic root $\alpha_1 = \alpha_2 = (\frac{a_1}{2})$. Eventually, substituting $N = 1000$, $\mu = 0.001$, $\beta = \varphi = 1$ into the general solution as described in eq. (2.22), the arbitrary constants A_1 and A_2 can be eliminated given two initial conditions for the $\{P_t\}$ sequence, namely, P_0 and P_1 . Simulating the $\{P_t\}$ sequence, as described in eq. (2.22) yields the line in Fig. 2.8. This figure shows the typical time path of the

²⁰The reader has to keep in mind that $z \neq 1/2$ is a further restriction that has been imposed on z , in order to ensure that eq. (2.6) is a second-order linear difference equation.

general solution when $d = 0$, for the case in which $\eta = 0.3$, $z = 0.646$ and t runs from 1 to 20.

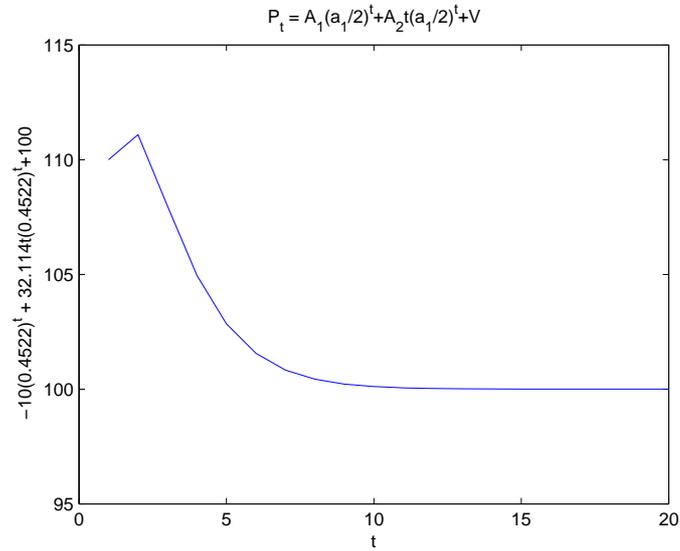


Figure 2.8: Behavior of the general solution when $\eta = 0.3$ and $z = 0.646$.

Although the system is found to converge, the behavior of the general solution is not monotonic. Indeed, it appears to explode before converging.

CASE 3: Imaginary characteristic roots.

With reasonable parameter values, namely, $\varphi > 0$, $\beta > 0$, $\mu > 0$ and $N > 0$, $d < 0$ if and only if:²¹

$$\begin{aligned}
 & i. \quad 0 < \eta < 1 \\
 & ii. \quad \frac{-N\beta\eta\mu - N\varphi\mu + N\eta\varphi\mu - 1}{2N(\eta - 1)\mu\varphi} - \sqrt{\frac{\beta\eta}{N(\eta - 1)^2\mu\varphi^2}} < z \leq 1
 \end{aligned}$$

For $N = 1000$, $\mu = 0.001$, $\beta = 1$ and $\varphi = 1$, condition *ii.* simplifies to:

$$-\sqrt{\frac{\eta}{(\eta - 1)^2}} - \frac{1}{\eta - 1} < z \leq 1$$

The conditions on the values of η and z that lead to $(a_1)^2 + 4a_2 < 0$ are summarized in Fig. 2.9.

When $a_1^2 + 4a_2 < 0$, d is negative and the characteristic roots are imaginary. Since $a_1^2 \geq 0$, imaginary roots can occur only if $a_2 < 0$ *i.e.*, if $z < \frac{1}{2}$. In this case, the two characteristic roots are:²²

$$\alpha_1, \alpha_2 = (a_1 \pm i\sqrt{-d})/2$$

where $i = \sqrt{-1}$. The homogeneous solution can be written as:²³

$$P_t^h = \beta_1 r^t \cos(\theta t + \beta_2) \tag{2.23}$$

²¹The reader has to keep in mind that a further restriction on z has been previously imposed *i.e.*, $z \neq 1/2$. This restriction still holds when $d < 0$.

²²See Enders (2003) for further technical details.

²³See Enders (2003) Appendix 1.1 for proof and further technical details.

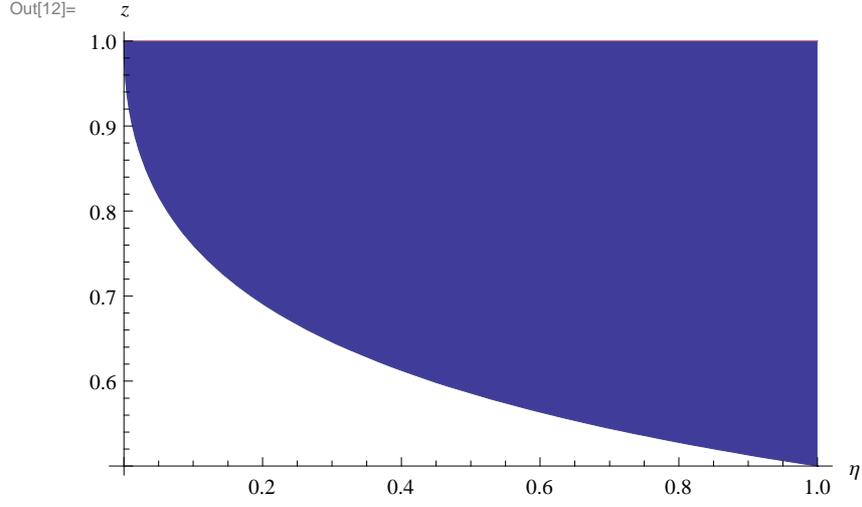


Figure 2.9: Possible combinations of η and z so that $d < 0$.

where β_1 and β_2 are arbitrary constants, $r = (-a_2)^{1/2}$, and the value of θ is chosen so as to simultaneously satisfy

$$\cos(\theta) = a_1/[2(-a_2)^{1/2}] \quad (2.24)$$

Since $\cos(\theta t) = \cos(2\pi + \theta t)$, the stability condition is determined only by the magnitude of $r = (-a_2)^{1/2}$. The homogeneous solution converges if $|a_2| < 1$. This condition holds if:

$$\frac{1}{2(1-\eta)} \left(\frac{-1}{\mu\varphi N} + 1 - \eta \right) < z < \frac{1}{2(1-\eta)} \left(\frac{1}{\mu\varphi N} + 1 - \eta \right) \quad (2.25)$$

With $N = 1000$, $\mu = 0.001$, $\beta = 1$ and $\varphi = 1$, eq. (2.25) simplifies to:

$$-\frac{\eta}{2(1-\eta)} < z < \frac{1}{2(1-\eta)} (2 - \eta) \quad (2.26)$$

With $0 < \eta < 1$, this condition actually implies that $0 \leq z \leq 1$, which is less restrictive than the condition on z found previously so that $d < 0$.

To sum up, given the values of N , μ , β and φ , the conditions on the values of η and z so that first, $d < 0$; second, the homogeneous solution is convergent, are the following:

$$\begin{aligned} 0 < \eta < 1 \\ -\sqrt{\frac{\eta}{(\eta - 1)^2}} - \frac{1}{\eta - 1} < z \leq 1 \end{aligned}$$

These conditions, as illustrated previously in Fig. 2.9, characterize the stability region when $d < 0$.

When the discriminant is negative, the general solution to eq. (2.6) is given by:

$$P_t = \beta_1 r^t \cos(\theta t + \beta_2) + \bar{V} \quad (2.27)$$

Given the foregoing findings, for any combination of η and z satisfying these conditions, the $\{P_t\}$ sequence, as described in eq. (2.27), is convergent and the asset price goes back to the asset fundamental value. However, since $d < 0$, convergence occurs through dampened oscillations. It is worth stressing that the frequency of the oscillations actually depends on the value of θ .

Knowing the possible values η and z so that the discriminant is negative and substituting $N = 1000$, $\mu = 0.001$, $\beta = \varphi = 1$ into the general solution, as described in eq. (2.27), it is possible to simulate the $\{P_t\}$ sequence, given two initial conditions P_0 and P_1 . Fig. 2.10 shows the time path of the general solution to eq. (2.6) when

$d < 0$, for the case in which $\eta = 0.3$, $z = 0.7$ and t runs from 1 to 20.

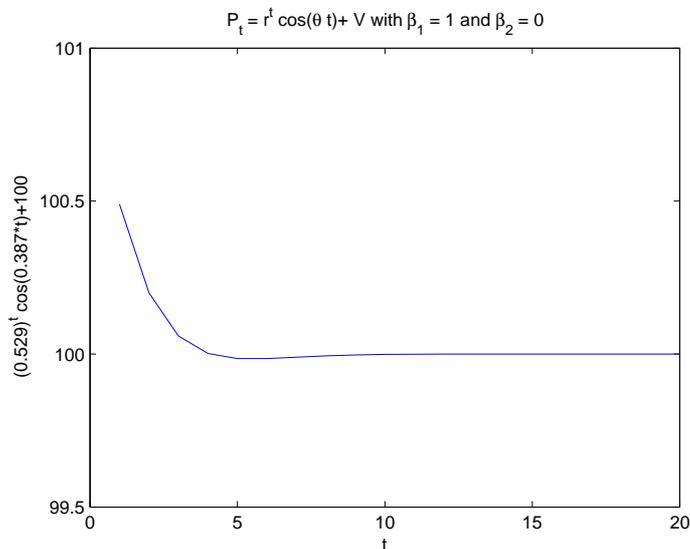


Figure 2.10: Behavior of the general solution when $\eta = 0.3$ and $z = 0.7$.

The $\{P_t\}$ sequence is in fact convergent but with such parameter values, the oscillations are weak.²⁴

Following Enders (2003), the general stability conditions can be summarized using the triangle ABC in Fig. 2.11.²⁵

Arc AOB is the boundary between cases 1 and 3 *i.e.*, it is the locus of points such that $a_1^2 + 4a_2 = 0$. The region above AOB corresponds to case 1 (since $d > 0$) and

²⁴As mentioned previously, while in this work, we mainly study the conditions which ensure that the system is stable, our model is able to generate unstable dynamics as well. In fact, if $\eta = 0$, $z = 1$ (*i.e.*, the chartist population is only composed of trend followers) and the $\{P_t\}$ sequence will never converge. The behavior of the $\{P_t\}$ sequence is rather explosive. Trend following strategies prevent the asset price from converging towards the fundamentals and tend to destabilize the market.

²⁵Source: Enders (2003) p. 28.

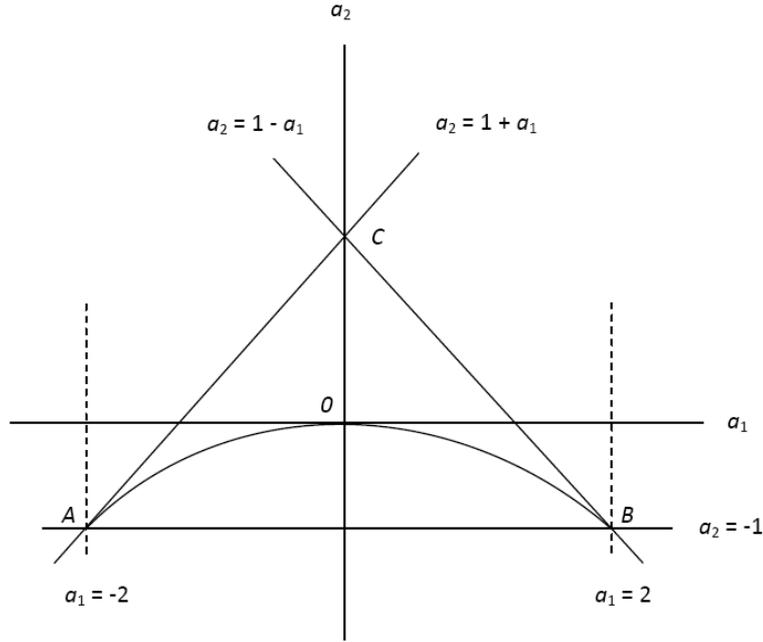


Figure 2.11: Characterizing stability conditions

the region below AOB corresponds to case 3 (since $d < 0$). Indeed in case 1, the region of stability consists of all points in the region bounded by $AOBC$. For any point in $AOBC$, conditions described by eq. (2.14) and eq. (2.16) hold and $d > 0$. In case 2, the region of stability consists of all points on arc AOB . In case 3, the region of stability consists of all points in region AOB . For any point in AOB , eq. (2.25) is satisfied and $d < 0$.

In order to investigate whether mispricing may persist in a market where arbitrageurs and chartists coexist, the foregoing work was a necessary first step. Indeed, this work has enabled us to identify the parameter values which ensure stationary price dynamics, so that any erratic price dynamics, characterized by

non-stationarity, are precluded from this work. More precisely, for any combinations of η and z satisfying the stability conditions identified in this section, the price dynamics are stable and the asset price converges to the asset fundamentals. It is already worth stressing that the system is stable for a very wide range of values of η and z .

2.4 Price Dynamics and Type of Convergence

In the previous section, the study of the model described in Section 2.2 as well as the stability conditions of the solution to the model was focused on *(i.)* deriving the analytical solution of the model; *(ii.)* identifying the parameter values which ensure that the price dynamics under study is stationary, so that the asset price converges to the asset fundamentals. In the following subsections, we turn to focus on the type of convergence of the $\{P_t\}$ sequence when arbitrageurs and chartists coexist in the market.²⁶ Indeed, we now carefully study the behavior of the general solutions determined in the previous section, which leads to a detailed examination of the possible parameter values within the stability regions. This enables us to clearly understand the relationships between the parameter values and the type of price dynamics that our model may generate as well as the price dynamics that may emerge in the market when arbitrageurs and chartists coexist.

²⁶In what follows, there is no need to study the situation in which there are only arbitrageurs, since in the previous section, we have established that in such a case, the convergence is immediate. It is thus pointless to study the type of convergence in this case. Furthermore, the first-order case in which the chartist population is split between trend followers and contrarian traders ($z = 1/2$) does not require further considerations, since from the previous study, the main finding has been already identified *i.e.*, mispricing duration is decreasing in the portion of arbitrageurs (η) in the market. Besides, with $N = 1000$, $\mu = 0.001$ and $\beta = \varphi = 1$, for any value of η , the convergence of the $\{P_t\}$ sequence is monotonic.

2.4.1 Real and Distinct Characteristic Roots.

In the previous section, we have identified the necessary conditions on η and z , so that the discriminant is positive and the homogeneous solution is convergent. When the discriminant is positive, as mentioned previously, there are two real and distinct characteristic roots, namely, α_1 and α_2 . The behavior of the homogeneous solution actually depends on the sign of these characteristic roots. If either α_1 or α_2 is negative, then the convergence of the $\{P_t\}$ sequence towards the long-run equilibrium will not be monotonic. We now turn to study the possible values of the characteristic roots within the relevant stability region in order to determine whether the convergence is monotonic.

For $N = 1000$, $\mu = 0.001$, $\beta = 1$ and $\varphi = 1$ and from the study of the analytical solution of the model presented in section 2.2, it can be derived that $0 \leq \alpha_1 < 1$, while $-1 < \alpha_2 < 1$. Fig. 2.12 shows the possible values of α_1 and α_2 according to the possible values of η and z when $d > 0$.

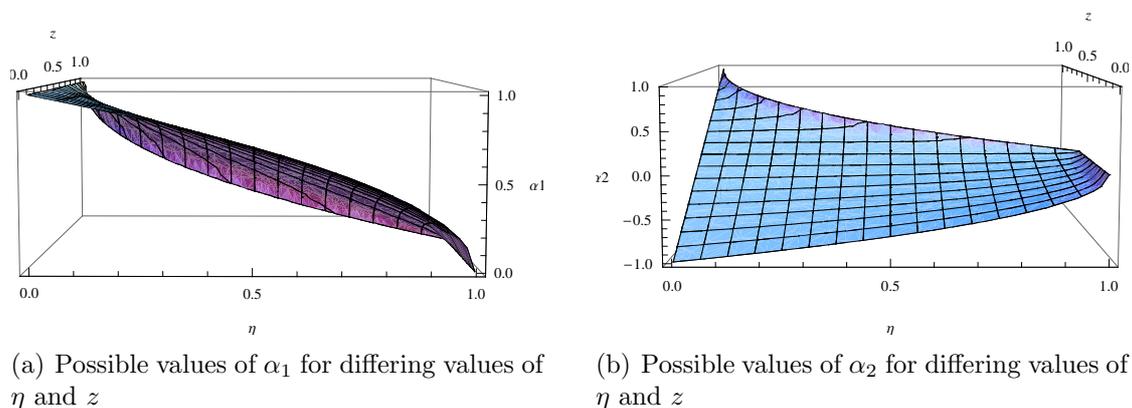


Figure 2.12: Values of the characteristic roots when $d > 0$

While from Fig. 2.12a, it is possible to observe that the value of α_1 is positive for any combination of η and z , Fig. 2.12b shows that the second characteristic root α_2 may rather be negative for some values of z . More precisely, α_2 is negative when $z < 1/2$ with $\eta < 1$. These conditions are summarized in Fig. 2.13.

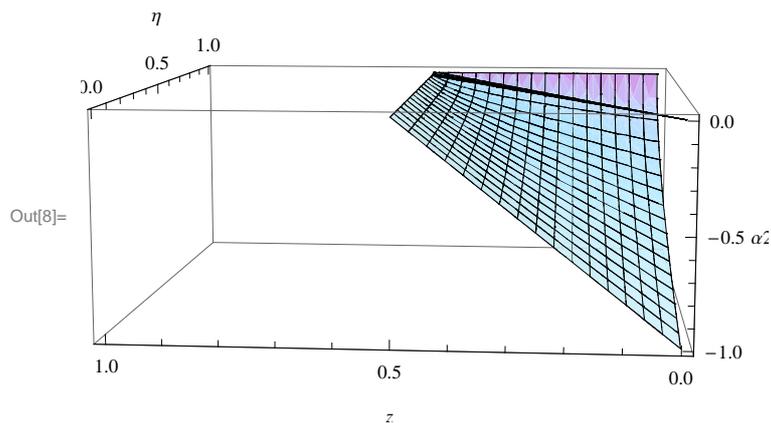


Figure 2.13: Values of η and z so that $\alpha_2 < 0$

Considering both the conditions on the values of η and z so that the discriminant is positive and the conditions for monotonic convergence of the $\{P_t\}$ sequence, it follows that:

1- the convergence of the $\{P_t\}$ sequence towards the long-run equilibrium is *monotonic* if and only if:²⁷

$$0 < \eta < 1$$

$$1/2 < z < -\sqrt{\frac{\eta}{(\eta-1)^2}} - \frac{1}{\eta-1}$$

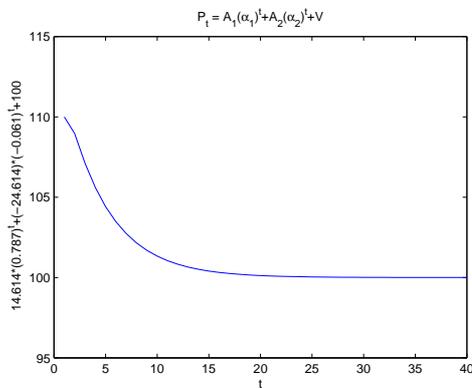
²⁷According to this study, the parameter z could actually equal 0.5, however if $z = 0.5$, as mentioned previously, the second-order linear difference equation becomes of the first order.

2- the convergence of the $\{P_t\}$ sequence towards the long-run equilibrium is not *monotonic* if and only if:

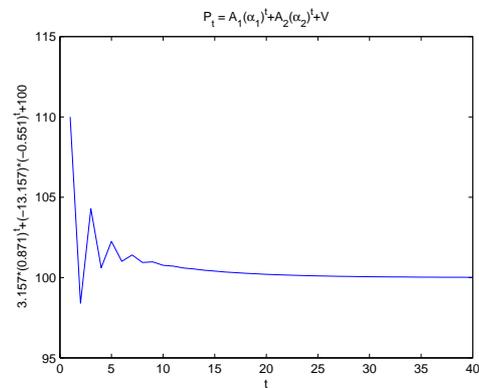
$$0 < \eta < 1$$

$$0 \leq z < 1/2$$

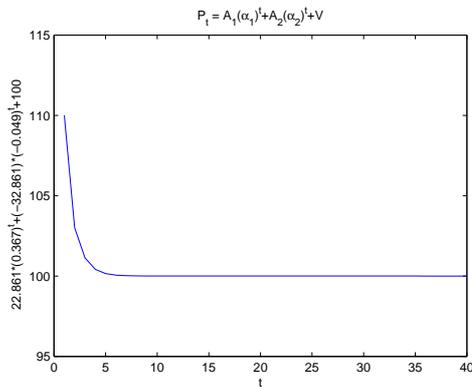
Fig. 2.14 shows the simulated time path of the general solution to eq. (2.6) when $d > 0$, for the cases in which $\eta = 0.2$ and $\eta = 0.7$.



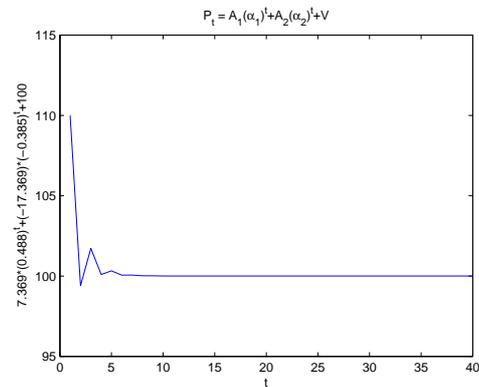
(a) When $\eta = 0.2$ and $z = 0.53$



(b) When $\eta = 0.2$ and $z = 0.2$



(c) When $\eta = 0.7$ and $z = 0.53$



(d) When $\eta = 0.7$ and $z = 0.2$

Figure 2.14: Behavior of the general solution when $d > 0$, for differing values of η and z .

Fig. 2.14a and Fig. 2.14c illustrate the cases in which the $\{P_t\}$ sequence monotonically converges towards the asset fundamentals (when $z = 0.53$). Fig. 2.14b and Fig. 2.14d rather show the cases in which the $\{P_t\}$ sequence when the convergence towards the asset fundamentals is not monotonic (when $z = 0.2$).

As a result, our study of the general solution to eq. (2.6), when the discriminant is positive, suggests that for any value of η ($0 < \eta < 1$), as shown in Fig. 2.14a and Fig. 2.14c, the $\{P_t\}$ sequence will monotonically converge towards the asset fundamentals when $1/2 < z < -\sqrt{\frac{\eta}{(\eta-1)^2}} - \frac{1}{\eta-1}$ *i.e.*, when the chartist population is mainly composed of trend followers (since $z > 0.5$). Rather, as shown in Fig. 2.14b and Fig. 2.14d, if the chartist population is mainly composed of contrarian traders ($0 \leq z < 0.5$), the convergence of the $\{P_t\}$ sequence towards the asset fundamentals is not monotonic.

Furthermore, from Fig. 2.14, we observe that mispricing duration is decreasing in the portion of arbitrageurs in the market. Indeed, the larger the portion of arbitrageurs, the shorter asset mispricing lasts.

This finding suggests that, under the conditions on the values of η and z so that $d > 0$, as depicted in Fig. 2.5, contrarian strategies tend to generate price oscillations, even when arbitrageurs dominate the chartist population ($\eta > 0.5$).

2.4.2 Real and Repeated Characteristic Roots.

In the previous section, we have identified the necessary conditions on η and z , so that the discriminant is zero and the homogeneous solution is convergent. When

the discriminant equals zero, as previously mentioned, there are two real but repeated roots, namely, $\alpha_1 = \alpha_2 = (a_1/2)$. Besides, there is a second homogeneous solution given by $t(a_1/2)^t$. In the previous section, the range of the parameter values which ensures that the homogeneous solution converges has been identified. However, it is worth noting that when the discriminant equals zero, convergence is never monotonic. The behavior of the homogeneous solution actually depends on the value of a_1 . For $0 < a_1 < 2$, the homogeneous solution explodes before eventually converging to zero. Instead, for $-2 < a_1 < 0$, the homogeneous solution oscillates before eventually converging to zero.

However, from the expression of $a_1 = 1 - \mu N \eta \beta + \mu N \varphi (1 - \eta)(2z - 1)$, with $N = 1000$, $\mu = 0.001$, $\beta = 1$ and $\varphi = 1$, it is possible to identify the range of possible values for a_1 . In our setting, the values of a_1 are actually included between 0 and 2. Fig. 2.15 shows the possible values of a_1 for any possible combination of η and z .

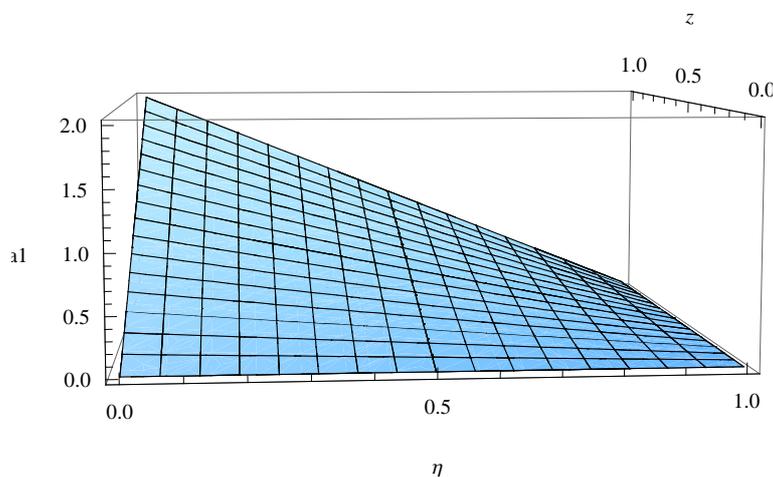


Figure 2.15: Possible values for a_1 varying η and z values

As a result, for any combination of η and z which ensures that the discriminant

equals zero, $0 < a_1 < 2$. The general solution to eq. (2.6) when $d = 0$, as described in eq. (2.22), explodes before ultimately converging to the asset fundamental value \bar{V} (see Fig. 2.8).

This finding suggests that, under the conditions on the values of η and z so that $d = 0$, as depicted in Fig. 2.7, any composition of the population tends to generate not monotonic price convergence.

2.4.3 Imaginary Characteristic Roots.

In the previous section, we have identified the necessary conditions on η and z , so that the discriminant is negative and the homogeneous solution is convergent. When the discriminant is negative, as mentioned previously, there are two imaginary characteristic roots, namely, $\alpha_1, \alpha_2 = (a_1 \pm i\sqrt{-d})/2$ where $i = \sqrt{-1}$. The general solution to eq. (2.6) when $d < 0$ is described by eq. (2.27). In this case, the trigonometric function reveals a wavelike pattern to the time path of the homogeneous as well as of the general solution given by eq. (2.27). When $d < 0$, the convergence of the $\{P_t\}$ sequence occurs through dampened oscillations. Fig. 2.16 and Fig. 2.17 show the time path of the simulated $\{P_t\}$ sequence for the cases in which $\eta = 0.3$ and $\eta = 0.7$, respectively, for differing values of z .

It can be seen that the greater is z , the more frequent price oscillations are. Indeed, it is worth mentioning that the frequency of the oscillations is actually determined by the value of the parameter θ , which itself depends on η and z . Indeed, given that

$$\cos(\theta) = \frac{a_1}{[2(-a_2)^{1/2}]} = \frac{1 - \mu N \eta \beta + \mu N \varphi (1 - \eta)(2z - 1)}{[2(\mu N \varphi (1 - \eta)(2z - 1))^{1/2}]} \quad (2.28)$$

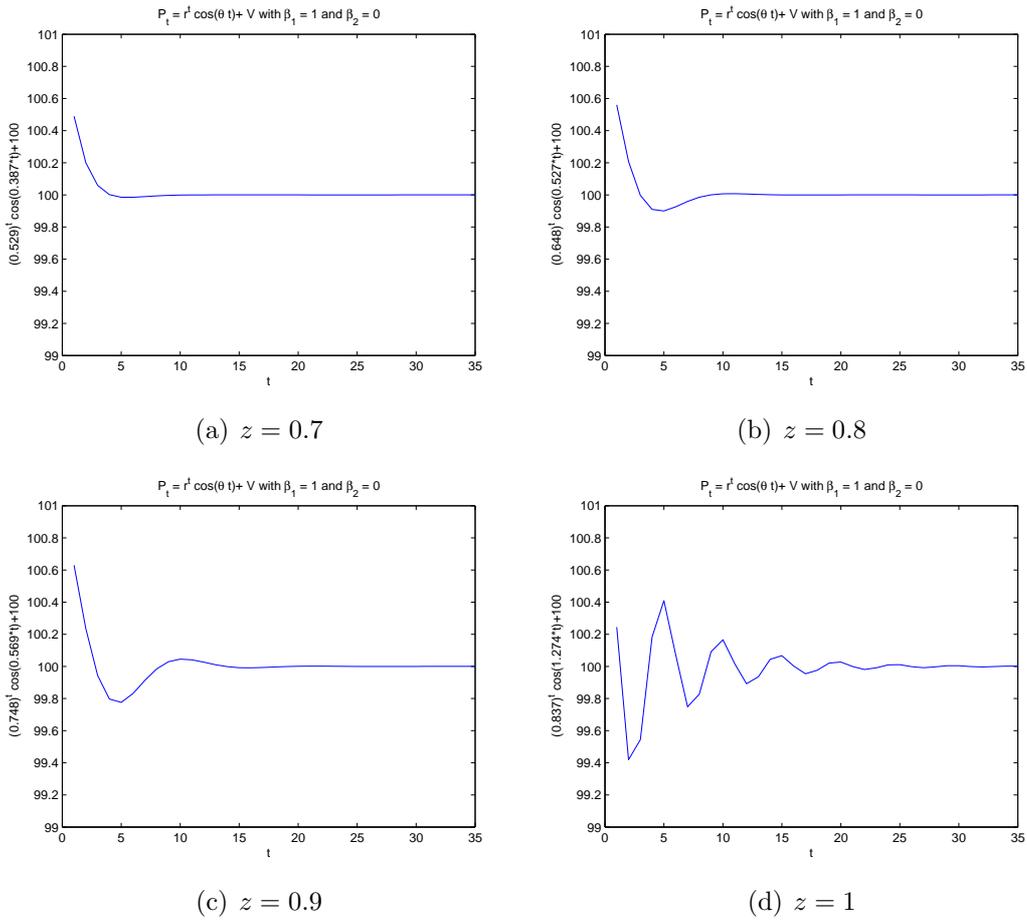
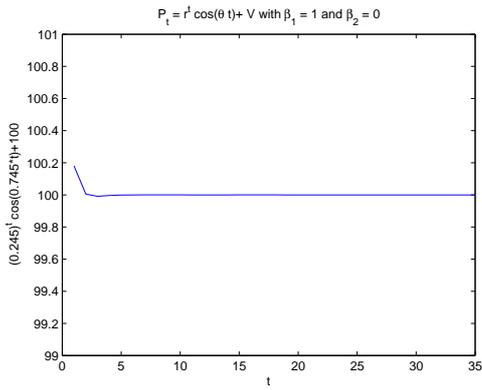
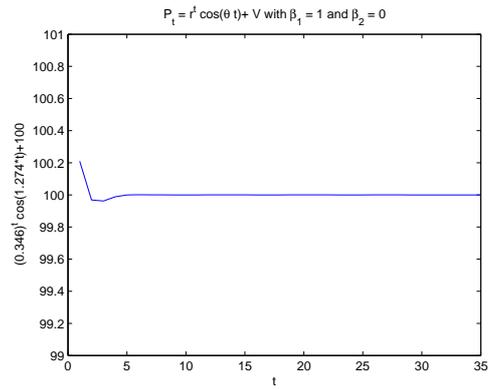


Figure 2.16: Behavior of the general solution when $\eta = 0.3$, for differing values of z .

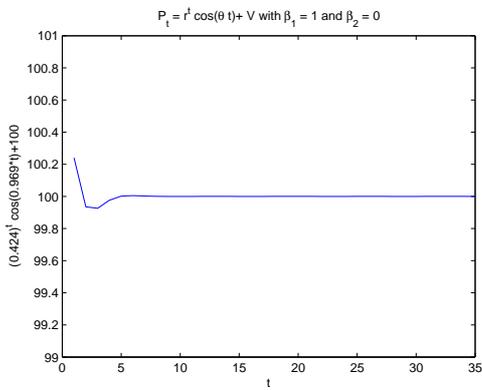
on the one hand, the greater z , the greater θ , so that price oscillations are more frequent, as illustrated in Fig. 2.16. On the other hand, the greater η , the smaller θ is, so that price oscillations are less frequent, as illustrated in Fig. 2.17. Mispricing duration is thus decreasing in the portion of arbitrageurs in the market. Mispricing duration is markedly lengthened especially when the portion of trend followers is high ($z > 0.5$) and the portion of arbitrageurs is low ($\eta < 0.5$) in the market.



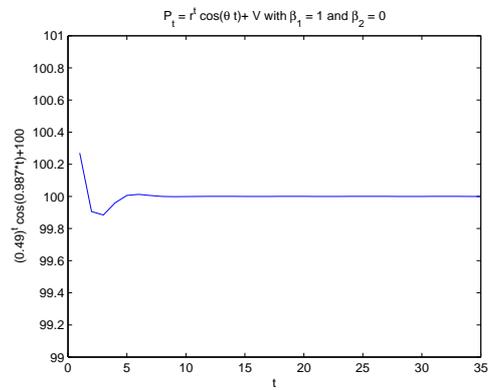
(a) $z = 0.6$



(b) $z = 0.7$



(c) $z = 0.8$



(d) $z = 0.9$

Figure 2.17: Behavior of the general solution when $\eta = 0.7$, for differing values of z .

This finding suggests that, under the conditions on the values of η and z so that $d < 0$, as depicted in Fig. 2.9, trend following strategies tend to generate price oscillations especially when the portion of arbitrageurs in the market is low ($\eta < 0.5$).

2.5 Comparative Statics

In the previous section, the study of our deterministic model driving the price dynamics was focused on the kind of price dynamics that may emerge in the market for differing compositions of the population. In the following subsections, we turn to study the effect of the other parameters of the model on the price dynamics. Indeed, in the investigation presented previously, the values of the reaction coefficients β in eq. (2.1) and φ in eq. (2.2) and eq. (2.3) as well as the value of the price adjustment parameter μ in eq. (2.4) were held constant. More precisely, the parameter values were set as follows: $\beta = 1$, $\varphi = 1$ and $\mu = 1/N$ with $N = 1000$. At this point, it is worth stressing that the foregoing parameter values have been carefully chosen so as to simultaneously satisfy the stability conditions of the system established in Section 2.3. Indeed, these parameter values, jointly with the conditions on η and z , ensure that the $\{P_t\}$ sequence always converges towards the asset fundamentals.

In order to complete the study of our model and check the robustness of the above-presented findings, it is worth studying the price dynamics which emerge when $\beta \neq 1$, $\varphi \neq 1$ and $\mu \neq 1/N$. This would enable us to assess whether the system is stable only for a limited range of these parameter values or rather, whether the findings presented above are robust.

2.5.1 The Role of the Reaction Coefficient in Arbitrageur Orders

In order to assess the effect of the value of the reaction coefficient β in eq. (2.1) on the price dynamics, in this subsection, we focus on the price dynamics that emerge in the market for differing values of β . Careful examination of the stability of the system reveals that, while when $\beta = 1$, the $\{P_t\}$ sequence, given by eq. (2.6b) and eq. (2.6), is convergent,²⁸ the system is also stable for a wide range of β values ($0.2 \leq \beta \leq 1.8$).²⁹ However, in what follows, we present the main findings of this study for some specific values of β , namely, 0.2, 0.5, 0.8, 1.2, 1.5 and 1.8.³⁰

For this purpose, we examine the most relevant settings³¹ which enable us to assess the effect of the value of β on the price dynamics. First, we concentrate on the setting where there are only arbitrageurs in the market (*i.e.*, $\eta = 1$). Indeed, with $\eta = 1$, we can focus on the effect of arbitrage strategies on the price dynamics and assess whether and when arbitrageurs are able to bring back the asset price towards the asset fundamentals, when chartists are not present in the market. Second, we concentrate on the setting where the chartist population is split between trend followers and contrarian traders (*i.e.*, $z = 0.5$). Indeed, with

²⁸The reader has to keep in mind that we have previously detected that when $\eta = 1$, eq. (2.6a) is no more a difference equation if $\beta = 1$.

²⁹Since when $\eta = 1$ and $\beta = 1$, eq. (2.6a) is no more a difference equation, such a value of β has been excluded from our study when $\eta = 1$. However, this restriction only concerns the particular case in which $\eta = 1$ and does not hold for any other composition of the population. Besides, $\beta = 1$, whenever $\eta \neq 1$, appears to be a useful benchmark when investigating the effect of the value of β on the price dynamics. We therefore deliberately choose to keep $\beta = 1$ as a key benchmark, even when $\eta = 1$, in order to ensure some uniformity when presenting the results.

³⁰We only focus on β values which ensure stability of the system. Values of β out of this range would lead to a non-convergent time path of the asset price, which are precluded from this study.

³¹These settings are chosen so that the parameter β can have an effect on price dynamics.

$z = 0.5$, since chartist strategies do not further affect the evolution of the asset price,³² we can assess the effect of arbitrage strategies on the price dynamics for differing values of η . The study of this setting also enables us to assess whether and when arbitrageurs are able to bring back the asset price towards the asset fundamentals, when chartists are present in the market.

First, when there are only arbitrageurs in the market ($\eta = 1$) and $\beta = 1$, given the other parameter values, as illustrated in Fig. 2.1, the asset price goes back towards the asset fundamentals and price correction arises almost immediately. However, for differing values of β , two distinct behaviors of the time path of the $\{P_t\}$ sequence, given by eq. (2.14a), emerge.

On the one hand, when $\beta < 1$, price correction is not immediate however first, the convergence towards the asset fundamental value is *monotonic*; second, mispricing duration is **decreasing** in the value of β . Fig. 2.18 shows the behavior of the general solution, given by eq. (2.14a),³³ when $\eta = 1$ and $\beta < 1$.

In fact, the *lower* the value of β , the longer the mispricing lasts. This is merely explained by the fact that $\beta < 1$ in eq. (2.1) dampens the effect of arbitrageur's orders on price dynamics, so that arbitrageurs need more time to correct the mispricing.

On the other hand, when $1 < \beta < 2$, price convergence towards the asset fundamental value is *not monotonic*. The time path of the general solution, given by eq. (2.14a), appears to oscillate explosively before the oscillations dampen and

³²The reader has to keep in mind the above-mentioned remark *i.e.*, when $z = 0.5$, trend follower orders and contrarian trader ones exactly compensate.

³³Indeed, when $\eta = 1$, eq. (2.6) becomes of the first order as described in eq. (2.6a).

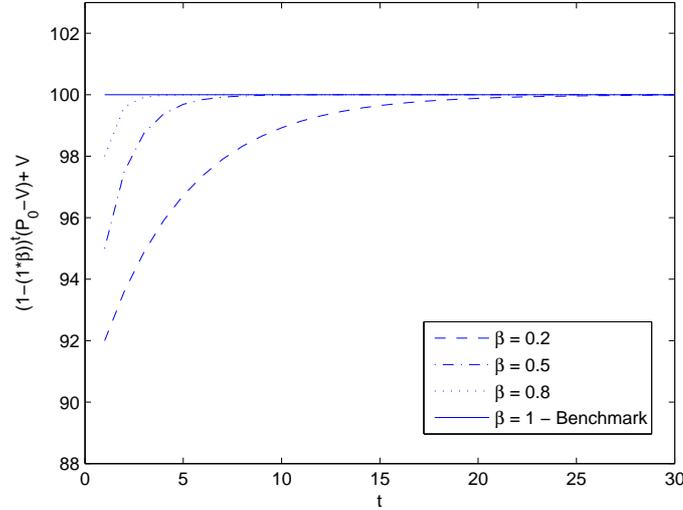


Figure 2.18: Behavior of the general solution when $\eta = 1$ for differing values of β ($\beta < 1$).

eventually converge to the asset fundamentals.³⁴ Fig. 2.19 illustrates the time path of the $\{P_t\}$ sequence when $\eta = 1$ and $\beta > 1$.

Furthermore, mispricing duration is now **increasing** in the value of β . The *greater* the value of β , the longer the mispricing lasts. This is merely explained by the fact that $\beta > 1$ in eq. (2.1) amplifies arbitrageur's orders, which leads to price oscillations in the short run. As a result, price convergence lasts longer.

Overall, mispricing duration appears to be lengthened as soon as the value of β departs from $\beta = 1$. This finding reveals that the value of β may markedly affect the price dynamics and may be crucial in explaining price oscillations in the short run. Nevertheless, in our model, the system is stable for a broad range of β values and the convergence of the $\{P_t\}$ sequence is robust for differing values of β .

³⁴When $\beta \geq 2$, the $\{P_t\}$ sequence never converges, since the stability conditions of the model are no more fulfilled, which justifies that these other cases are not considered in this work.

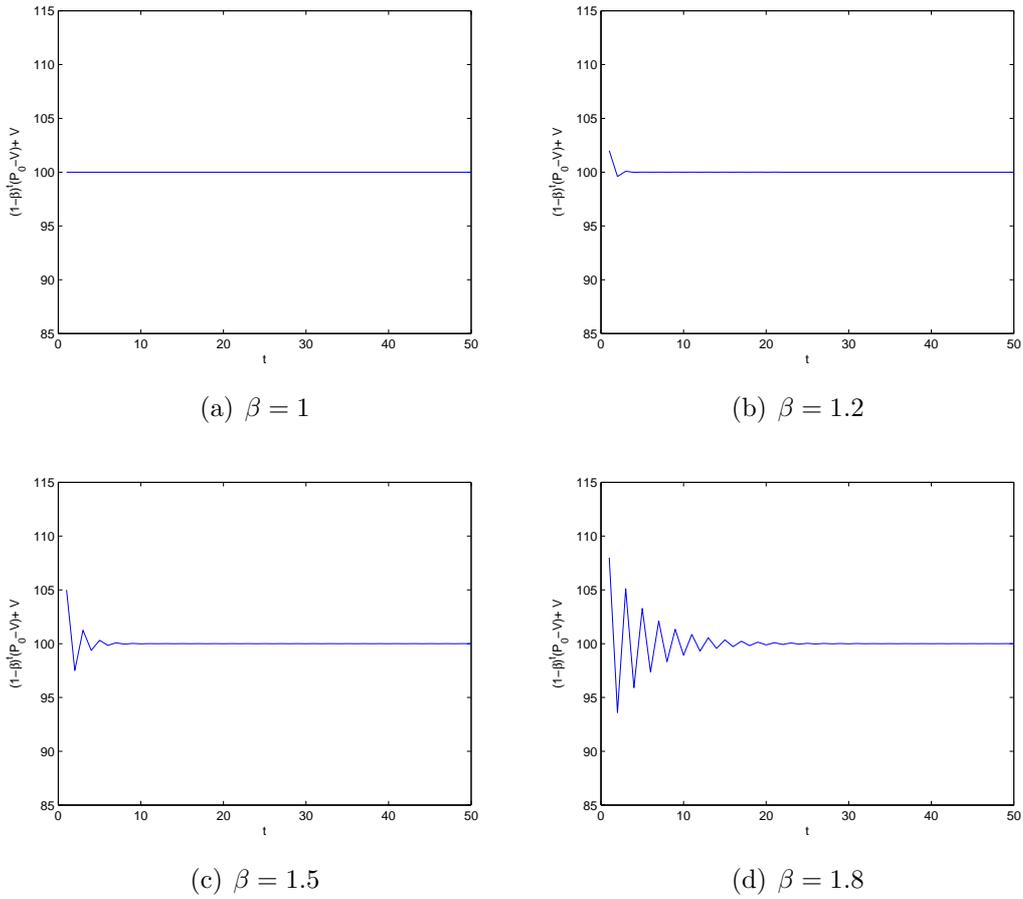


Figure 2.19: Behavior of the general solution when $\eta = 1$ for differing values of β ($\beta > 1$).

Second, when the chartist population is split between trend followers and contrarian traders ($z = 0.5$), as shown in the study of the stability conditions of the model, eq. (2.6) becomes of the first order. The general solution is given by eq. (2.6b) with $a_1 = 1 - \mu N \eta \beta$. So when $z = 0.5$, given the values of μ , N , β and φ , mispricing duration mainly depends on the portion of arbitrageurs (η) in the market. Careful examination of the time path of the $\{P_t\}$ sequence when $z = 0.5$ reveals that, as illustrated in Fig. 2.2, mispricing duration is **decreasing** in the

portion of arbitrageurs in the market. The *larger* the portion of arbitrageurs in the market, the shorter the mispricing lasts.

We now turn to investigate the effect of the value of β on the price dynamics when the chartist population is split between trend followers and contrarian traders. In this case, the main findings actually fall down to three cases.

First, when $\beta < 1$, as well as when $\beta = 1$, mispricing duration is decreasing in the portion of arbitrageurs in the market. However, mispricing duration is **decreasing** in the value of β , so that mispricing correction occurs later than in the benchmark ($\beta = 1$). Fig. 2.20 shows the behavior of the general solution when $z = 0.5$ and $\eta = 0.5$, for differing values of β .

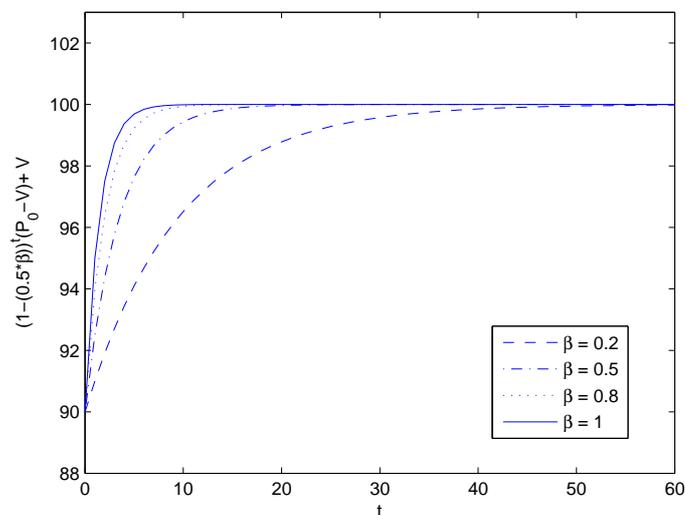


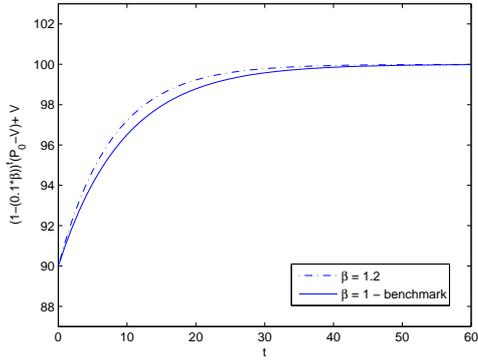
Figure 2.20: Behavior of the general solution for differing values β when $z = 0.5$ and $\eta = 0.5$.

The smaller β , the longer the mispricing lasts in the market.³⁵ More precisely, careful comparison of mispricing duration when $\beta < 1$ and when $\beta = 1$ shows that while when $\beta = 0.2$ ($\beta = 0.5$), mispricing duration is multiplied by almost three (two), when $\beta = 0.8$, mispricing duration is only slightly greater than when $\beta = 1$. Second, when $\beta > 1$, two main patterns emerge. On the one hand, when $1 < \beta \leq 1.5$, as when $\beta = 1$, mispricing duration is decreasing in the portion of arbitrageurs in the market. However, it is worth noting that whatever the portion of arbitrageurs in the market, mispricing correction occurs before than in the benchmark (*i.e.*, when $\beta = 1$). Furthermore, mispricing duration tends to be decreasing in the value of β . Fig. 2.21 and Fig. 2.22 show the behavior of the general solution to eq. (2.6b) when $\beta = 1.2$ and $\beta = 1.5$, respectively, for differing values of η .

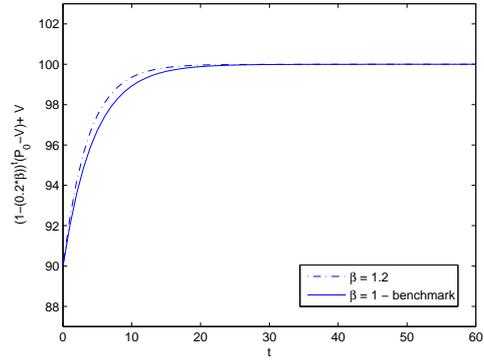
Mispricing duration is in fact decreasing in η as well as in β . The greater the value of β , the shorter the mispricing lasts. When $\beta > 1$, mispricing duration may be markedly reduced.

On the other hand, when $1.5 < \beta < 2$, a distinct and singular pattern emerges. First, while in previous cases, mispricing duration was decreasing in the portion of arbitrageurs in the market, for $\beta > 1.5$, this is no longer the case. Instead, mispricing duration is now increasing in the portion of arbitrageurs when $\eta > 0.5$. Second, when $1.5 < \beta < 2$, mispricing duration is markedly shorter than in the benchmark ($\beta = 1$) when the portion of arbitrageurs in the market is low ($\eta \leq 0.5$). However, when $\beta \leq 1.5$, mispricing duration is decreasing in the portion of arbitrageurs. Nevertheless, when $\eta > 0.5$, mispricing duration is longer than in the benchmark ($\beta = 1$), as illustrated in Fig. 2.23.

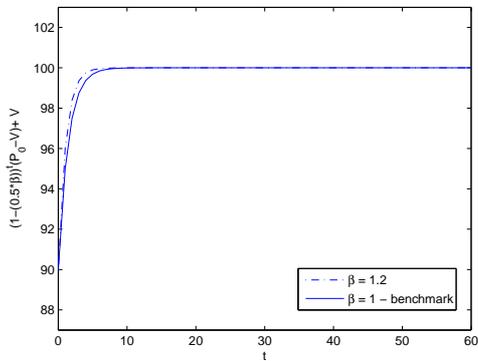
³⁵The rationale for the negative relationship between mispricing duration and β is actually the same as when $\eta = 1$.



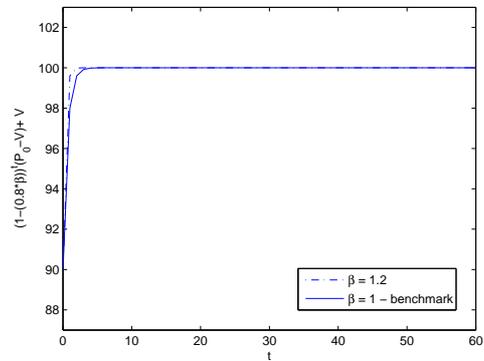
(a) $\eta = 0.1$



(b) $\eta = 0.2$



(c) $\eta = 0.5$



(d) $\eta = 0.8$

Figure 2.21: Behavior of the general solution when $\beta = 1.2$ and $\beta = 1$, for differing values of η .

The increase in mispricing duration when $1.7 < \beta < 2$ and $\eta > 0.5$ is explained by a noteworthy phenomenon that emerges for these parameter values. In fact, in this case, the general solution to eq. (2.6b) appears to oscillate explosively before the oscillations dampen and eventually converge to the asset fundamentals. Dampened oscillations are even more pronounced when the portion of arbitrageurs is large ($\eta > 0.5$). This specificity is the reason for the increase in mispricing duration when $1.7 < \beta < 2$. Our model is therefore able to reproduce price overshooting,

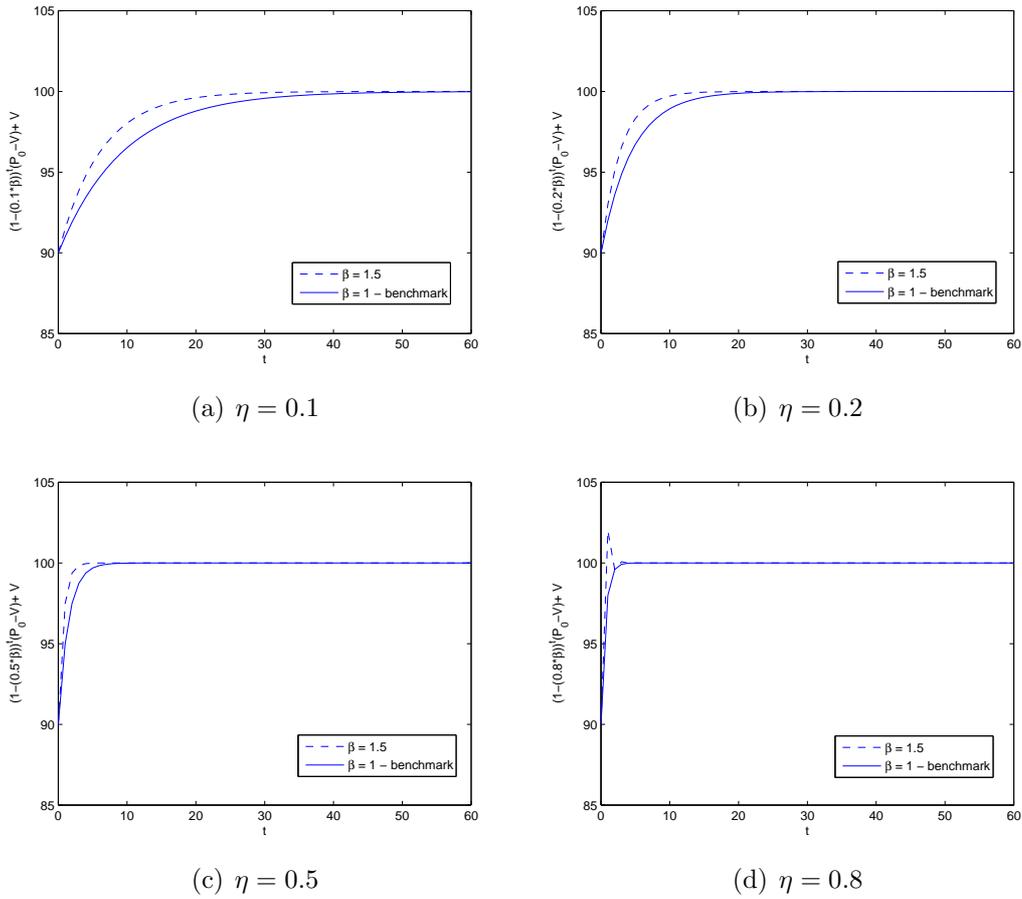
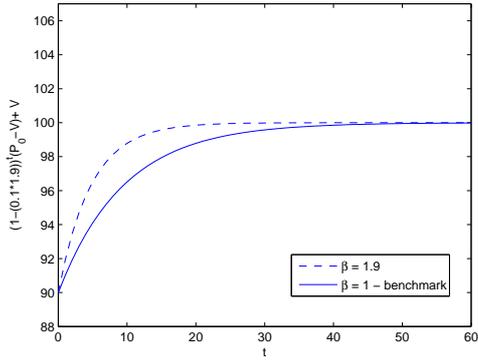


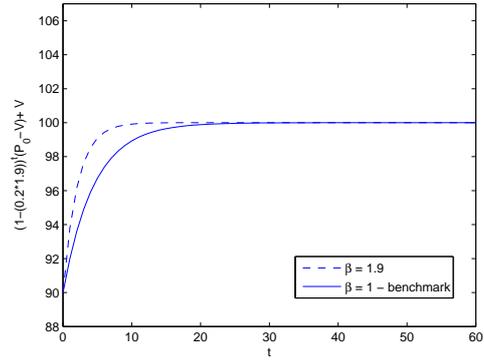
Figure 2.22: Behavior of the general solution when $\beta = 1.5$ and $\beta = 1$, for differing values of η .

which is often observed in financial time series. Lastly, the relationship between mispricing duration and the value of β is no more linear. More precisely, for low portions of arbitrageurs ($\eta < 0.5$), mispricing is **decreasing** in the value of β . Rather when the portion of arbitrageurs in the market is large ($\eta > 0.5$), mispricing duration tends to be **increasing** in the value of β .

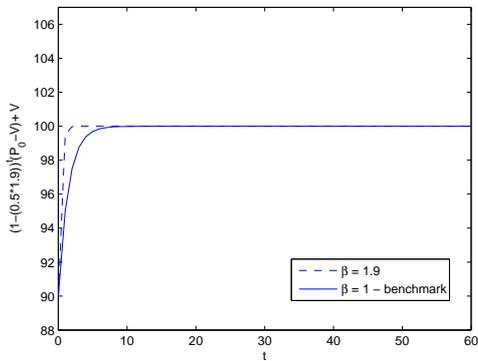
Overall, from the study of the effect of the value of β on the price dynamics, we can assert that first, the parameter β can markedly affect price dynamics and



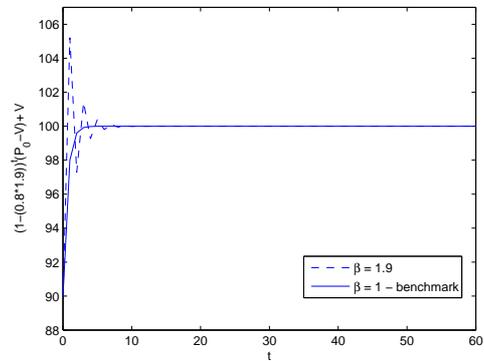
(a) $\eta = 0.1$



(b) $\eta = 0.2$



(c) $\eta = 0.5$



(d) $\eta = 0.8$

Figure 2.23: Behavior of the general solution when $\beta = 1.9$ and $\beta = 1$, for differing values of η .

amplify the effect of arbitrage strategies; second, the system is stable for a wide range of β values. However, since this work aims at trying to understand the effect of the interplay between arbitrageurs and chartists on the price dynamics, any further effect that may be brought about by exogenous factors, such as for instance β values, has to be precluded from this work. Indeed, this would constitute a bias in our analysis. This finding further justifies the use along this work of $\beta = 1$; value which ensures that such an exogenous bias does not arise.

2.5.2 The Role of the Reaction Coefficient in Chartist Orders

In the previous subsection, we have examined the price dynamics that may emerge in the market for a large range of β values. We have seen that *(i.)* the system is stable not only when $\beta = 1$; *(ii.)* the parameter β may play a key role in explaining mispricing duration. We now turn to assess the effect of the value of φ on the price dynamics. Indeed, in previous sections, the value of φ was set to 1. With this purpose in mind, we study the price dynamics that emerge in the market for differing values of the positive reaction coefficient φ from eq. (2.2) and eq. (2.3). Careful examination of the stability of the system reveals that, while when $\varphi = 1$, the $\{P_t\}$ sequence, given by eq. (2.6a), eq. (2.6b) and eq. (2.6), is convergent for a large range of η and z values, the system is actually also stable for differing values of φ . More precisely, the values of φ that ensure that the system is stable range from 0.2 to 1.5.³⁶

In this investigation, the most relevant settings, determined by the composition of the population in the market, on which we concentrate are: first, the setting in which the chartist population is composed only of trend followers (*i.e.*, $z = 1$). Indeed, when $z = 1$, we can focus on the effect of trend following strategies as well as assess the effect of the parameter φ on the price dynamics. This enables us to investigate the price dynamics induced by the interplay between arbitrageurs and

³⁶As for the study of the effect of the value of β on the price dynamics, in what follows, we only consider parameter values that satisfy the stability conditions identified in Section 2.3 and 2.4. At this point, it is worth mentioning that the range of φ values considered is smaller than the range of β values. Indeed, when $\varphi > 1.5$, the $\{P_t\}$ sequence is explosive and never converges to the asset fundamentals.

trend followers. Second, the setting where the chartist population is composed only of contrarian traders (*i.e.*, $z = 0$). Indeed, when $z = 0$, we can focus on the effect of contrarian strategies as well as assess the effect of the parameter φ on the price dynamics. This enables us to investigate the price dynamics induced by the interplay between arbitrageurs and contrarian traders.

First, when the chartist population is only composed of trend followers ($z = 1$), the general solution to eq. (2.6) is given by eq. (2.27).³⁷ The examination of the behavior of this general solution, when $z = 1$ and $\varphi = 1$, clearly reveals that mispricing duration is decreasing in the portion of arbitrageurs in the market. The smaller the portion of arbitrageurs (η), the longer the mispricing lasts. Besides, across all possible values of the parameter η , price convergence towards the asset fundamental value occurs through dampened oscillations, due to the fact that the asset price appears to explode before eventually converging. Trend following strategies (since $z = 1$) thus tend to generate oscillations in prices and, as a result, markedly increase mispricing duration. Fig. 2.24 shows the time path of the general solution when $\varphi = 1$ and $z = 1$, for differing values of η .

For differing values of φ , as when $\varphi = 1$, mispricing duration is decreasing in the portion of arbitrageurs in the market. However, when $\varphi \neq 1$, two additional results are detected: (*i.*) when $\varphi < 1$, mispricing correction occurs before than in the benchmark ($\varphi = 1$); (*ii.*) when $1 < \varphi < 2$, mispricing correction occurs after than in the benchmark ($\varphi = 1$).

³⁷Indeed, when $z = 1$, the discriminant is negative and the homogeneous solution is given by eq. (2.23).

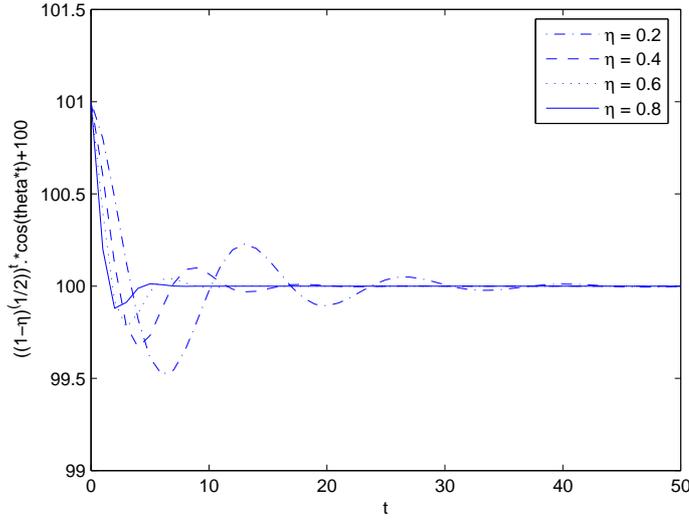


Figure 2.24: Behavior of the general solution for differing values η when $\varphi = 1$ and $z = 1$.

On the one hand, when $\varphi < 1$, it is worth mentioning that, while mispricing correction occurs before than in the benchmark, the behavior of the general solution to eq. (2.6), given by eq. (2.27), exhibits two distinct patterns. First, for $\varphi < 0.6$, dampened oscillations towards convergence completely disappear. Fig. 2.25 illustrates the behavior of the general solution, given by eq. (2.27), when $\eta = 0.2$ for differing values of φ ($\varphi < 1$).³⁸ Fig. 2.25a shows the time path when $\varphi < 0.6$. Rather Fig. 2.25b shows the time path when $\varphi > 0.5$.

Indeed, even in the case where the dampened oscillations are the most pronounced (*i.e.*, $\eta = 0.2$), from Fig. 2.25a, it can be seen that when $\varphi < 0.6$, oscillations in price almost disappear. This is likely to explain the reduction in mispricing duration when $\varphi < 1$ with respect to the benchmark ($\varphi = 1$). This is merely explained by the fact that when $\varphi < 1$, chartist - more important in this case trend follower

³⁸This value of η has been chosen knowing that the lower the portion of arbitrageurs, the larger the dampened oscillations are.

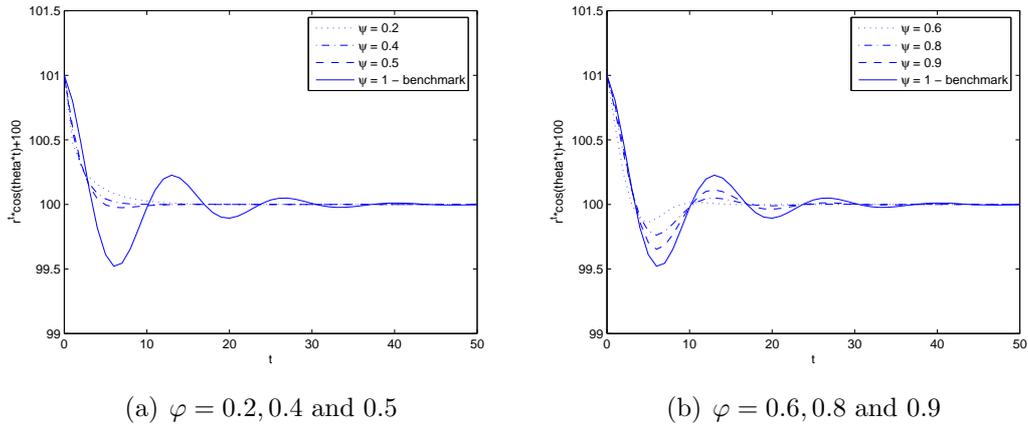


Figure 2.25: Behavior of the general solution for differing values of φ ($\varphi < 1$) and $\eta = 0.2$.

- orders are markedly reduced, so that the effect of trend following strategies is noticeably lessened. Furthermore, the smaller φ , the greater the reduction in mispricing duration is.

Second, from $\varphi = 0.6$, as shown in Fig. 2.25b, the time path of the $\{P_t\}$ sequence exhibits anew dampened oscillations for low values of η . This is likely to explain that, when the portion of arbitrageurs is low in the market, mispricing duration decrease, with respect to the benchmark ($\varphi = 1$), is weaker than when the portion of arbitrageurs is larger.

On the other hand, when $\varphi > 1$, mispricing duration tends to be increasing in the value of φ . Fig. 2.26 illustrates the behavior of the general solution to eq. (2.6), given by eq. (2.27), when $\eta = 0.5$, for differing values of φ ($\varphi > 1$).

Indeed, the greater φ , the longer the mispricing lasts. This is mainly explained by the fact that when $\varphi > 1$, the behavior of the general solution to eq. (2.6) is widely erratic, as illustrated in Fig. 2.26. The time path of the $\{P_t\}$ sequence

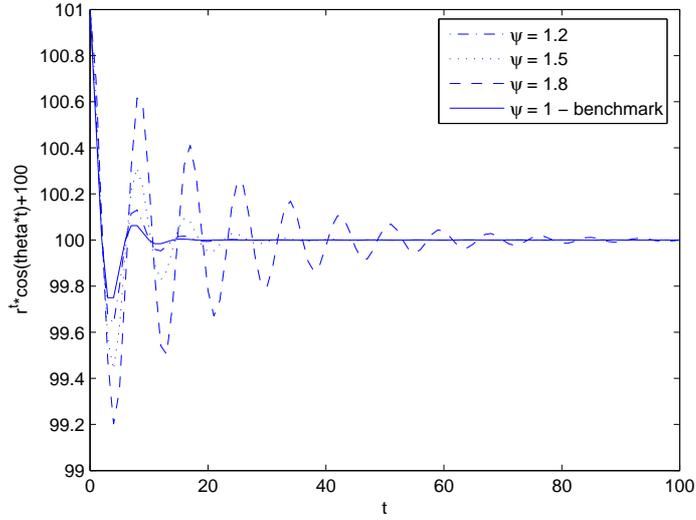


Figure 2.26: Behavior of the general solution for differing values of φ ($\varphi > 1$) and $\eta = 0.5$.

oscillates explosively before the oscillations dampen and finally converge to the asset fundamentals. Price oscillations then markedly lengthen mispricing duration.³⁹ Overall, the behavior of the general solution is merely explained by the fact that when $\varphi > 1$, chartist - more important in this case trend follower - orders are noticeably strengthened, so that the effect of trend following strategies on the price dynamics is markedly amplified. As a result, mispricing duration increases.

Second, when the chartist population is only composed of contrarian traders ($z = 0$), the general solution to eq. (2.6) is rather given by eq. (2.13).⁴⁰ Careful examination of the behavior of this general solution when $\varphi = 1$ clearly reveals

³⁹It is worth noting that for low values of η (*i.e.*, $0.1 \leq \eta \leq 0.2$), the behavior of the asset price over time is now explosive. This observation suggests an additional restriction on the stability conditions identified in subsection 2.3.2.

⁴⁰Indeed, when $z = 0$, the discriminant is positive and the homogeneous solution is given by $A_1(\alpha_1)^t + A_2(\alpha_2)^t$.

that mispricing duration is decreasing in the portion of arbitrageurs in the market. The smaller the portion of arbitrageurs (η), the longer the mispricing lasts. Fig. 2.27 shows the behavior of the general solution given by (2.13) when $\varphi = 1$, for differing values of η .

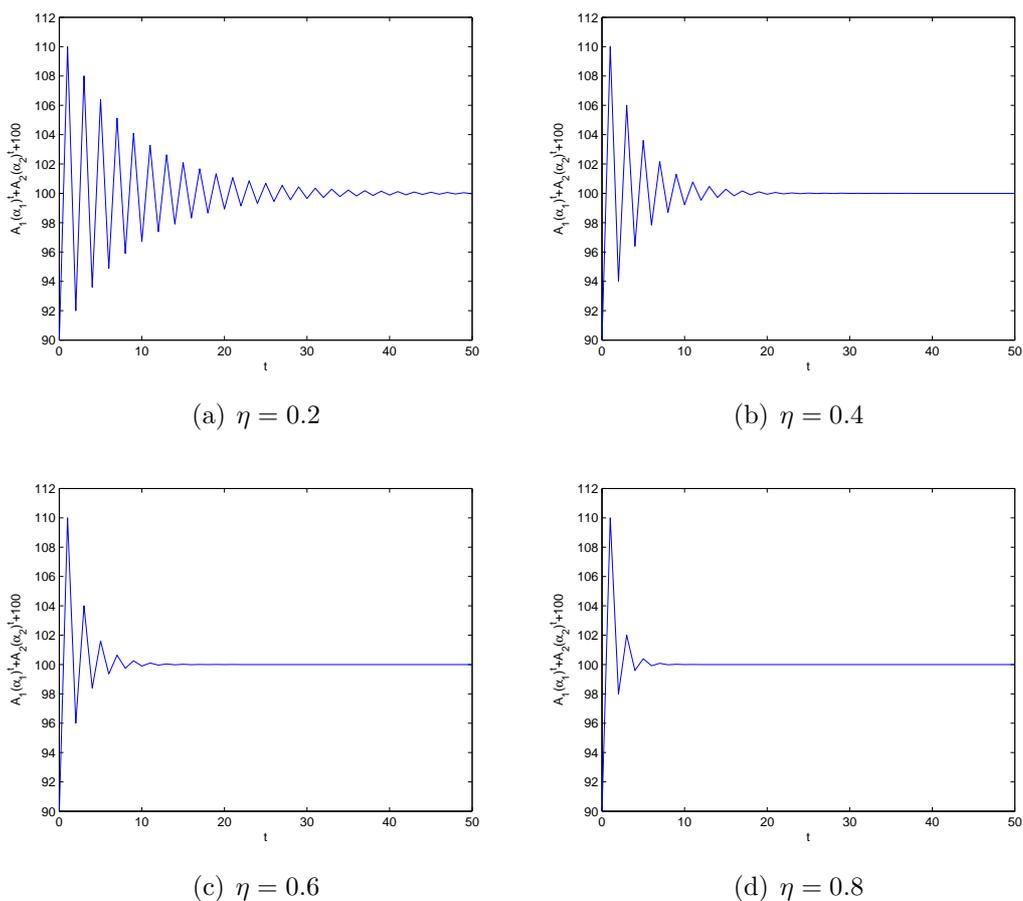


Figure 2.27: Behavior of the general solution for differing values of η and $\varphi = 1$.

As identified in Section 2.3, for these parameter values, the general solution converges. However, the convergence of the $\{P_t\}$ sequence is not monotonic. Contrarian strategies appear to generate price oscillations. Furthermore, from Fig. 2.27,

it is possible to observe that the extent of price oscillations is clearly decreasing in η . Arbitrageur strategies tend to dampen price oscillations and favor mispricing correction.

For differing values of φ , as when $\varphi = 1$, careful examination of the behavior of the general solution (2.13) clearly reveals that mispricing duration is decreasing in the portion of arbitrageurs in the market. More important, when $\varphi < 1$, mispricing correction tends to occur before than in the benchmark ($\varphi = 1$); rather when $\varphi > 1$, mispricing correction occurs later than in the benchmark. Fig. 2.28 shows the time path of the $\{P_t\}$ sequence when for instance $\eta = 0.5$, for differing values of φ ($\varphi < 1$).

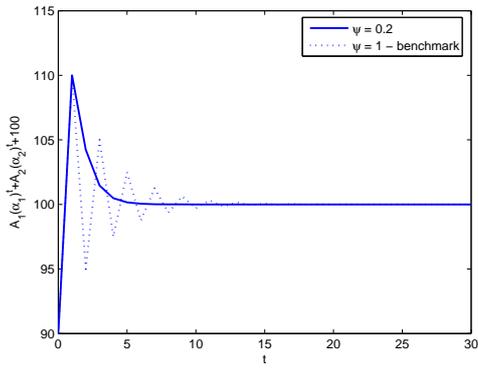
Mispricing duration is markedly reduced when $\varphi < 0.5$. When $\varphi > 0.5$, mispricing duration is almost the same as in the benchmark ($\varphi = 1$).

Fig. 2.29 rather shows the time path of the $\{P_t\}$ sequence when for instance $\eta = 0.5$, for differing values of φ ($\varphi > 1$).⁴¹

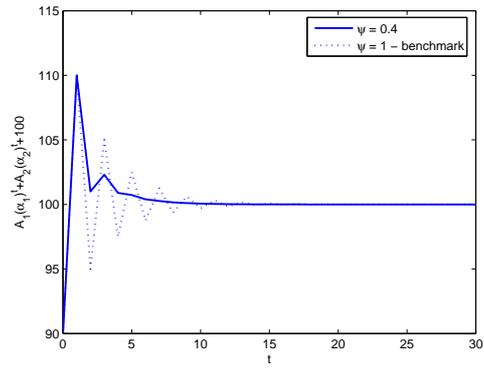
When $\varphi > 1$, mispricing duration is increasing in the value of φ and mispricing correction tends to occur later than in the benchmark ($\varphi = 1$). Indeed, when $\varphi > 1$, chartist - more important in this case contrarian trader - orders are noticeably strengthened, so that the effect of contrarian strategies on the price dynamics is markedly amplified.

Overall, from the study of the effect of the value of φ on the price dynamics, we assert that first, the parameter φ can markedly affect the price dynamics and

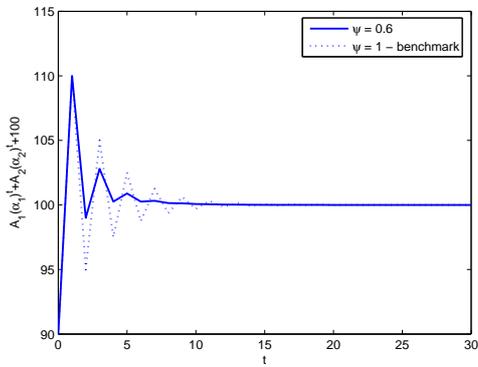
⁴¹However, it is worth mentioning that when $\varphi > 1$ and $\eta < 0.5$, the time path of the asset price is explosive and never converges towards its fundamental value.



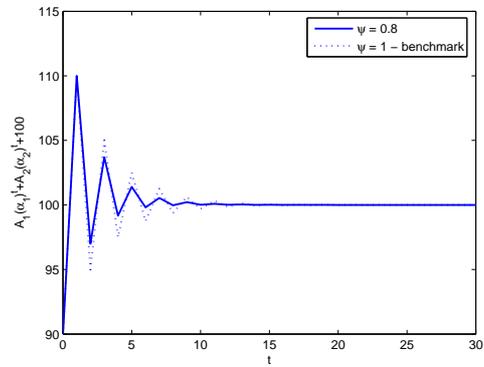
(a) $\varphi = 0.2$



(b) $\varphi = 0.4$



(c) $\varphi = 0.6$



(d) $\varphi = 0.8$

Figure 2.28: Behavior of the general solution for differing values of φ ($\varphi < 1$) and $\eta = 0.5$.

amplify the effect of chartist strategies; second, the system is stable for a broad range of φ values. However, since the aim of this work is to try to understand the effect of the interplay between arbitrageurs and chartists, any exogenous factor that could affect the price dynamics would represent a bias in our analysis. This further justifies the use along this work of $\varphi = 1$, which ensures that such an exogenous bias on the price dynamics is precluded from our study.

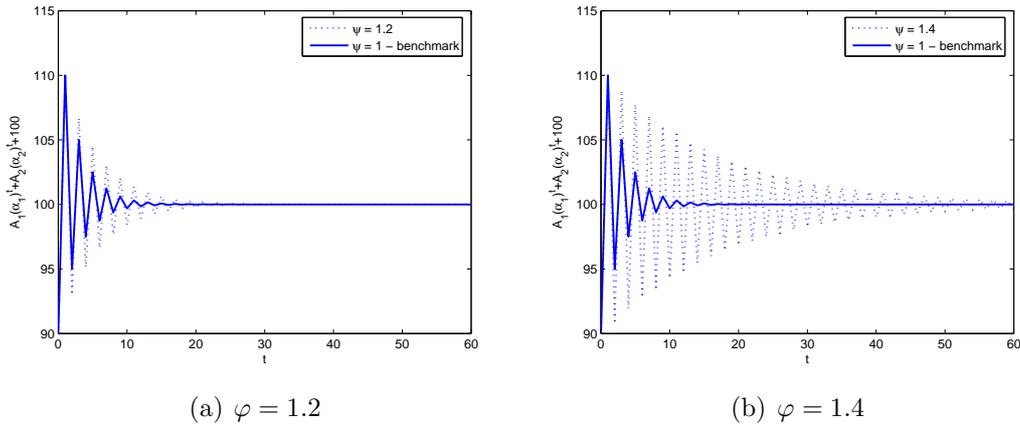


Figure 2.29: Behavior of the general solution for differing values of φ ($\varphi > 1$) and $\eta = 0.5$.

2.5.3 The Role of the Price Adjustment Parameter

In the previous subsection, we have examined the price dynamics that may emerge in the market for a large range of φ values. We have seen that *(i.)* the system is stable not only when $\varphi = 1$; *(ii.)* the parameter φ plays a key role in explaining price oscillations and mispricing duration. We now turn to assess the effect of the value of the price adjustment parameter μ , from eq. (2.4), on price dynamics. Indeed, in the previous sections, the value of μ was set to $1/N$ with $N = 1000$. To this end, we study the price dynamics that emerge in the market for differing values of the parameter μ . Careful examination of the effect of the parameter μ on the price dynamics reveals that the system is stable not only when $\mu = 1/N$. Rather when $0.0005 \leq \mu \leq 0.0015$, the system is still stable and the $\{P_t\}$ sequence converges towards the asset fundamentals.⁴²

⁴²As for the study of the effect of the value of β and φ on price dynamics, we only consider parameter values that satisfy the stability conditions identified in Section 2.3 and 2.4. Indeed, μ values that do not satisfy the stability conditions of the system would lead to non-convergent time path of the asset price, which is out of the scope of this study.

In this investigation, the most relevant settings, determined by the composition of the population in the market, on which we concentrate are: first, the setting in which there are only arbitrageurs in the market (*i.e.*, $\eta = 1$). Indeed, when $\eta = 1$, we can focus on the effect of arbitrage strategies on the price dynamics as well as the effect of μ . Second, the setting in which the chartist population is composed only of trend followers (*i.e.*, $z = 1$). Indeed, we can focus on the effect of trend following strategies on price dynamics as well as the effect of μ . Third, the setting where the chartist population is composed only of contrarian traders (*i.e.*, $z = 0$). Lastly, we concentrate on the setting where the chartist population is split between trend followers and contrarian traders. When $z = 0.5$, this enables us to focus on the effect of arbitrage strategies, while chartists are present in the market.

First, when there are only arbitrageurs in the market and $\mu = 0.001$,⁴³ as shown in subsection 2.3.1 and illustrated in Fig. 2.1, the asset price goes back towards the asset fundamental value. From eq. (2.14a), $P_t = \bar{V}$. Price correction is thus immediate.

However, for differing values of μ , price correction is no longer instantaneous. Fig. 2.30 shows the time path of the $\{P_t\}$ sequence when $\eta = 1$, for differing values of μ . Indeed, when $\mu < 0.001$, the convergence of the $\{P_t\}$ sequence slowly converges towards V_t and price convergence is monotonic. However, when $\mu > 0.001$, the asset price tends to overshoot before eventually converge. Price convergence now occurs through dampened oscillations.

⁴³It is worth stressing that this value of the price adjustment parameter is not completely exogenous, rather depends on the number of agents in the market *i.e.*, $\mu = 1/N$ with $N = 1000$. This is a condition which ensures that the value of the price adjustment parameter does not introduce an exogenous effect on the price dynamics.

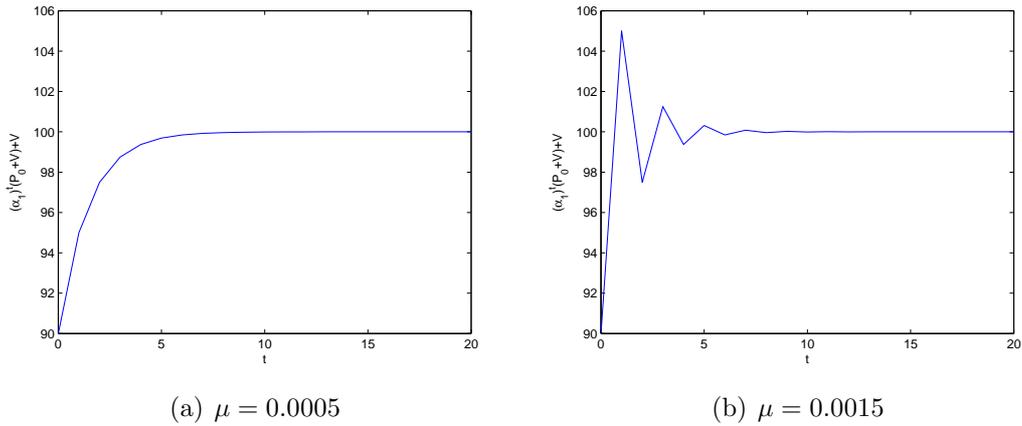


Figure 2.30: Behavior of the general solution for differing values of μ and $\eta = 1$.

However, it is worth noting that whereas the behavior of the general solution to eq. (2.14a) crucially depends on the value of μ , mispricing duration is almost the same whenever $\mu > 1/N$ or $\mu < 1/N$. Mispricing duration rather increases when $\mu \neq 1/N$.

Second, when the chartist population is only composed of trend followers ($z = 1$) and $\mu = 0.001$, as explained previously, mispricing duration is decreasing in the portion of arbitrageurs in the market. The smaller the portion of arbitrageurs, the longer the mispricing lasts. However, in this case, price convergence occurs through dampened oscillations, as illustrated in Fig. 2.24.

For differing values of μ , as when $\mu = 0.001$, mispricing duration is decreasing in the portion of arbitrageurs in the market. However, it is possible to observe that, while when $\mu < 0.001$, dampened oscillations completely disappear. Dampened oscillations however reappear when $\mu > 0.001$. Fig. 2.31 shows the behavior of the general solution to eq. (2.27) when $z = 1$, for differing values of μ .

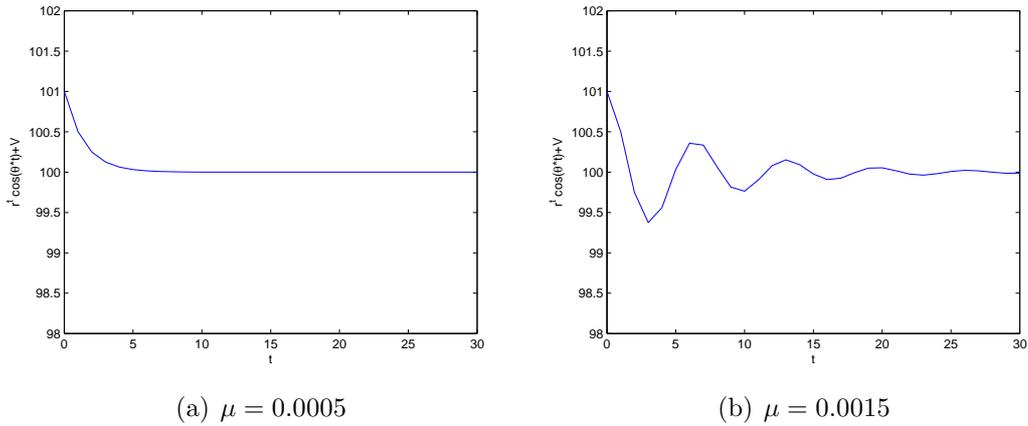


Figure 2.31: Behavior of the general solution for differing values of μ and $z = 1$.

The emergence of dampened oscillations, as shown in Fig. 2.31b, is mainly due to the fact that a greater value of the reaction coefficient μ in eq. (2.2) markedly amplifies the effect of trend following strategies, which tend to generate price oscillations on price dynamics.

Third, when the chartist population is only composed of contrarian traders ($z = 0$) and $\mu = 0.001$, mispricing duration is decreasing in the portion of arbitrageurs in the market. However, as explained in subsection 2.4.1, when $z = 0$, the convergence of the $\{P_t\}$ sequence towards the asset fundamental value is not monotonic. Fig. 2.32 shows the time path of the $\{P_t\}$ sequence when, for instance, $\eta = 0.5$. Indeed, the time path of the general solution to eq. (2.22) explodes before converging.

For differing values of μ , as for $\mu = 1/N = 0.001$, mispricing duration is decreasing in the portion of arbitrageurs in the market, as illustrated in Fig. 2.27. However, when $\mu < 0.001$, as illustrated in Fig. 2.33a, price convergence is much slower

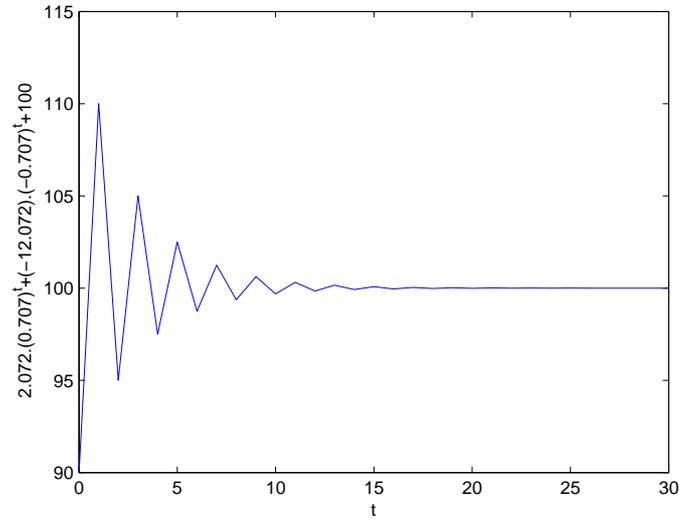
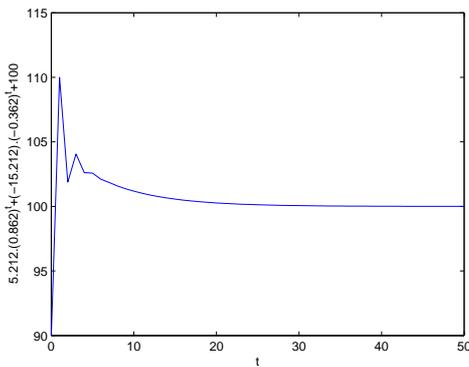
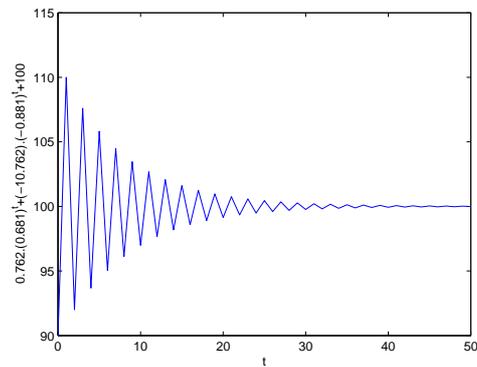


Figure 2.32: Behavior of the general solution when $\mu = 0.001$, $z = 0$ and $\eta = 0.5$.

than in the benchmark ($\mu = 0.001$). Rather, when $\mu > 0.001$, as illustrated in Fig. 2.33b, the behavior of the general solution to eq. (2.13) is widely erratic, and convergence towards the asset fundamental value is markedly lengthened with respect to the benchmark ($\mu = 0.001$).



(a) $\mu = 0.0005$



(b) $\mu = 0.0012$

Figure 2.33: Behavior of the general solution for differing values of μ and $z = 0$.

This is explained by the fact that, when $\mu > 1/N$, the effect of contrarian strategies on the price dynamics is markedly amplified.

Lastly, when the chartist population is split between trend followers and contrarian traders ($z = 0.5$) and $\mu = 0.001$, mispricing duration is decreasing in the portion of arbitrageurs in the market and price convergence is monotonic.

For differing values of μ , while for $\mu < 0.001$, price convergence is monotonic, for $\mu > 0.001$, price dynamics exhibit dampened oscillations towards V_t . Fig. 2.34 shows the time path of the $\{P_t\}$ sequence when $z = 0.5$ for differing values of μ , namely, $\mu = 0.0005$ (see Fig. 2.34a) and $\mu = 0.0015$ (see Fig. 2.34b).

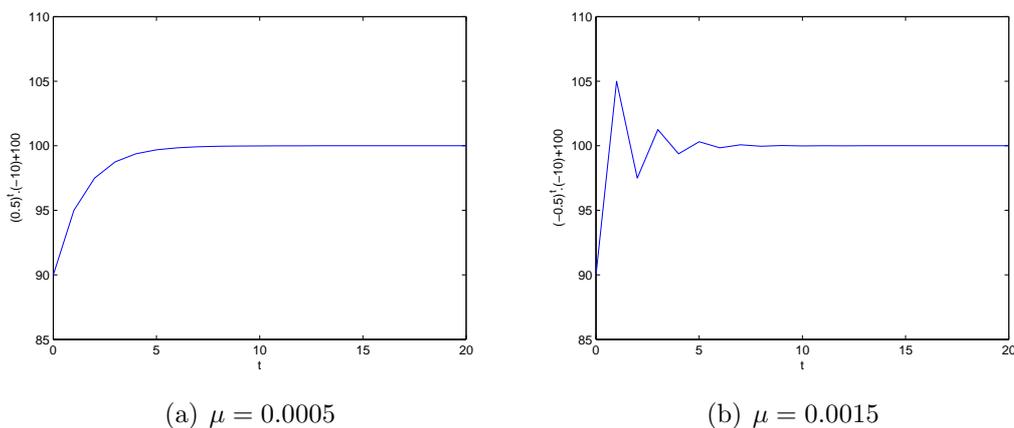


Figure 2.34: Behavior of the general solution for differing values of μ and $z = 0.5$.

Overall, from the study of the effect of the value of μ on the price dynamics, we assert that first, the parameter μ can markedly affect the price dynamics and amplify agents' orders; second, the system is stable for a large range of μ values. However, since this work aims at understanding the effect of the interplay between

arbitrageurs and chartists, any further effect that may be brought about by exogenous factors, such as μ values for instance, has to be precluded from this work. Indeed, this would introduce a bias in our analysis. This further justifies the use along this work of $\mu = 1/N$, which ensures that such an exogenous bias does not arise.

2.6 Summary and Discussion of Results

In order to investigate whether arbitrage strategies could bring the asset price back towards the asset fundamentals, while arbitrageurs have homogeneous beliefs about the asset fundamentals and chartists - both trend followers and contrarian traders - are present in the market, we have explored several situations. These settings have enabled us to independently investigate the effect of each trading strategy on the price dynamics and eventually, starting from well-documented results, the price dynamics induced by the interplay between arbitrageurs and chartists. In discussing the results of the foregoing sections, we go through the main similarities and differences among the settings examined in this chapter.

First, when there are only well-funded and well-informed arbitrageurs in the market, price correction is immediate and asset mispricing does not persist *i.e.*, arbitrage *does* work. Our model actually reproduces one of the backbone of the efficient market hypothesis according to which asset prices should reflect any new information.

Second, when there are both arbitrageurs and chartists - both trend followers

and contrarian traders, but $z = 0.5$ - in the market, mispricing is decreasing in the portion of arbitrageurs in the market. Mispricing duration is markedly delayed only when chartists widely dominate the market. Arbitrageurs appear to play a key role in explaining mispricing correction, even when chartists are present in the market.

Third, when there are both arbitrageurs and chartists - both trend followers and contrarian traders, but $z \neq 0.5$ - in the market, mispricing is still decreasing in the portion of arbitrageurs in the market. However, while arbitrageurs still dominate ($\eta > 0.5$), trend followers as well as contrarian traders may markedly affect the price dynamics. More precisely, although trend followers are widely considered to destabilize markets (Lux, 1995, 1998; Lux and Marchesi, 1999, 2000; Farmer and Joshi, 2002), our settings enable us to extend such a result to contrarian strategies. Indeed, both trend following strategies and contrarian ones may generate price oscillations and so destabilize markets. More important, they can do so, even when arbitrageurs dominate the market.

While arbitrageurs play a crucial role in correcting asset mispricing when *(i.)* there is no chartist in the market; *(ii.)* when chartists do not really further affect the evolution of the asset price; when chartists - both trend followers and contrarian traders - markedly affect the price dynamics, the former may prevent arbitrageurs from correcting asset mispricing. Our settings are able to reproduce the emergence in the market of overvalued/undervalued assets which can be explained by the composition of the population and more important, by the presence of trend followers or contrarian traders.

Lastly, we have confirmed that our findings are robust. Indeed, the system is stable for a wide range of values of β , φ and μ . Furthermore, we have established that all these parameters have a significant influence on the price dynamics that emerge in the market. However, since this work aims at understanding the effect of the interplay between arbitrageurs and chartists on the induced price dynamics, any further effect that could affect the price dynamics would represent a bias in our analysis. Such an exogenous effect, once identified, has definitely to be precluded from our study. More precisely, the study of the role played by the parameters β , φ and μ on the price dynamics has enabled us to clearly identify the parameter values that ensure that no such exogenous influence is introduced in our analysis. The main findings are summarized in Table 2.1.

2.7 Conclusions

In this chapter, we have proposed a simple heterogeneous agent models, based on the bounded-rationality hypothesis. In line with earlier heterogeneous agent models, we make explicit the distinction between arbitrageurs (or *fundamentalists*) and chartists. The purpose of this work has been to assess one of the predictions of the efficient market hypothesis according to which asset prices should be consistent with fundamentals. Within a highly stylized model, we have been able to provide a qualitative description of asset price behaviors.

Our framework is able to reproduce, at least partially, the foregoing prediction of the efficient market hypothesis. More precisely, when there are only well-funded

Main features	Main findings
<p>Benchmark 1</p> <p>Arbitrageurs only</p>	<ul style="list-style-type: none"> • Mispricing correction occurs almost immediately;
<p>Benchmark 2</p> <p>Arbitrageurs and chartists</p> <p>Chartist strategies do not further affect the price dynamics (<i>i.e.</i>, $z = 0.5$).</p>	<ul style="list-style-type: none"> • Mispricing duration is decreasing in the portion of arbitrageurs in the market; • Mispricing correction is markedly delayed only if chartists dominate.
<p>Arbitrageurs and chartists</p> <p>Chartist strategies further affect the price dynamics (<i>i.e.</i>, $0 \leq z \leq 1$).</p>	<ul style="list-style-type: none"> • Mispricing duration is still decreasing in the portion of arbitrageurs in the market; • Mispricing duration is increasing in the portion of trend followers as well as the portion of contrarian traders in the market; • Both trend following strategies and contrarian ones may generate price oscillations, even when arbitrageurs dominate the market.

Table 2.1: Main findings when arbitrageurs have homogeneous beliefs.

and fully informed arbitrageurs, asset prices instantaneously reflect new information about the fundamentals. In contrast, when chartists are present in the market, this is no more the case. However, the extent of market efficiency crucially depends on the portion of arbitrageurs in the market. Indeed, when the chartist population is split between trend followers and contrarian traders, mispricing duration is decreasing in the portion of arbitrageurs in the market. Besides, mispricing duration is markedly delayed only when chartists widely dominate. This finding actually supports earlier works which suggest that the presence of fundamentalists tends to stabilize market prices (e.g., Lux, 1995; Lux and Marchesi, 1999; Farmer and Joshi, 2002). Lastly, our framework is able to reproduce the occurrence of over- and under-valued assets, which can be explained by the presence of trend following as well as contrarian strategies.

In the following chapters, we relax some of the assumptions of the model discussed in this chapter. In chapter 3, for instance, we rather assume that arbitrageurs are not immediately informed about new information regarding the fundamentals. With this in mind, the above-mentioned findings constitute the crucial starting point and baseline of the following analyses. Second, the findings regarding the comparative statics experiments *i.e.*, on the role of the parameters β , φ and μ , constitute the backbone of the subsequent investigations, wherein numerical simulations will be required.

Chapter 3

A Model of Price Formation with Partially Informed Traders

3.1 Introduction

In the previous chapter, our simple heterogeneous agent model with fully informed, boundedly rational arbitrageurs and chartists - both trend followers and contrarian traders - enabled us to derive some well-known results as well as identify under which conditions, fully informed arbitrageurs may not be able to correct misalignment in asset prices, when chartists are present in the market:

(i.) if there are only well-informed and well-funded arbitrageurs in the market, arbitrage *does* work and asset mispricing is immediately corrected.

(ii.) mispricing duration is decreasing in the portion of arbitrageurs in the market *i.e.*, the greater the portion of arbitrageurs in the market, the shorter the asset mispricing lasts.

(iii.) chartist strategies - both trend following and contrarian strategies - may destabilize asset prices, by preventing arbitrageurs from correcting asset mispricing.

Our framework from chapter 2 is able to reproduce price oscillations and slow convergence of the asset price towards the fundamentals. In our settings, the aforementioned asset price behaviors can be explained by trend following as well as contrarian strategies.

In this chapter, we investigate the statistical properties of generated time series, mainly regarding autocorrelation patterns of returns, through a heterogeneous agent model with partially informed arbitrageurs. More precisely, we assess whether positive serial correlations of returns over short horizons, often observed in financial time series (*e.g.*, Lo and MacKinlay, 1988; Jegadeesh and Titman,

1993; Rouwenhorst, 1998),¹ can emerge from the interplay between arbitrageurs and chartists - both trend followers and contrarian traders. Behind this work, the main theoretical issues that we explore are whether *(i.)* the presence of partially informed arbitrageurs as well as chartists - both trend followers and contrarian traders - can explain significant misalignments in asset prices that persists over time (*i.e.*, mispricing persistence); *(ii.)* whether trend followers destabilize the market, only if they dominate; *(iii.)* whether price underreaction can be explained by the interaction between partially informed arbitrageurs and chartists.

For this purpose, starting from the model discussed in chapter 2, we modify the behavior of arbitrageurs by allowing them to be partially informed. Arbitrageurs rather become sequentially aware of any news about asset fundamentals. Due to sequential awareness, arbitrageurs may not be immediately able to recognize that the asset price does not reflect the asset fundamental value. Sequential awareness has also been a key element in many earlier works (*e.g.*, Hong and Stein, 1999; Abreu and Brunnermeier, 2002, 2003).

The motivation of this work is twofold. First, most of the heterogeneous agent models based on the distinction between fundamentalists and chartists (as mainly presented in Section 1.4.1) suggest that trend followers tend to destabilize markets. A common condition for this finding to hold is that the effect of trend followers on the price dynamics is significant enough. This is often characterized by either a large fraction of chartists in the market with respect to fundamentalists, or a high reaction coefficient, so that the impact of trend followers orders on the asset

¹Further references are provided in Section 1.2.2.

price is greater than the effect of fundamentalists ones (*e.g.*, Lux, 1995; Lux and Marchesi, 1999; De Grauwe and Grimaldi, 2005).

In this chapter, we precisely investigate whether this finding holds whenever arbitrageurs are partially informed about news regarding asset fundamentals. In other words, we try to assess whether the presence of chartists is a sufficient condition to explain mispricing persistence.

Second, a large volume of empirical works has actually documented the tendency of asset prices to underreact to news in the short run, which is actually characteristic of many misalignments in asset prices (*e.g.*, the momentum effect on individual as well as industry portfolios). However, as discussed in chapter 1, these patterns are hardly rationalized based on the classical asset pricing theory *i.e.*, through measured variations in risk. Evidence of underreaction often implies that returns tend to exhibit positive serial correlations at short horizons (*e.g.*, Chan, Jegadeesh, and Lakonishok, 1996; Hong and Stein, 1999; Hong, Lim, and Stein, 2000).

With this in mind, we propose a heterogeneous agent model with partially informed arbitrageurs and chartists - both trend followers and contrarian traders - to explain underreaction phenomena, through the emergence of positive serial correlations of returns over short horizons.

In the next chapter, we rather propose a heterogeneous agent model in order to investigate predictability of returns as well as volatility of returns in financial markets. In this model, the behavioral rule describing fundamentalists decisions is more sophisticated, so that fundamentalist's memory may affect the price dynamics.

The chapter is organized as follows. In Section 3.2, we present the model

of linear price formation with partially informed arbitrageurs. In Section 3.3, we analyze the results of the numerical simulations. The work presented in this section enables us to investigate under which conditions the interplay between arbitrageurs and chartists can generate price underreaction as well as mispricing persistence. Section 3.4 summarizes the main findings and Section 3.5 presents some concluding remarks.

3.2 The Model

In the model discussed in this chapter, we consider a market in which there is a single risky asset with price P_t and fundamental value V_t . The fundamental value of the asset is based on the asset future payoffs *i.e.*, on the prospect of future cash flows only. While in the previous chapter, we assumed that the value of the asset fundamental value was constant over time, in this chapter, we relax such an assumption and take account for frequent and repeated new information that could actually affect the intrinsic value of the asset over time. The evolution of the asset fundamental value over time is thus formalized as a simple *random walk*:

$$V_t = V_{t-1} + \epsilon_t \quad (3.1)$$

where $\{\epsilon_t\}$ is a sequence of i.i.d. random variables with $E[\epsilon_t] = 0$, $var[\epsilon_t] = \sigma_\epsilon^2$ and $cov(\epsilon_t, \epsilon_{t-j}) = 0$ for $j \geq 0$. So that $E[V_t] = V_{t-1}$. In this work, we assume that any changes in the asset fundamental value cannot be predictable, since they should be intrinsically stochastic.

Under the efficient market hypothesis, stock price movements should actually coincide with asset fundamentals ones, so that changes in stock prices should be only brought about by new information (see further discussion in Section 1.2). This assumption implies that beliefs about future prices independent of cash flows are precluded from this work. The prediction on which we mainly focus in this work is whether returns cannot be predictable, which implies that no sign of autocorrelation should be observable in financial time series.

In the market, there are N agents, who are assumed to be of two types, namely, arbitrageurs (or *fundamentalists*) and chartists (for detailed discussion and references on heterogeneous agent models based on the distinction between fundamentalists and chartists, see Section 1.4 and 1.4.1). The number of arbitrageurs and chartists in the market is denoted n_a and n_c respectively, with $n_a + n_c = N$. The portion of arbitrageurs is denoted $\eta \equiv n_a/N$. The chartist population is composed of two subgroups, namely, trend followers and contrarian traders (for detailed discussion and references on the behavioral and empirical motivations of the presence of contrarian traders in this work, see Section 2.2). Their respective numbers are denoted n_{TF} and n_{CT} with $n_{TF} + n_{CT} = n_c$. The portion of trend followers among the chartist population is denoted $z \equiv n_{TF}/n_c$.

In each period, agents can place buy or sell orders in the market. First, in order to make their trading decisions, arbitrageurs observe the asset price which prevails in the market and know in each period the expected value of the asset fundamentals *i.e.*, $E[V_t] = V_{t-1}$. They base their trading strategy upon any differential between the observed asset price and its fundamental value. Arbitrageur

orders are captured as:

$$X_{t+1}^A = \beta(E[V_t] - P_t) \quad (3.2)$$

where the term β is a positive reaction coefficient.

Second, in the market there are also chartists. We recall the equations for the orders of both types of chartists, namely, trend followers and contrarian traders:

$$X_{t+1}^{TF} = \varphi(P_t - P_{t-1}) \quad (3.3)$$

$$X_{t+1}^{CT} = -\varphi(P_t - P_{t-1}) \quad (3.4)$$

where the term φ is a positive reaction coefficient.

In each period, the market maker mediates transactions by matching agents' demand and supply and sets the asset price according to aggregate excess demand in the market, as follows:

$$P_{t+1} = P_t + \mu(n_a X_{t+1}^A + n_{TF} X_{t+1}^{TF} + n_{CT} X_{t+1}^{CT}) \quad (3.5)$$

where the term μ is a positive price adjustment parameter.² Substituting eq. (3.2), eq. (3.3) and eq. (3.4) into eq. (3.5) yields:

$$P_{t+1} = \mu n_a \beta V_{t-1} + (1 - n_a \mu \beta + \mu \varphi(n_{TF} - n_{CT})) P_t + \mu \varphi(n_{CT} - n_{TF}) P_{t-1} \quad (3.6)$$

which constitutes, due to the evolution of the fundamental value of the asset as

²The terms n_a , n_{TF} and n_{CT} represent the number of arbitrageurs, trend followers and contrarian traders, respectively. It worth reminding that the portion of arbitrageurs and trend followers in the market is rather η and z , respectively.

described in eq. 3.1, our stochastic model driving the price dynamics.

In the market at some point in time, namely, t_0 , there is a positive³ shock on the asset fundamental value. This shock captures any new positive information which markedly alters future cash flows of the asset and consequently its intrinsic value. The asset price which prevails in the market may then markedly deviate from the asset fundamental value. Fig. 3.1 illustrates the evolution of the asset fundamental value over time.

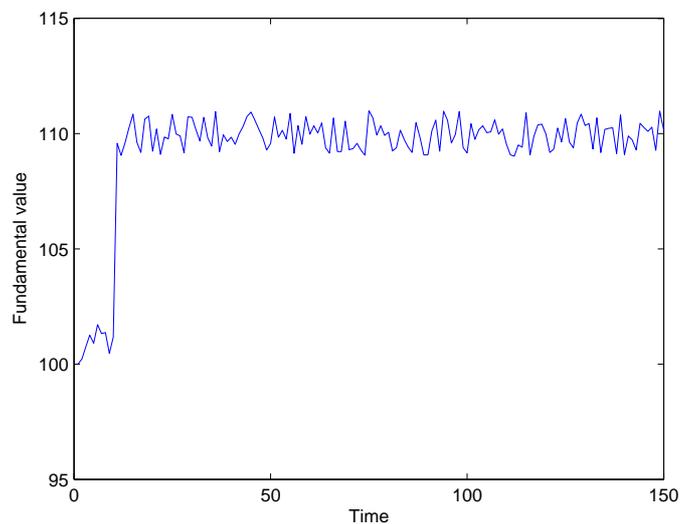


Figure 3.1: Evolution of the asset fundamental value.

However, in the model discussed in this chapter, all arbitrageurs may not be immediately able to recognize that the asset fundamentals have markedly changed.

³Similar results of the model may actually be derived if a *negative* shock on V_t is considered.

Indeed, if asset fundamentals happen to change drastically, arbitrageurs only become sequentially aware of the true asset fundamental value. This was done by using a uniform rule, so that in each period, an additional fraction of arbitrageurs (n_{inc}) becomes informed of the true asset's fundamental value. Consequently, for a while, arbitrageurs may have heterogeneous beliefs about asset future payoffs. This assumption may be justified by the fact that information about the asset fundamental value diffuses slowly in the market.⁴ Arbitrageurs' perceived asset fundamental value may thus not coincide with the true one. In this case, on the one hand, some arbitrageurs know the true asset fundamental value (*i.e.*, arbitrageurs with “*realistic beliefs*”) and are immediately able to recognize that the asset is mispriced (*i.e.*, either overvalued or undervalued). On the other hand, others stick to a misleading asset fundamental value (*i.e.*, arbitrageurs with “*unrealistic beliefs*”) which does not enable them to recognize that the asset is mispriced. As a result for a while, arbitrageurs with “realistic” and “unrealistic beliefs” about the asset fundamentals coexist in the market.

3.3 Numerical Analysis

In this section, we now consider the model presented above and seek to investigate through such a simple framework - with linear behavioral rules and linear price formation rule - under which conditions asset mispricing may persist when arbitrageurs and chartists coexist in the market. More precisely, we investigate whether the fact that chartists dominate the market is still a necessary condition

⁴The foregoing assumption may also capture situations in which there is asymmetric information among arbitrageurs or differential interpretation of such an information.

for destabilizing asset prices and for preventing arbitrageurs from bringing the asset price towards the asset fundamentals.

At this point, it is worth clarifying the meaning of mispricing persistence in this work. With this in mind, first, in line with Singal (2006),⁵ in what follows, mispricing is intended to be any discrepancy between an asset price and its fundamental value. In other words, whenever asset prices do not reflect the fundamentals ($P_t \neq V_t$). Second, and more important, in what follows, mispricing persistence is detected by the emergence of positive serial correlations in returns over short horizons. In fact, the crucial issue behind the existence of any asset price misalignment is whether it implies predictability in returns, so that some traders could earn excess returns.

With the above-mentioned purpose in mind, we have first to define mispricing persistence, although arbitrageurs may not be all immediately aware that asset fundamentals have changed. Consequently, as a preliminary step in this study, we implement some benchmark cases in which arbitrageurs may have heterogeneous beliefs about future payoffs. These benchmarks include first, a setting in which there are only arbitrageurs (*benchmark 1*). This setting enables us to determine when arbitrage strategies can correct the mispricing *i.e.*, can bring back the asset price towards its fundamentals, while there is no chartist in the market. Second, we study a setting where chartists are present as well in the market, although their trading activity does not further affect the price dynamics (*benchmark 2*).

⁵More precisely, Singal (2006) proposes to define a mispricing as “any *predictable* deviation from a normal or expected return” (p. 7).

This is done by setting the portion of trend followers equal to the portion of contrarian traders (*i.e.*, $z = 0.5$).⁶ This setting enables us to focus on the effect of arbitrageur strategy on the price dynamics, when chartists are present in the market. Eventually, in order to investigate whether asset mispricing may persist while both well-funded arbitrageurs and chartists - both trend followers and contrarian traders - may influence the price dynamics, we study the setting of major interest to us *i.e.*, the situation in which chartist strategies actually affect the price dynamics. This is done by varying the portion of trend followers versus contrarian traders (*i.e.*, $0 \leq z \leq 1$).

In the previous chapter, our objective was to identify the long-run equilibrium of the price dynamics and to derive the conditions for the stationarity of the price dynamics. In this chapter, we turn to focus on the price dynamics within the time window in which arbitrageurs discover the true asset fundamental value. Computer simulations are thus convenient to grasp the price dynamics over short horizons.

For this purpose, we simulate the model⁷ with 500 time steps. The parameter values used in the simulations are reported in Table 3.1. It is worth stressing that the main criterion for choosing parameter values was to match one of the crucial efficient market hypothesis prediction according to which stock price should reflect asset fundamentals and arbitrage works. The study presented in the previous

⁶Indeed, in this case, according to eq. (3.3) and eq. (3.4) from Section 3.2, trend follower's orders and contrarian trader ones exactly compensate, so that chartist trading activity does not further affect the evolution of the asset price.

⁷The code, written in Java, is available from the author upon request.

chapter provides the required background to set the appropriate framework as well as parameter values. Indeed, *(i.)* in the model discussed in the previous chapter, when there are only well-informed and well-funded arbitrageurs, arbitrage works; *(ii.)* from the study of the model discussed in the previous chapter, we have identified the parameter values (η and z) which ensure that the system is stable, so that any erratic behavior of the $\{P_t\}$ sequence in the long run is precluded from this work; *(iii.)* we have excluded any exogenous effect on the price dynamics which would constitute a bias in our investigation of the effect of the interaction between arbitrageurs and chartists on the price dynamics. This is done by setting $\beta = 1$, $\varphi = 1$ and $\mu = 1/N$ with $N = 1000$.

Parameters	Values
Number of periods (t)	500
Number of agents (N)	1000
Date of the shock (t_0)	10
Date at which all arbitrageurs are informed (t_i)	20
Additional number of informed arbitrageurs in each period (n_{inc})	$N/(t_i - t_0)$
Portion of arbitrageurs (η)	from 0 to 1
Portion of trend followers (z)	from 0 to 1
Arbitrageur reaction coefficient (β)	1
Chartist reaction coefficient (φ)	1
Market maker price adjustment parameter (μ)	$1/N$
Initial price (P_0)	100
Initial fundamental value (V_0)	100
Size of the shock (b)	0.1

Table 3.1: Parameters for the simulations.

Our investigation of the statistical properties of artificially generated series is presented in the following subsections. In what follows, we mainly focus on autocorrelation patterns in returns from the simulated time series. Indeed, the emergence of positive serial correlations over short horizons is the witness of the presence of short-term trends in prices, which are characteristic of mispricing persistence. To this end, we mainly try to assess whether significant positive autocorrelation values over short horizons are detected when arbitrageurs and chartists - both trend followers and contrarian traders - are present in the market.

We begin our study by presenting the results regarding return predictability when only the arbitrageurs are present in the market. Then we account for the presence of chartists, even if their strategies do not further affect the price dynamics. Eventually, we present the results regarding the setting in which both arbitrage strategies and chartist strategies further affect the price dynamics.

3.3.1 A Market with only Arbitrageurs

In this section, we focus on return predictability when only the arbitrageurs are present ($\eta = 1$) *i.e.*, **benchmark 1**. As explained in the previous section, after the shock on V_t , all arbitrageurs do not immediately realize that the asset fundamentals have markedly changed. Instead, they only become sequentially aware of it. So for a certain time period,⁸ arbitrageurs have heterogeneous beliefs about the asset fundamental value.

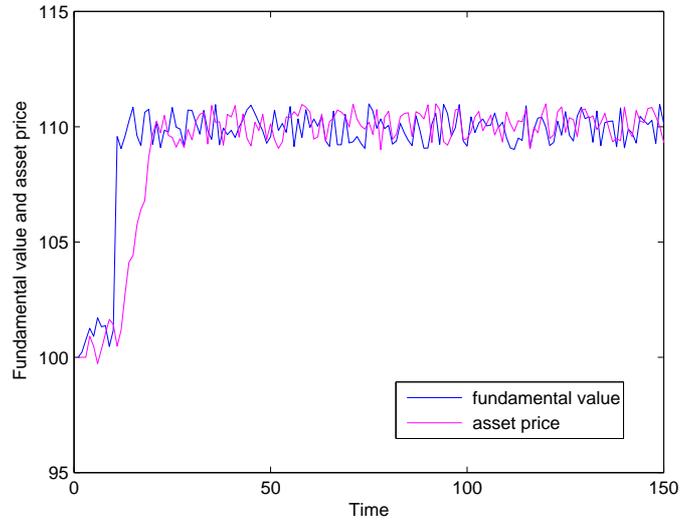
⁸That is the period of time through which arbitrageurs learn the true asset fundamental value (*i.e.*, the *awareness window*). At this point, it is worth stressing that the latter is (*i.*) exogenous and (*ii.*) determined by both the date of the shock (t_0) and the date at which all arbitrageurs know the true fundamental value of the asset (t_i).

Fig. 3.2 shows the simulated time series when only the arbitrageurs are present in the market ($\eta = 1$). More precisely, Fig. 3.2a shows the evolution of the asset fundamental value as well as the simulated price time series when $\eta = 1$. Fig. 3.2b rather shows the size of the mispricing over time.

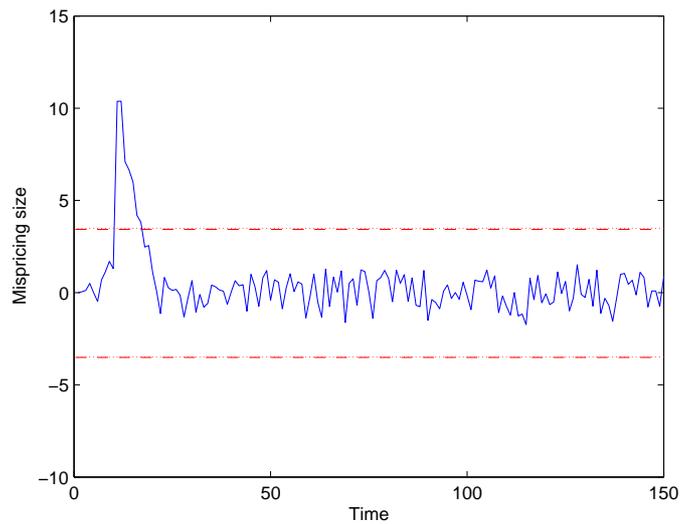
As illustrated in these figures, simulations reveal that after the shock, the asset price departs from the asset fundamental value as long as all arbitrageurs do not know the true asset fundamental value (*i.e.*, $t_i = 20$). Indeed, although there is no chartist in the market, the asset price does not immediately go back to the asset fundamental value. Fig. 3.2b clearly shows that in fact mispricing size is significantly different from zero for that period of time. This is merely explained by the fact that first, the trading activity of arbitrageurs with “unrealistic” beliefs supports asset mispricing, since their trades are based on a misleading asset fundamental value. Second, arbitrageurs with “realistic” beliefs about asset future payoffs, who actually trade against the mispricing are fewer all along the awareness window,⁹ so that arbitrage does enable the price to immediately reflect the asset fundamentals. As a result, mispricing correction takes more time than if all arbitrageurs were immediately aware of the fact that the asset fundamentals have markedly changed.

Fig. 3.3, showing the sample autocorrelation functions of mispricing size, clearly supports this finding *i.e.*, mispricing lasts as long as arbitrageurs become sequentially aware of the asset fundamentals. Indeed, the positive coefficients that emerge over small lags indicate that the mispricing does not vanish immediately.

⁹The reader has to keep in mind that within the awareness window, in each period, an additional number of arbitrageurs (n_{inc}) is informed about the true asset fundamental value.



(a) Evolution of the asset fundamental value and simulated time series



(b) Mispricing size evolution

Figure 3.2: Simulated time series when $\eta = 1$.

The number of lags for which positive coefficients arise actually corresponds to the size of the awareness window. Furthermore, the positive coefficients for small lags

indicate short-term trends in prices.

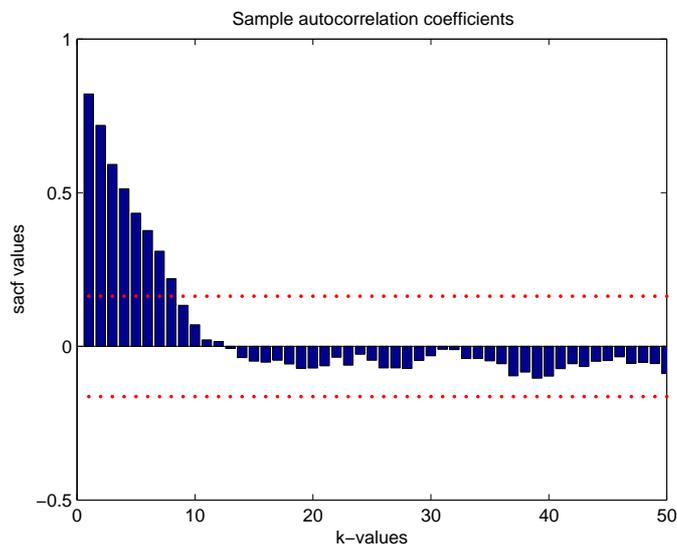


Figure 3.3: Sample autocorrelation function of mispricing size when $\eta = 1$.

However when only the arbitrageurs are present, while mispricing is not immediately corrected, simulations bring out that returns from the simulated time series cannot be predictable. Fig. 3.4 shows the sample autocorrelation functions of returns from the simulated time series when $\eta = 1$.

Indeed, while it is not surprising that changes in the fundamental value cannot be predictable, returns from the simulated time series do not reveal as well any significant sign of autocorrelation. In consequence, given the assumption on the learning process, we can assert that the presence of arbitrageurs only, even if they are not immediately able to bring back the price towards the asset fundamental value, is not sufficient to explain mispricing persistence. Indeed, positive serial correlations in returns over short horizons that are often observed in real financial

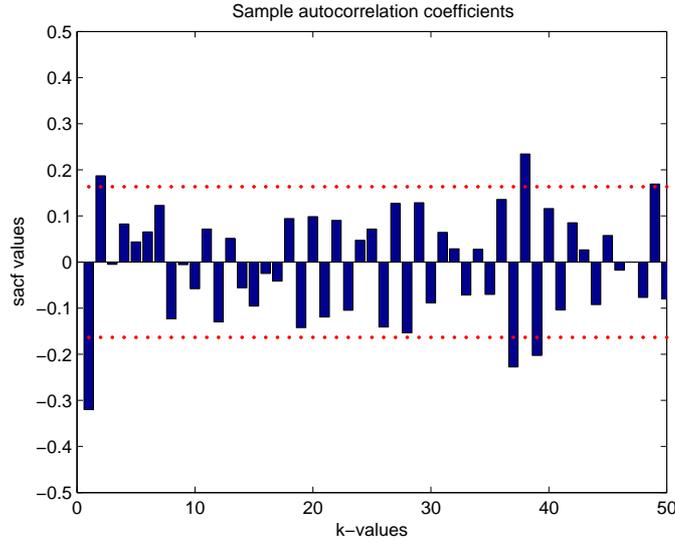


Figure 3.4: Sample autocorrelation functions of returns when $\eta = 1$.

time series do not emerge when only the arbitrageurs are present in the market. This can be explained by the fact that, while arbitrageurs are not immediately able to correct the mispricing, the effect of arbitrage strategies on price dynamics is not enough pronounced (or the convergence process too smooth) to markedly generate trends in prices over short horizons, which are characteristic of mispricing persistence.

3.3.2 A Market with Arbitrageurs and “Neutral” Chartists

In the previous subsection, we have seen that arbitrageurs only cannot explain mispricing persistence. In contrast, as described in Section 3.2, in our model there are not only arbitrageurs. We now turn to focus on the effect of arbitrage strategies on the price dynamics when chartists - both trend followers and contrarian traders - are present in the market. To this end, as a first step, we study the

price dynamics that emerge in the market when chartist trading activity does not further affect the evolution of the asset price *i.e.*, **benchmark 2**. As mentioned previously, this is implemented by setting $z = 0.5$. The foregoing setting enables us to focus on the effect of arbitrage strategies on the price dynamics, for differing portions of arbitrageurs in the market.

Fig. 3.5 shows the evolution of the mispricing size when $z = 0.5$, for differing portions of arbitrageurs (η).

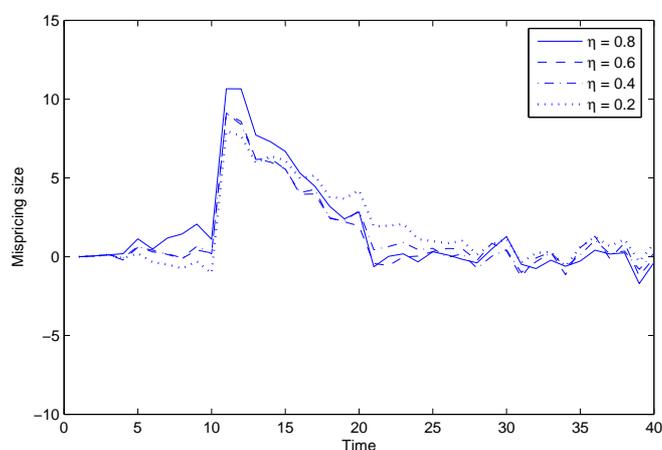


Figure 3.5: The evolution of the size of the mispricing when $z = 0.5$, for differing values of η .

As illustrated in Fig. 3.5, simulations unveil that first, when the portion of arbitrageurs is large ($0.6 \leq \eta < 1$), even though chartists are present in the market, mispricing correction occurs as soon as all arbitrageurs are informed about the true asset fundamental value ($t_i = 20$). Second, while chartists may dominate (for instance, $\eta = 0.4$), arbitrageurs are still able to quickly bring back the asset price

towards the asset fundamental value. Indeed, mispricing correction still occurs almost as soon as all arbitrageurs know the true asset fundamental value. However, mispricing correction tends to be delayed only when the portion of arbitrageurs is low ($\eta = 0.2$). This is further illustrated in Fig. 3.6, which shows the values of the autocorrelation coefficients at lag 1 regarding the size of the mispricing for differing values of η . Indeed, from this figure, mispricing duration is clearly decreasing in the portions of arbitrageurs in the market,¹⁰ although non-linearly. More precisely, the greater the portion of arbitrageurs in the market, the shorter the mispricing is, suggests that the presence of arbitrageurs is crucial to explain mispricing correction.

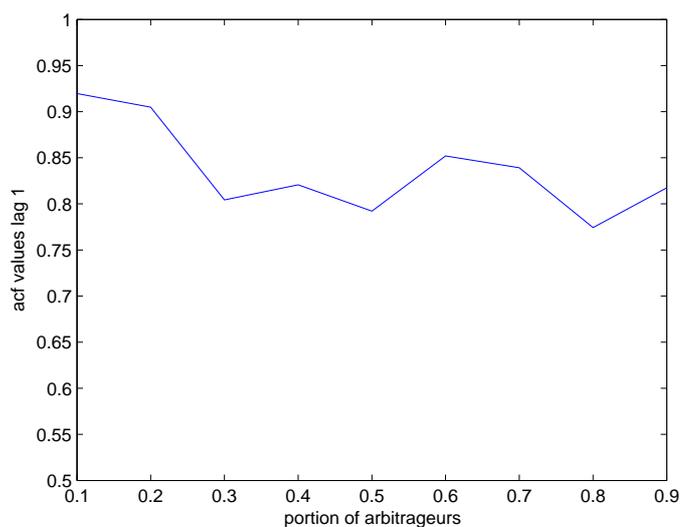


Figure 3.6: Relationship between mispricing duration and η when $z = 0.5$.

In contrast, mispricing correction is only markedly delayed when the portion of

¹⁰For instance, when the portion of arbitrageurs (η) decreases from 0.8 to 0.2, the autocorrelation coefficients at lag 1 regarding mispricing size increases by more than 2%.

chartists in the market is extremely high ($0.1 \leq \eta \leq 0.2$). In consequence, when chartists widely dominate the market, they can prevent arbitrageurs from bringing back the price towards the asset fundamental value.

In order to assess the effect of arbitrage strategy on mispricing persistence, we now turn to examine whether autocorrelation patterns in returns from the simulated time series emerge, even when chartist strategies do not further affect the price dynamics ($z = 0.5$). Simulations actually bring out that autocorrelation patterns emerge only for low portion of arbitrageurs ($\eta < 0.5$). Fig. 3.7 shows the sample autocorrelation function of returns for differing values of η .

More precisely, as shown in Fig. 3.7c and Fig. 3.7d, positive coefficients over small lags, which are characteristic of trends in prices over short horizons, only arise when $\eta < 0.5$. Indeed, when arbitrageurs dominate the market ($\eta > 0.5$), as shown in Fig. 3.7a and Fig. 3.7b, autocorrelation patterns rather do not emerge and no sign of autocorrelations in returns is detected.

As a result, while for any composition of the population, mispricing correction is not immediate, this is not sufficient to systematically generate positive serial correlations in returns over short horizons *i.e.*, mispricing persistence. More important, the fact that arbitrageurs need time to learn the true asset fundamental value is not sufficient to explain mispricing persistence. Rather, when $z = 0.5$, a low portion of arbitrageurs in the market appears to be a necessary condition for the emergence of trends in prices over short time window.

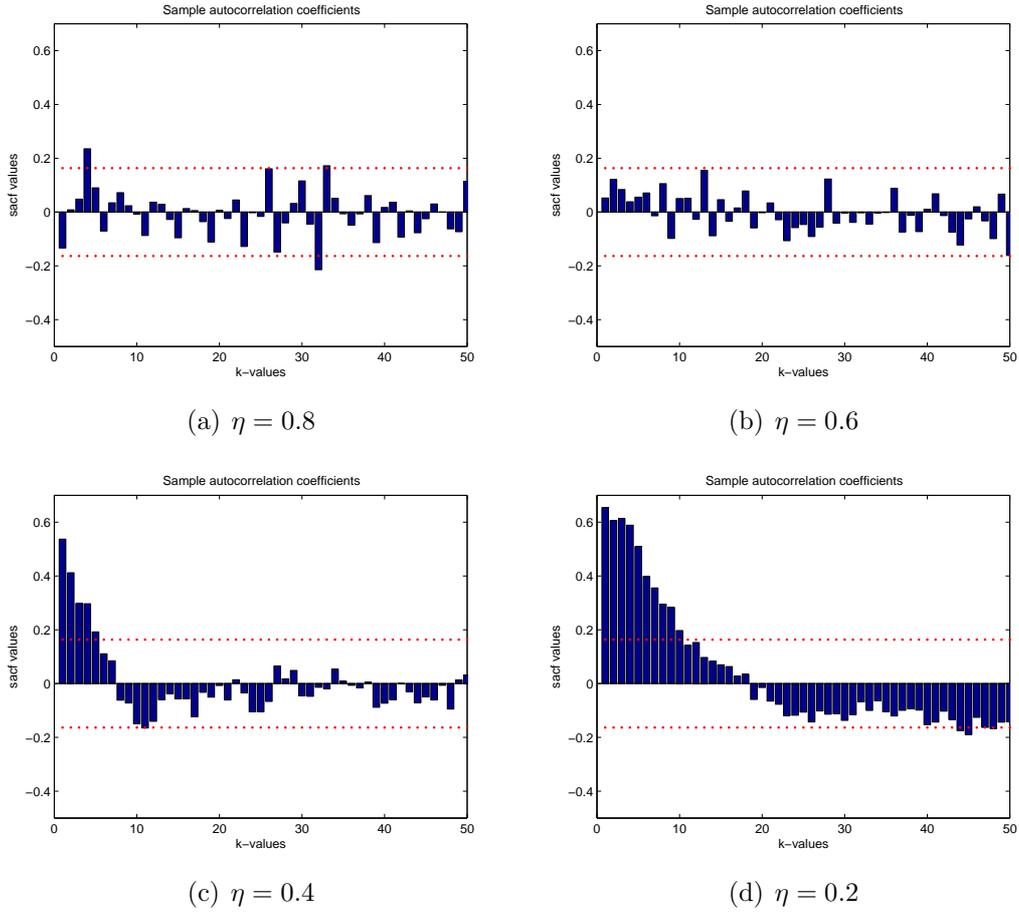


Figure 3.7: Sample autocorrelation function of returns for differing values of η and $z = 0.5$.

3.3.3 A Market with Arbitrageurs and “Active” Chartists

In the previous subsection, we have seen that when chartists are present in the market, mispricing may persist if the portion of arbitrageurs in the market is low ($\eta < 0.5$). As a result, arbitrageurs play a key role in correcting asset mispricing in the market. In order to further study the effect of chartist strategies on the evolution of the asset price and the duration of the mispricing, we now examine the price dynamics that emerge in the market for differing portions of trend followers

versus contrarian traders. This is done by varying the portion of trend followers from 0 to 1.

First, simulations reveal that, as long as arbitrageurs widely dominate ($0.7 \leq \eta < 1$), whatever the portion of trend followers in the market ($0 \leq z \leq 1$), the asset price goes back to the asset fundamental value almost as soon as all arbitrageurs know the true asset fundamental value ($t_i = 20$). Fig. 3.8 shows, for instance, the evolution of the size of the mispricing over time when $\eta = 0.7$, for differing values of z .

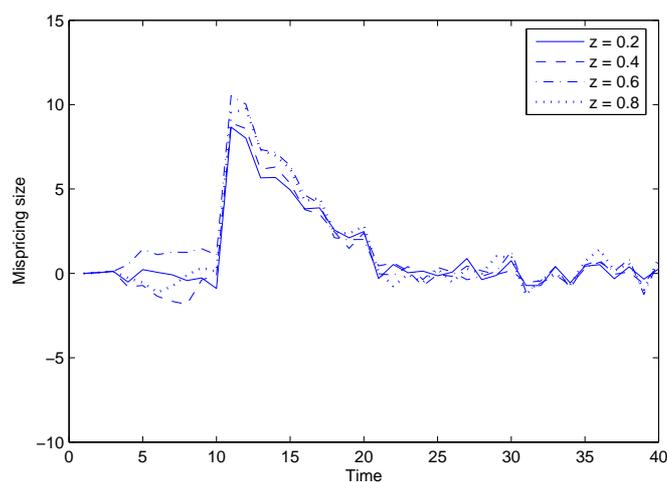
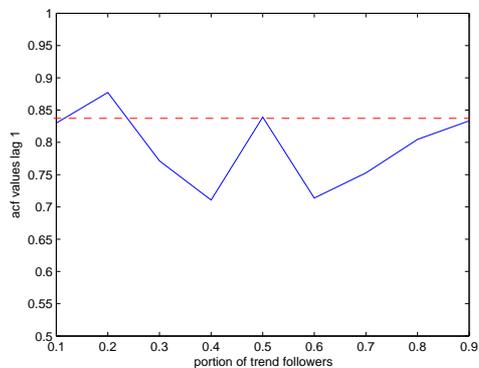


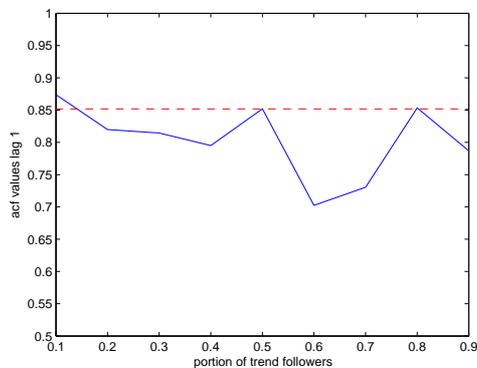
Figure 3.8: Evolution of the size of the mispricing for $\eta = 0.7$, for differing values of z .

However, simulations also bring out that, while arbitrageurs dominate (*e.g.* $\eta = 0.7$), the presence of chartists - both trend followers and contrarian traders - may actually reduce mispricing duration. Fig. 3.9 shows the relationship between the autocorrelation coefficient at lag 1 regarding mispricing and the portion of trend

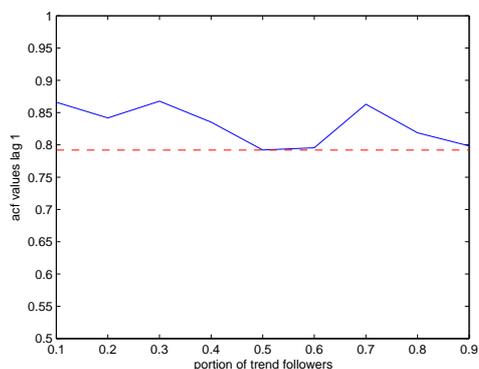
followers in the market ($0 \leq z \leq 1$) for differing portions of arbitrageurs (η).



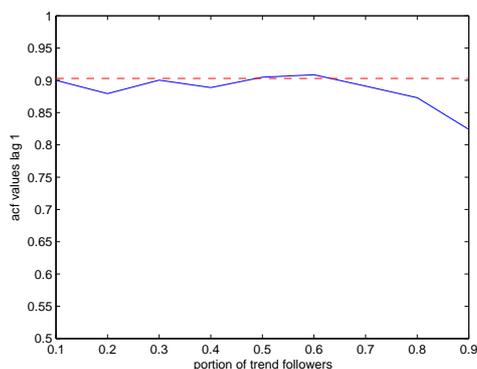
(a) $\eta = 0.7$



(b) $\eta = 0.6$



(c) $\eta = 0.5$



(d) $\eta = 0.2$

Figure 3.9: Relationship between mispricing duration and the values of z for differing values of η .

It is worth mentioning that first, whatever the portion of arbitrageurs in the market, the relationship between mispricing duration and the portion of trend followers is clearly not linear. However, when $\eta = 0.7$, as illustrated in Fig. 3.9a, the effect of trend followers and contrarian traders on mispricing duration is almost symmetric. More important, Fig. 3.9a shows for instance that, when $z = 0.6$ and $z = 0.4$,

mispricing duration is seemingly reduced with respect to *benchmark 2*, which is depicted by the dash line. Indeed, when z increases (*decreases*) from 0.5 to 0.6 (0.4), mispricing duration is reduced by almost 15% with respect to *benchmark 2*. As a result, trend following strategies (when $z > 0.5$) as well as contrarian ones (when $z < 0.5$) can markedly reduce mispricing duration.¹¹ However, as illustrated in Fig. 3.9a, contrarian strategies may also increase mispricing duration with respect to *benchmark 2*. This is especially the case when $z = 0.2$, for which mispricing duration is lengthened by almost 5%.

Nevertheless, simulations unveil that when the portion of arbitrageurs is high, no matter the effect of chartists on mispricing duration, mispricing does not persist. In fact, Fig. 3.10, which indicates the sample autocorrelation functions of returns from the simulated time series when $z = 0.3$ and $z = 0.7$, shows that autocorrelation patterns in returns do not emerge when arbitrageurs widely dominate.

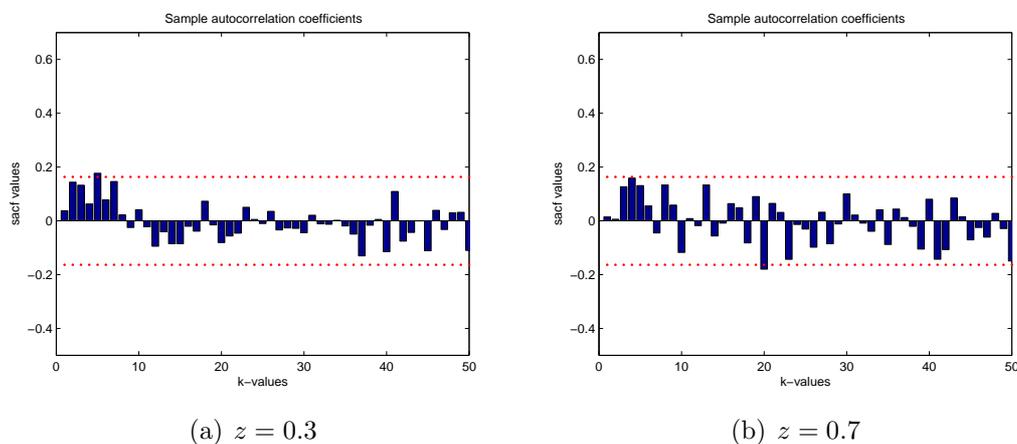


Figure 3.10: Simulated time series for $\eta = 0.7$ and for differing values of z .

¹¹It is however worth mentioning that when $z = 0.9$, mispricing duration is almost the same as in *benchmark 2* ($z = 0.5$).

Second when $\eta = 0.6$, as illustrated in Fig. 3.9b, the effect of trend followers on mispricing duration is much more pronounced than the effect of contrarian traders. More precisely, trend following strategies markedly reduce mispricing duration. Indeed, although with respect to *benchmark 2* the largest decrease in mispricing duration due to contrarian traders is only 7% (from $z = 0.5$ to 0.4), when z increases from 0.5 to 0.6, mispricing duration is reduced by almost 18%. Furthermore, contrary to existing results (see for instance, Lux, 1995, 1998; Farmer and Joshi, 2002; De Grauwe and Grimaldi, 2004, 2005), simulations suggest that, while arbitrageurs still dominate (*e.g.*, $\eta = 0.6$), asset mispricing may markedly persist when the chartist population is mainly composed of trend followers. Fig. 3.11 shows the sample autocorrelation functions of returns when $\eta = 0.6$, for differing portions of trend followers.

This figure actually illustrates that positive serial correlations in returns over short horizons tend to emerge when trend followers dominate the chartist population *i.e.*, when z tends to 1. Rather, contrarian traders (when $z < 0.5$) tend to generate both positive as well as negative serial correlations in returns over different horizons. Negative coefficients are characteristic of oscillations in prices.

Fig. 3.9c, which indicates the relationship between mispricing duration and the portion of trend followers in the market when $\eta = 0.5$, rather shows that, while chartists do not dominate, both trend followers and contrarian traders are now able to markedly lengthen mispricing duration with respect to *benchmark 2* ($z = 0.5$). Indeed, for any portion of trend followers, the autocorrelation coefficient at lag 1 regarding mispricing is greater than in *benchmark 2*. More precisely, for instance, when $z = 0.7$ (*0.3*), mispricing duration is increased by almost 9% (*10%*) with respect to *benchmark 2*. Furthermore, Fig. 3.9c suggests that the effect of

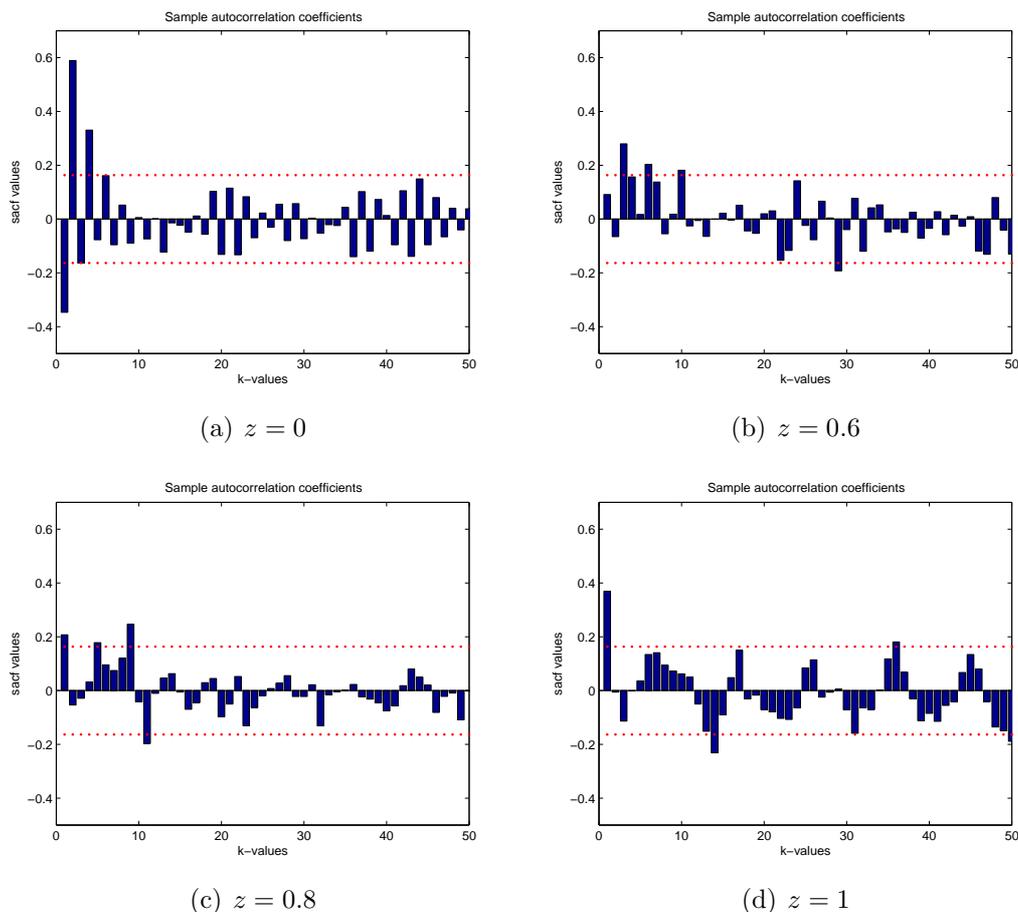


Figure 3.11: Sample autocorrelation function of returns when $\eta = 0.6$ for differing values of z .

contrarian traders ($z < 0.5$) on mispricing duration is more pronounced than the effect of trend followers ($z > 0.5$).¹²

Nevertheless, Fig. 3.12, illustrating the sample autocorrelation functions of returns from the simulated time series for differing portions of trend followers versus contrarian traders, further supports the previous finding regarding the effect of chartists on mispricing persistence, while arbitrageurs dominate. Indeed, mainly

¹²More precisely, contrarian strategies tend to increase mispricing duration by almost 2% on average (when z decreases from 0.5 to 0.1), while the effect of trend following strategies on mispricing duration is on average 10 times smaller (when z increases from 0.5 to 0.9).

trend following strategies tend to induce positive serial correlations in returns over short horizons (see Fig. 3.12c and Fig. 3.12d).

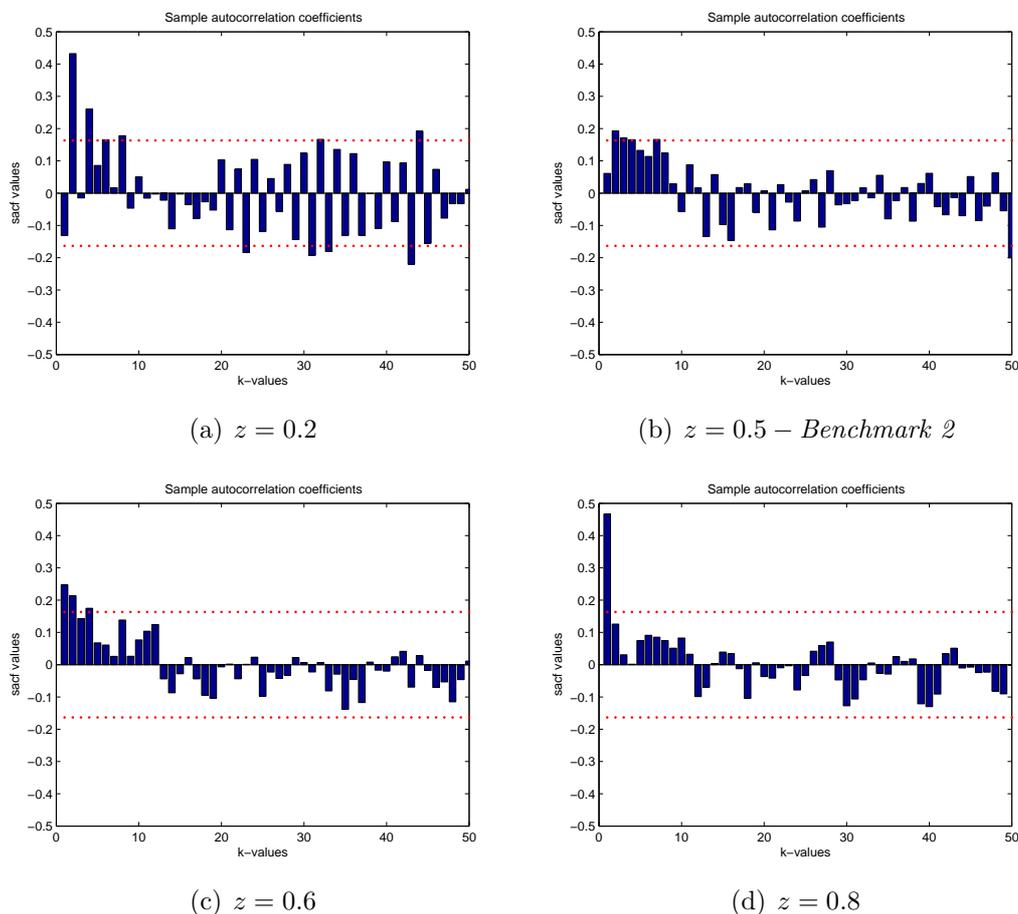


Figure 3.12: Sample autocorrelation function of returns when $\eta = 0.5$ for differing values of z .

Furthermore, when $z > 0.5$, mispricing persists more than in *benchmark 2*.¹³ However, comparison of Fig. 3.12b, Fig. 3.12c and 3.12d actually brings out that mispricing persistence tends to be increasing in the portion of trend followers. In

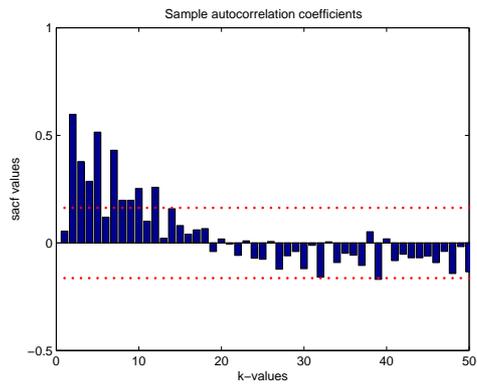
¹³For instance, when $z = 0.5$, the autocorrelation coefficient at lag 2 (since at lag 1, the value of the coefficient not significant) is 0.1928, while when, for instance, $z = 0.8$ ($z = 0.6$), the autocorrelation coefficient is 0.4669 (0.2471), which represents an increase by about 142% (28%).

other words, the greater z , the longer mispricing persists. Rather, when contrarian traders dominate the chartist population, no clear autocorrelation patterns in returns from the simulated time series is detected (see Fig. 3.12a).

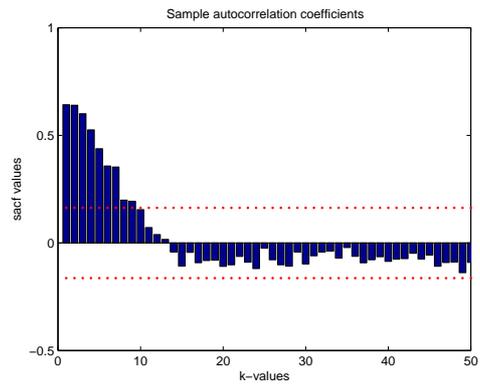
As a result, while chartists do not dominate the market, first, the presence of trend followers may destabilize the market; second, mispricing persistence can be explained by trend following strategies.

Lastly, when chartists widely dominate (*e.g.*, $\eta = 0.2$), Fig. 3.5, presented previously, clearly shows that, in this case, mispricing duration is markedly lengthened, even if the chartist population is split between trend followers and contrarian traders ($z = 0.5$). This is mainly explained by the fact that the portion of arbitrageurs in the market is low. However, when $z \neq 0.5$, simulations suggest that, as illustrated in Fig. 3.9d, contrarian strategies but mainly trend following ones tend to shorten mispricing duration with respect to *benchmark 2*. This is especially the case when $z > 0.8$. For instance, when $z = 0.9$, mispricing duration is shortened by almost 9% with respect to *benchmark 2*. Furthermore, from $z = 0.6$, mispricing duration is decreasing in the portion of trend followers. The larger the portion of trend followers in the market, the smaller mispricing duration is.

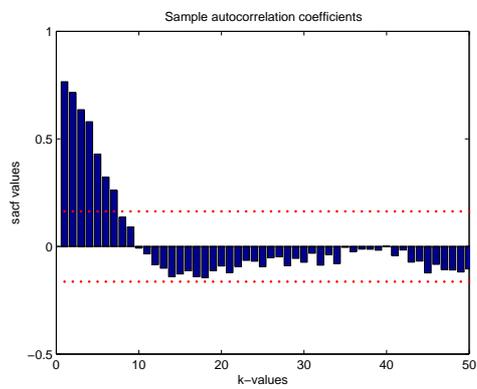
We now turn to assess the effect of trend followers and contrarian traders on mispricing persistence. Simulations reveal that when chartists dominate the market, mispricing persists. Fig. 3.13 shows the sample autocorrelation functions of returns when $\eta = 0.2$, for differing portions of trend followers. The returns from the simulated time series actually exhibit positive serial correlations over short horizons, whatever the portion of trend followers versus contrarian traders in the



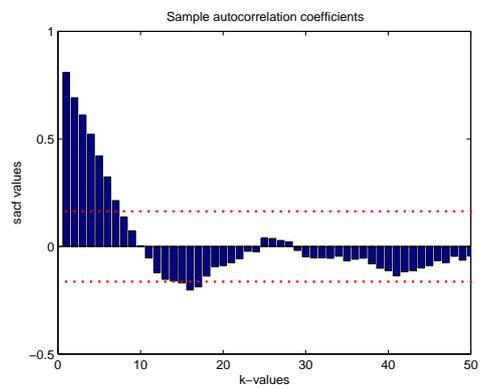
(a) $z = 0.2$



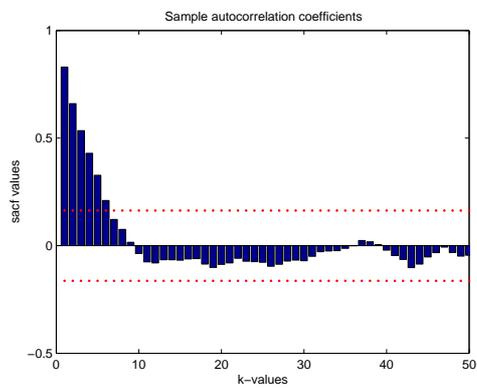
(b) $z = 0.4$



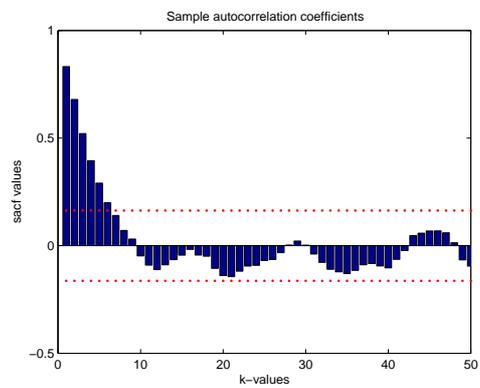
(c) $z = 0.5 - \text{Benchmark 2}$



(d) $z = 0.7$



(e) $z = 0.8$



(f) $z = 0.9$

Figure 3.13: Sample autocorrelation function of returns when $\eta = 0.2$ for differing values of z .

market.¹⁴ However, first when the chartist population is mainly composed of contrarian traders ($z < 0.5$), positive serial correlations in returns over short horizons tend to vanish when z tends to 0.¹⁵ In other words, the greater the portion of contrarian traders in the market, the less persistent the mispricing is. As a result, contrarian traders may actually shorten mispricing persistence. When the chartist population is, in contrast, mainly composed of trend followers (*i.e.*, $z > 0.5$), the value of the autocorrelation coefficients (*i.e.*, mispricing strength) is increasing in the portion of trend followers.¹⁶ This means that the greater the portion of trend followers in the market, the more persistent the mispricing is. Trend following strategies rather tend to lengthen mispricing persistence.

As a result, when chartists widely dominate, while both contrarian traders and trend followers tend to reduce mispricing duration, mispricing markedly persists. This is above all explained by the fact that, in this case, the portion of arbitrageurs in the market is low. However, assessing the effect of trend following strategies and contrarian ones on mispricing persistence, simulations reveal that, with respect to *benchmark 2*, trend followers tend to lengthen mispricing persistence; contrarian traders rather tend to shorten it.

Furthermore, simulations bring out that when there are only trend followers within the chartist population (*i.e.*, $z = 1$), the price overshoots before converging towards

¹⁴It is worth reminding that the reference point now is *benchmark 2* ($z = 0.5$). More precisely, when $z = 0.5$, the autocorrelation coefficient at lag 1 is 0.7658 and significant positive coefficients emerge until lag 7.

¹⁵The value of the autocorrelation coefficients decreases as the portion of contrarian traders increases. For instance, when $z = 0.4$ ($z = 0.2$), the autocorrelation coefficient at lag 1 is now 0.6423 (0.5976 at lag 2 since the value at lag 1 is not significative), which represents a reduction of almost 17% (22%) with respect to *benchmark 2*.

¹⁶For instance, when $z = 0.9$ ($z = 0.8$), the autocorrelation coefficient at lag 1 is now 0.8322 (0.8297), which represents an increase of about 9% (8%) with respect to *benchmark 2*.

the asset fundamental value. Fig. 3.14 shows the simulated time series as well as the sample autocorrelation function of returns from the simulated time series when $\eta = 0.2$ and $z = 1$.

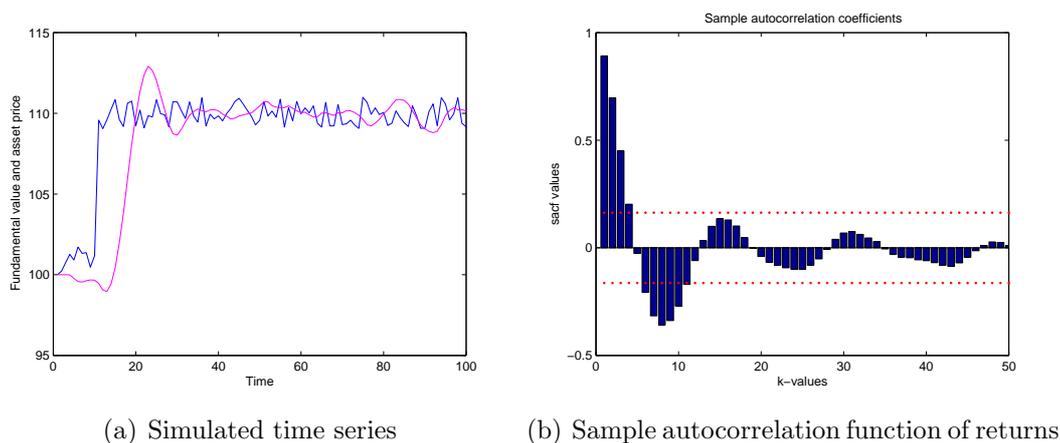


Figure 3.14: Price dynamics when $\eta = 0.2$ and $z = 1$.

In Fig. 3.14b, convergence towards the asset fundamentals through dampened oscillations, which are characteristic of price overshooting, is detected by the emergence of significant positive as well as negative serial coefficients over differing horizons. Negative coefficients are characteristic of oscillations in prices.

Therefore, when the portion of arbitrageurs is low, trend following strategies tend to generate price overshooting, as clearly illustrated in Fig. 3.14a. This is likely to explain the fact that, in these cases, mispricing persists longer in the market. In fact, when arbitrageurs are present in the market, trend following strategies, which primarily induce trends in price (INSERT REF that support this), amplify the foremost trend in prices initiated by arbitrageurs, leading in extreme cases to price overshooting. This kind of price dynamics is also likely to be the reason why

when the chartist population is mainly composed of trend followers, mispricing tends to further persist than when the chartist population is mainly composed of contrarian traders.

3.4 Summary and Discussion of Results

In order to investigate whether arbitrage strategies could bring the asset price back towards the asset fundamentals, while arbitrageurs have heterogeneous beliefs about the asset fundamentals and chartists - both trend followers and contrarian traders - are present in the market, we have explored several situations. These settings have enabled us to independently investigate the effect of each trading strategy on the price dynamics and eventually, starting from well-documented results (worked out from benchmarks 1 and 2), the price dynamics induced by the interplay between arbitrageurs and chartists. In discussing the results of the foregoing sections, we stress the main similarities and differences among the settings examined in this chapter.

First, when there are only arbitrageurs in the market (**benchmark 1**), numerical simulations reveal that mispricing correction occurs as soon as all arbitrageurs are informed that the asset fundamentals have changed. However, careful examination of the statistical properties of artificially generated time series reveals that when there are only arbitrageurs in the market, autocorrelation patterns do not emerge. As a result, even when information about asset fundamentals diffuses slowly among arbitrageurs and mispricing is not immediately corrected, asset returns do not exhibit any sign of autocorrelation, so that returns are not predictable.

Second, when there are both arbitrageurs and chartists - trend followers as well as contrarian traders, given that $z = 0.5$ - (**benchmark 2**)¹⁷ in the market, numerical simulations reveal that arbitrage strategies play a crucial role in explaining asset mispricing correction. Indeed, mispricing duration is decreasing in the portion of arbitrageurs in the market. Furthermore, careful examination of the statistical properties of artificially generated time series reveals that when the portion of arbitrageurs is low in the market ($\eta < 0.5$), stock returns exhibit positive serial correlations over short horizons.

Lastly, when both arbitrageurs and chartists may further affect the price dynamics ($z \neq 0.5$), numerical simulations unveil that, contrary to existing results (Lux, 1995, 1998; Farmer and Joshi, 2002; De Grauwe and Grimaldi, 2004, 2005), while arbitrageurs dominate the market, trend following strategies can induce positive serial correlations in returns over short horizons, which are characteristic of asset mispricing persistence. Accordingly, trend followers can prevent arbitrageurs from correcting asset mispricing. In our setting, mispricing persistence can thus be explained by trend following strategies, even when arbitrageurs are present in the market.

The main findings of each benchmark are summarized in Table 3.2.

¹⁷The reader has to keep in mind that, in this setting, chartist strategies does not further affect the price dynamics, because trend follower orders and contrarian trader ones cancel out each other

Main features	Main findings
<p>Benchmark 1</p> <p>Arbitrageurs only</p>	<ul style="list-style-type: none"> • Mispricing correction occurs as soon as all arbitrageurs are informed of the true asset fundamental value; • But mispricing persistence does not emerge.
<p>Benchmark 2</p> <p>Arbitrageurs and chartists</p> <p>Chartists trading activity does not further affect the evolution of the asset price (<i>i.e.</i>, $z = 0.5$).</p>	<ul style="list-style-type: none"> • Mispricing duration is decreasing in the portion of arbitrageurs in the market; • Mispricing persists when chartists dominate.
<p>Arbitrageurs and chartists</p> <p>Chartists trading activity may further affect the evolution of the asset price (<i>i.e.</i>, $0 \geq z \geq 1$).</p>	<ul style="list-style-type: none"> • While chartists do not dominate, mispricing persistence can be explained by trend following strategies; • When chartists dominate the market, contrarian strategies tend to shorten mispricing persistence; while trend following strategies tend to lengthen mispricing persistence.

Table 3.2: Main findings when arbitrageurs have heterogeneous beliefs.

3.5 Conclusions

In this chapter, we model a financial market wherein arbitrageurs (or *fundamentalists*) have heterogeneous beliefs about the true fundamental value of the asset. This is done by allowing arbitrageurs to be sequentially informed about any new information regarding the asset fundamentals. In line with earlier works (for instance Hong and Stein, 1999; Abreu and Brunnermeier, 2002, 2003), we assume sequential awareness. The purpose of this work is to assess whether positive serial correlation of returns can be explained by the interaction of arbitrageurs and chartists - both trend followers and contrarian traders. Our model offers a qualitative description of asset prices behaviors and enables us to propose an alternative explanation of misalignments in asset prices, that persist over time as well as the momentum effect.

Our framework is in fact able to reproduce some stylized facts observed in real financial markets, namely, positive serial correlations of returns over short horizons. Contrary to existing results, according to which trend following strategies tend to destabilize market prices only when they dominate the market (for instance Lux, 1995; Farmer and Joshi, 2002; De Grauwe and Grimaldi, 2005), we have found that even when arbitrageurs dominate, trend following strategies can cause mispricing persistence. When arbitrageurs, altogether, are not able to trade against the asset mispricing, mispricing can persist for some period of time. Slow diffusion of news and/or dispersion of opinions among arbitrageurs play a crucial role in explaining the occurrence of positive serial correlations in returns over short horizons, which characterize mispricing persistence. This finding supports earlier

works which suggest that *synchronized risk* tends to prevent arbitrageurs from bringing back asset prices towards the fundamentals (Abreu and Brunnermeier, 2002, 2003).

However, first, while Abreu and Brunnermeier (2002, 2003) assume that arbitrageurs are rational, we relax this hypothesis by assuming boundedly rational arbitrageurs. This departure is justified by the extensive works on bounded rationality, which seems highly relevant in a heterogeneous world. Arthur (1995); Hommes (2001, 2006) have, for instance, emphasized that “in a *heterogeneous* world a rational agent has to know the beliefs of *all* other, *non-rational* agents, which seems highly unrealistic” (Hommes, 2006, p. 1114). Second, we have found that the extent of mispricing persistence crucially depends on the presence of trend following strategies, which can markedly lengthen mispricing correction.

Furthermore, our framework is also able to reproduce negative serial correlations in returns over long horizons, which are explained by a low portion of arbitrageurs and an extremely large portion of trend followers.

In the next chapter, we relax the assumption according to which arbitrageurs (or *fundamentalists*) end up by knowing the true fundamental value of the asset. We rather model an asset market wherein fundamentalists forecast future prices cum dividend through an adaptive learning rule. This framework enables us to assess, for instance, the effect of fundamentalists memory on the price dynamics. Besides, we investigate whether such a simple heterogeneous agent model can explain both positive and negative serial correlations in returns over differing horizons as well as excess volatility.

Chapter 4

Fundamentalists, Chartists and Asset Pricing Anomalies

4.1 Introduction

In the previous chapter, we investigated the statistical properties of artificially generated time series - mainly regarding autocorrelation patterns of returns - through a model of linear price adjustment with partially informed arbitrageurs. Numerical simulations reveal that our model is able to reproduce positive serial correlations of returns over short horizons. Furthermore, our analysis suggests that *(i.)* even when arbitrageurs dominate the market, positive serial correlations of returns over short horizon, which characterize mispricing persistence, can be explained by trend following strategies; *(ii.)* when chartists dominate the market, positive serial correlations of returns over short horizons tend to vanish as contrarian traders dominate the chartist population. As a result, arbitrage as well as contrarian strategies may favor mispricing correction. Trend following strategies, in contrast, generate positive serial correlations of returns over short horizons, which further confirms that mispricing persistence can be explained by trend following strategies.

In this chapter, we build a heterogeneous agent model in order to investigate predictability as well as volatility of returns in financial markets. While under the efficient market hypothesis, excess returns should not be predictable and no sign of autocorrelation should be observed in financial time series, many studies have supported the fact that returns are predictable. Excess returns are negatively correlated over long horizons (*e.g.*, Fama and French, 1988) and they are positively correlated over short horizons (*e.g.*, Lo and MacKinlay, 1988). Besides, many works have suggested that stock prices exhibit *excess volatility* (*e.g.* LeRoy and Porter, 1981; Shiller, 1981, 1992) and that stock price movements may not

coincide with fundamental news (Frankel and Froot, 1986; Cutler, Poterba, and Summers, 1989; Ofek and Richardson, 2003). These empirical “anomalies” are not fully explained by the classical asset pricing theory (Shiller, 1992). The agent-based approach to finance (see, for instance, LeBaron (2000) and Barberis and Thaler (2003) for extensive surveys of the literature) has however tried to overcome the weaknesses of the classical asset pricing theories in explaining anomalies in financial markets. In agent-based models, markets are populated by heterogeneous boundedly rational agents (*e.g.*, Cutler, Poterba, and Summers, 1991; Lakonishok, Shleifer, and Vishny, 1994), using rules of thumb strategies.

This chapter contributes to the ongoing debate about the explanation of the above-described anomalies by studying asset price dynamics in an agent-based model with heterogeneous trading strategies. Specifically, we investigate whether serial correlations in returns as well as excess volatility can be explained by the interplay of fundamentalists, who set their strategies from the inference of the asset fundamentals, and chartists, who set their strategies on the observation of past price movements.¹ The main novelty of our study consists in the assumption regarding the knowledge of fundamentals. In fact, most of the works based on such an approach have so far concentrated on the assumption that fundamentalists know the true value of the fundamentals (see, for instance, Zeeman, 1974; Frankel and Froot, 1986; Farmer and Joshi, 2002; Westerhoff, 2003). In our analysis, we depart from this benchmark, by assuming that fundamentalists do not have complete knowledge of the fundamentals, rather they can only try to predict the evolution of some related statistics, namely, dividends. More precisely, we assume

¹Fundamentalists are considered arbitrageurs in previous chapters.

that fundamentalists update their expectations through a first-order autoregressive learning rule: today's expectation of tomorrow's price is a convex combination of yesterday's expectation of today's price and yesterday's price (see Hommes, 1994; Barucci, 2000; Barucci, Monte, and Renò, 2004, for similar attempts in this direction). The starting point of our study is the work by Barucci, Monte, and Renò (2004). In a framework similar to ours, they establish that the presence of boundedly rational fundamentalists is able to simultaneously generate positive correlations in returns over short horizons and negative ones over long horizons. Our work, in contrast, consists in investigating the induced price dynamics when chartists are introduced in the market. The aforementioned departure is actually supported by empirical evidence (for detailed discussion and references on survey data works, which have highly influenced the development of heterogeneous agent models based on the distinction between fundamentalists and chartists, see Section 1.4 and 1.4.1).

Numerical simulations reveal that the interplay of fundamentalists and chartists is able to robustly generate both trends in price over short horizons and oscillations in price over long horizons, as well as excess volatility, hence reproducing the anomalies observed in the empirical literature. First, short-run dependencies in financial time series may be explained by the presence of trend followers in the market. Moreover, agents' memory seems to play a significant role. Trends in price tend to vanish when fundamentalists have short memory, and to be strengthened for higher memory values. It follows that in the presence of chartists, fundamentalist trading strategies may weaken predictability of returns over short horizons when fundamentalists have short memory. Lastly, we find that excess volatility

of returns tends to vanish when fundamentalists have long memory. This finding suggests that short memory can explain excess volatility of returns.

The chapter is organized as follows. Section 4.2 presents the stochastic model of price formation with boundedly rational agents. Section 4.3 analyzes the results of the numerical simulations. In this section, we investigate *(i.)* whether our model can reproduce serial correlations of returns and excess volatility; *(ii.)* what is the role played by the composition of the population in the market as well as the memory of fundamentalists. Lastly, Section 4.4 summarizes the main findings of this work and concludes.

4.2 The Model

Consider a market in which there are two assets: a risk-free asset and a risky one. The risk-free rate, which is assumed to be constant over time, is r_f . The risky asset delivers dividends D_t that are modeled through a trend stationary AR(1) process:

$$D_{t+1} = \theta + \beta(t + 1) + \gamma D_t + \sigma Z_{t+1} \quad (4.1)$$

where θ , β , γ and σ are constant parameters ($\gamma < 1$, $\sigma > 0$) and $(Z_t)_{t \geq 1}$ is a sequence of i.i.d. normal random variables with $E[Z_t] = 0$ and $E[Z_t^2] = 1$ for $t \geq 0$ (see Timmermann, 1996).

In the market, there are two types of boundedly rational agents,² namely,

²The economy is not common knowledge, *i.e.*, agents do not know the true dividend process as described in eq. (4.1). In this respect, agents are not fully rational.

fundamentalists and chartists. Fundamentalists employ a recursive learning mechanism to update their beliefs about the price cum dividend. In order to derive fundamentalist behavioral rule, we closely follow Barucci, Monte, and Renò (2004). In a rational expectation environment, with risk neutral rational traders aiming at exploiting arbitrage opportunities, the no arbitrage condition entails that today's price be equal to the discounted expectation of tomorrow's price plus dividend.³ The expectation is given by the conditional expectation and therefore:

$$S_t = \frac{1}{r_f} \mathbb{E}[S_{t+1} + D_{t+1} | F_t] \quad (4.2)$$

Under full rationality, there is a unique rational expectation solution to (4.1) - (4.2) (see Timmermann, 1996):

$$S_t = \frac{\gamma}{r_f - \gamma} D_t + \frac{r_f}{(r_f - 1)(r_f - \gamma)} \left(\theta + \frac{r_f}{r_f - 1} \beta + \beta t \right) \quad (4.3)$$

In what follows, though, fundamentalists are not assumed fully rational. More precisely, they compute the expected price cum dividend, denoted $X_t = S_t + D_t$ at time t , according to an adaptive learning mechanism as a smoothed average of observed prices (see Barucci, 2000). Let \hat{X}_t denote the expectation at time t of the price cum dividend at time $t + 1$. According to the adaptive-learning scheme, it follows that:

$$\hat{X}_t = \hat{X}_{t-1} + \alpha_t (X_{t-1} - \hat{X}_{t-1}) \quad (4.4)$$

where α_t is the learning coefficient ($0 \leq \alpha_t \leq 1$). In this work, the coefficient α_t is assumed to be constant (α). The coefficient α describes the memory of the

³The discount factor is given by the risk-free rate.

learning mechanism: α near 0 means that agents have a long memory, in this case remote and recent observations are weighted almost the same way ($1/t$). On the contrary, when α is near 1, agents have a short memory: recent observations have a weight larger than remote ones. Memory is thus decreasing in the value of α , *i.e.*, agents have longer memory as α decreases.

The no arbitrage condition described in eq. (4.2) applied to the current environment, with boundedly rational agents, gives us:

$$\hat{S}_t = \frac{1}{r_f} \hat{X}_t \quad (4.5)$$

In addition, in this work, not all of the agents in the market try to predict dividends. Indeed, chartists merely base their trading strategies on the observation of prices that have prevailed in the market. They try to predict future prices from past price movements. In what follows, we consider two types of chartists, namely, trend followers and contrarian traders (for detailed discussion and references on the behavioral and empirical motivations of the presence of contrarian traders in this work, see Section 2.2). Trend followers believe that any trend in past prices will repeat in the future. Contrarian traders rather believe that any trend in past prices will revert in the future.

In each period, agents can place buy or sell orders in the market. Fundamentalist orders are captured as:

$$d_t^F = \bar{\beta} \left(\hat{S}_t - P_{t-1} \right) \quad (4.6)$$

where $\bar{\beta}$ is a positive reaction coefficient.

Trend follower orders are expressed as:

$$d_t^T = \varphi (P_{t-1} - P_{t-2}) \quad (4.7)$$

where φ is a positive reaction coefficient.

Lastly, contrarian trader orders are expressed as:

$$d_t^C = -\varphi (P_{t-1} - P_{t-2}) \quad (4.8)$$

where φ is a positive reaction coefficient.

Following Farmer and Joshi (2002), we assume that in each period, a market maker mediates all transactions and sets the price according to aggregate excess demand in the market:⁴

$$P_t = P_{t-1} + \mu(d_t^F q^F + d_t^T q^T + d_t^C q^C) + \varepsilon_t \quad (4.9)$$

where μ is a positive price adjustment parameter. The terms q^F , q^T and q^C represent the portion of fundamentalists, trend followers and contrarian traders, respectively. It is worth mentioning that the sum of these terms is always equal to 1. The term ε_t captures any remaining random elements that may affect the market maker's price setting decision.⁵

⁴A market maker based method of price formation enables one to study the price dynamics induced by each trading strategy as well as by the interplay of agents following different trading strategies.

⁵The term ε_t may also represent the activity of noise traders in the market.

Substituting eq. (4.6), eq. (4.7) and eq. (4.8) into eq. (4.9) yields:

$$P_t = \mu \bar{\beta} q^F \hat{S}_t + (1 - \mu (\bar{\beta} q^F + \varphi(q^C - q^T))) P_{t-1} + \mu \varphi(q^C - q^T) P_{t-2} + \varepsilon_t \quad (4.10)$$

which constitutes our stochastic model driving the price dynamics.

4.3 Numerical Analysis

We consider the model described above and seek to investigate whether such a simple framework - with linear behavioral rules and linear price formation rule - based on the bounded-rationality hypothesis may reproduce some empirical regularities that are not fully explained within the classical asset pricing theory with rational expectations, namely, serial correlations of returns and excess volatility. The work by Barucci, Monte, and Renò (2004) establishes, in a framework similar to ours, that the presence of boundedly rational fundamentalists is able to simultaneously generate positive correlations in returns over short horizons and negative ones over long horizons. Our work rather consists in investigating the induced price dynamics when chartists are introduced in the market.

Computer simulations are required because the learning rule introduces a high degree of nonlinearity into the solution, so that the model cannot be solved analytically. We simulate the model⁶ with 5,000 time steps and using the following parameter values as estimated in Timmermann (1996) on the Standard & Poor 500 time series for the period (1873-1992): $\theta = 0.47$, $\beta = 0.022$, $\gamma = 0.9$ and

⁶The code, written in MATLAB, is available from the author upon request.

$\sigma^2 = 0.25$.⁷ The remaining parameter values used in the simulations are reported in Table 4.1.

Parameters	Values
Fundamentalist reaction coefficient ($\bar{\beta}$)	1
Chartist reaction coefficient (φ)	1
Market maker price adjustment parameter (μ)	1
Memory of the learning mechanism (α)	From 0 to 1

Table 4.1: Parameters for the simulations.

Our investigation of the statistical properties of artificially generated series is presented in the following two subsections.⁸ We begin by presenting results regarding return predictability. For this purpose, we are interested in evaluating whether *(i.)* significant autocorrelation values are detected and *(ii.)* autocorrelation patterns emerge at different horizons. Next, we move to study excess volatility of returns. In this regard, we assess whether excess volatility emerges in the market when both fundamentalists and chartists are present. This is done by comparing the asset price bounded rationality dynamics with the one obtained under rational expectations.

⁷As suggested in Barucci, Monte, and Renò (2004), we set $\beta = 0$ in order to have a stationary time series.

⁸A preliminary simulation analysis shows that the variability across Monte Carlo simulations is very weak. In addition, results are robust to changes in initial conditions.

4.3.1 Explaining Serial Correlations in Returns

The role of market composition

As a first step in our investigation, we first examine the price dynamics induced by each trading strategy as well as by the interplay of fundamentalists and chartists. Therefore, the value of the parameter α is held constant at the benchmark level *i.e.*, $\alpha = 0.5$.

First, it is worth focusing on the effect of fundamentalist strategies on the price dynamics, while chartists are present in the market. At this point, we examine a setting where chartists are split between trend followers and contrarian traders (*i.e.*, $q^T = q^C$). In such a setting, according to eq. (4.7) and (4.8), trend follower's orders and contrarian trader's ones exactly compensate, so that chartist trading activity does not further affect the evolution of the asset price. Fig. 4.1 shows the sample autocorrelation function of returns when there are only chartists in the market (*i.e.*, $q^F = 0$). As illustrated in Fig. 4.1, autocorrelation patterns do not emerge.

However, when fundamentalists are introduced in the market, as indicated in Fig. 4.2, simulations reveal that positive serial correlations in returns over short horizons and negative ones over long horizons emerge.

Our setting is actually able to reproduce the anomalies regarding serial correlations in returns, namely, positive serial correlations over short horizons and negative serial correlations over long horizons. Nevertheless, it is worth noting that the presence of chartists plays a key role. Indeed, while their trading activity does not further affect the evolution of the asset price (because $q^T = q^C$), chartist strategies

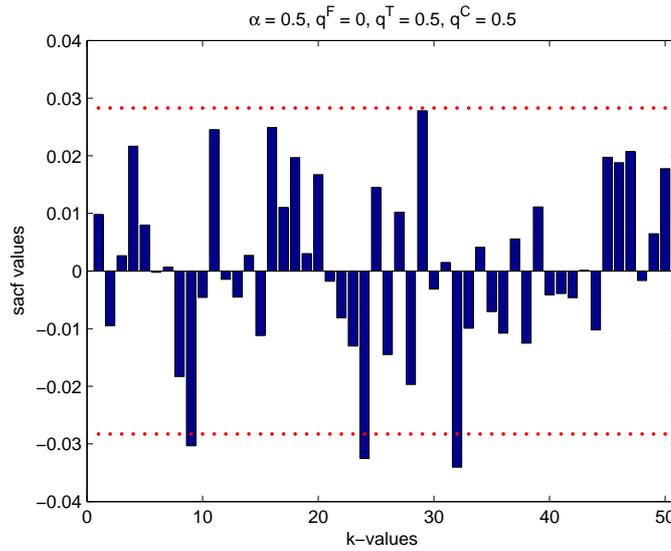


Figure 4.1: Sample autocorrelation function of returns when there are only chartists in the market and chartists are split between trend followers and contrarian traders.

strengthen positive serial correlations over short horizons as well as negative serial correlations over long horizons. In fact, from Fig. 4.2, it can be observed that the extent of positive coefficients over small lags is decreasing in the portion of fundamentalists. The presence of fundamentalists tends to slightly reduce predictability of returns over short horizons. Furthermore, when the portion of fundamentalists is markedly lower than the portion of chartists (*e.g.*, $q^F = 0.2$ and q^T and q^C equals 0.4), chartist strategies tend to strengthen oscillations in prices over long horizons. From Fig. 4.2a, it can be noticed that negative coefficients over large lags persist for larger time window. The presence of chartists in the market tends to induce a greater degree of dependence in returns over long horizons (*i.e.*, mean-reversion effect is stronger).

We now examine the price dynamics that emerge in the market for differing

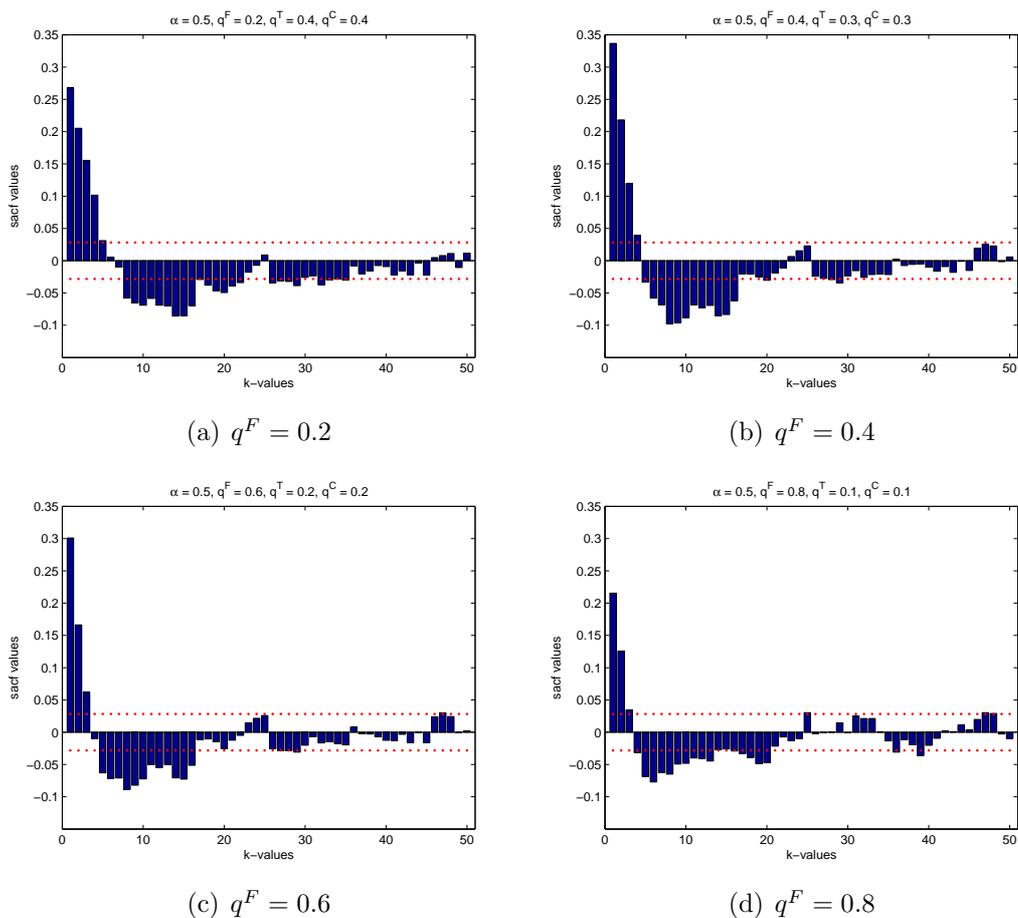


Figure 4.2: Sample autocorrelation functions of returns for differing portions of fundamentalists (q^F).

portions of trend followers versus contrarian traders. Simulation results are shown in Fig. 4.3 and 4.4. First, from Fig. 4.3, serial correlations in returns seem not crucially dependent on the portions of trend followers versus fundamentalists in the market. However, as illustrated in Fig. 4.4, positive (*negative*) serial correlations in returns over short (*long*) horizons tend to vanish as the portion of contrarian traders outweighs the portion of trend followers in the market.

This is mainly explained by the fact that while trend following strategies tend

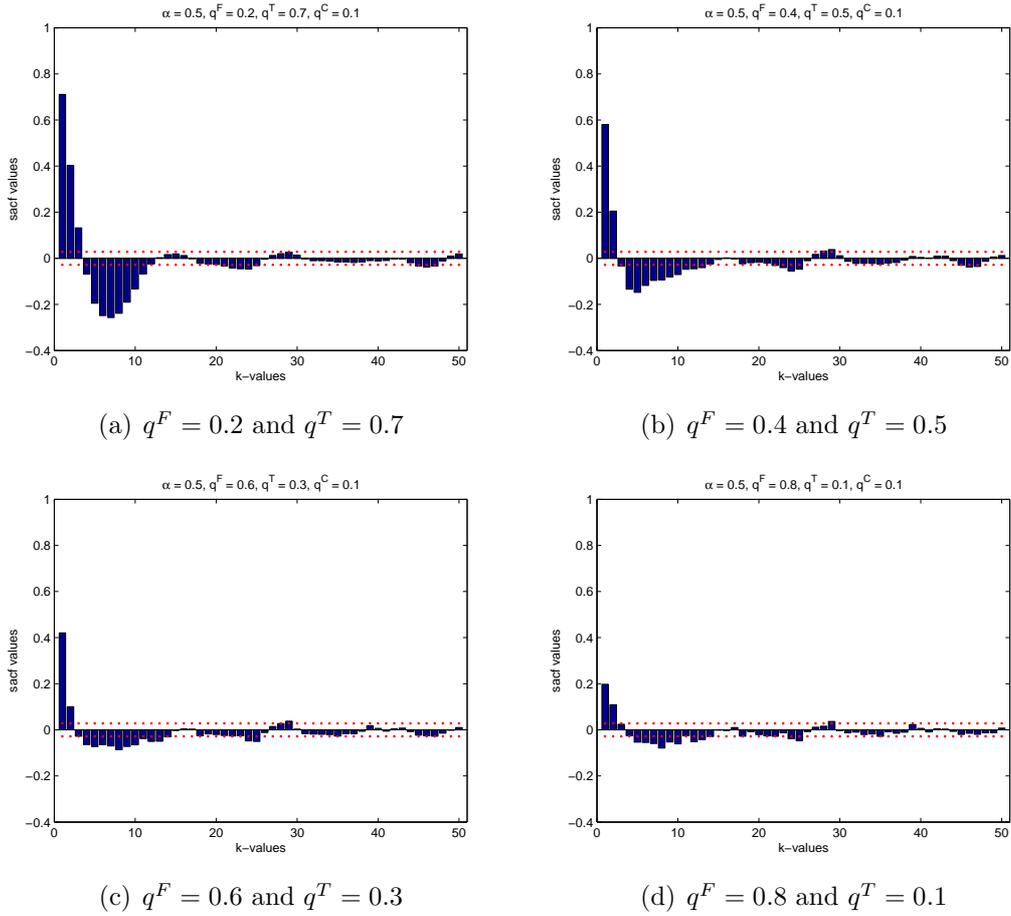


Figure 4.3: Sample autocorrelation functions of returns for differing values of q^F and q^T .

to induce short-term trends in prices (e.g., Farmer and Joshi, 2002), contrarian strategies tend to induce long-term price mean reversion. As long as the portion of trend followers outweighs the portion of contrarian traders in the market, trends in price induced by trend following strategies dominate oscillations in price induced by contrarian strategies (*i.e.*, weaker mean-reversion effect), so that returns tend to exhibit positive serial correlations over short horizons.

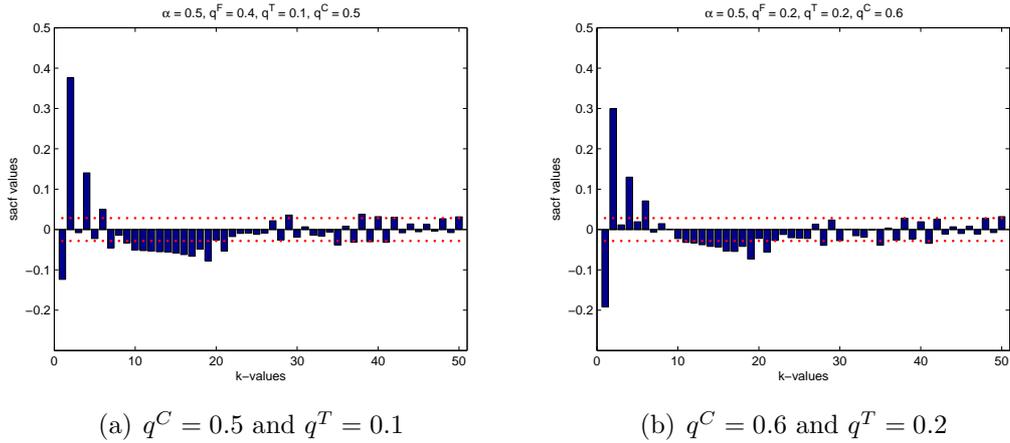


Figure 4.4: Sample autocorrelation functions of returns when q^C is greater than q^T .

The role of fundamentalists' memory

While with $\alpha = 0.5$, we have seen that the presence of chartists, especially trend followers, tends to strengthen positive (*negative*) serial correlations in returns over short (*long*) horizons, in order to complete this study, it is worth investigating the effect of the learning coefficient α on the patterns identified above. With this purpose in mind, we study the price dynamics that emerge for differing values of α .⁹

First, when chartist trading activity does not further affect the evolution of the asset price (*i.e.*, $q^T = q^C$), as shown in Fig. 4.5, simulations reveal that trends in price over short horizons tend to vanish as fundamentalists have short memory (*i.e.*, $\alpha \rightarrow 1$) (see Fig. 4.5a and Fig. 4.5b). Indeed, positive coefficients over small lags are both smaller and less persistent as α increases. As a result, when chartists are present in the market, fundamentalist strategies may weaken predictability of returns over short horizons, when fundamentalists have short memory. Conversely,

⁹This is done by varying the value of α from 0 to 1 with step 0.1.

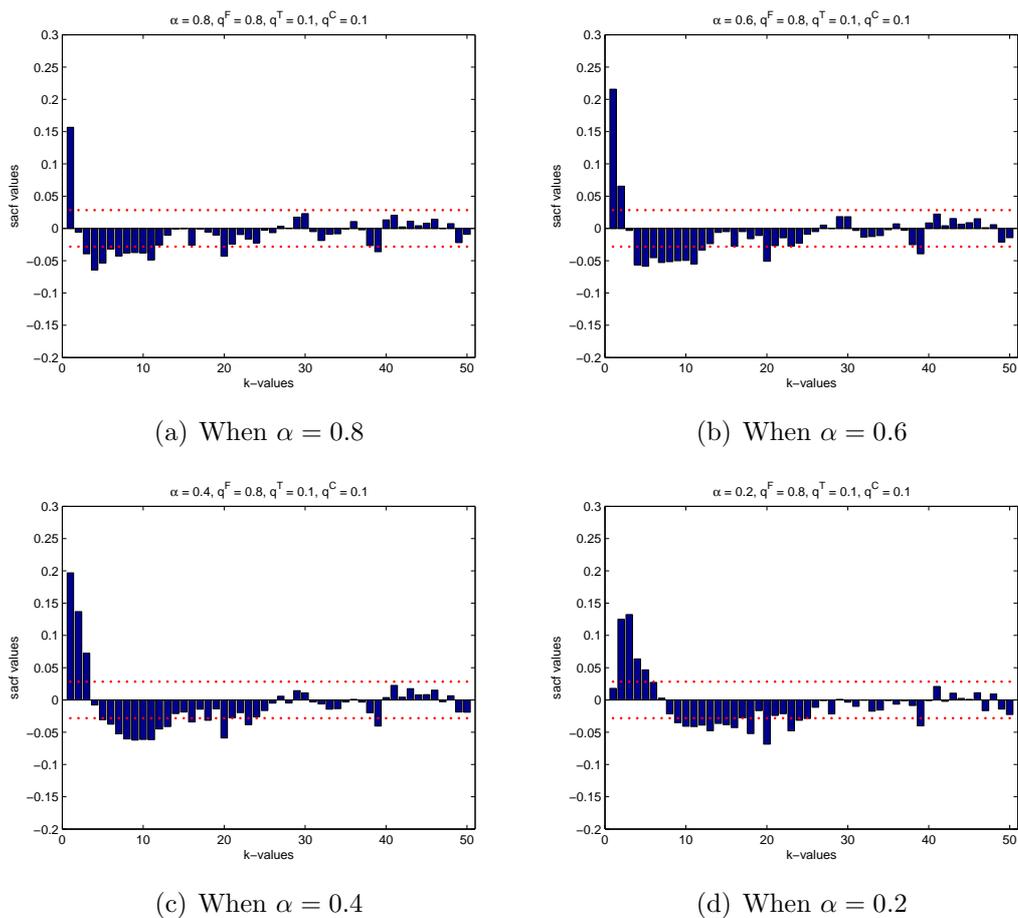


Figure 4.5: Sample autocorrelation functions for differing values of α and $q^T = q^C$.

when fundamentalists have long memory (*i.e.*, $\alpha \rightarrow 0$), returns exhibit more persistent positive serial correlations over short horizons (see Fig. 4.5c and Fig. 4.5d). When chartists are present in the market, longer memory values tend to induce both trends in price over short horizons and oscillations in price over long horizons.¹⁰

¹⁰With differing portions of trend followers versus contrarian traders, simulation results do not crucially differ from the aforementioned ones.

From the above considerations, our setting actually reproduces simultaneously positive serial correlations over short horizons and negative serial correlations over long horizons. Furthermore, we have established that trends in price can be explained by long memory in the learning mechanism of fundamentalists. Our setting might thus offer a valid framework in order to explain some stylized facts that emerge in real time series.

4.3.2 Explaining Excess Volatility of Returns

In the previous subsections, the investigation of the statistical properties of artificially generated series was focused on autocorrelation patterns. In the following subsections, we turn to focus on excess volatility of returns. Indeed we now examine whether excess volatility of returns that often emerges in financial time series (Shiller, 1981) can emerge in our framework. We mainly compare the asset price bounded rationality evolution to the one obtained under rational expectations as described in eq. (4.3).

The role of market composition

As a first step in this investigation, as before, we first focus on the price dynamics that emerge in the market when the value of α is held constant at the benchmark level *i.e.*, $\alpha = 0.5$.

First, as in the previous subsection, it is worth focusing on the effect of fundamentalist strategies on the price dynamics, while chartists are present in the market but do not further affect the evolution of the asset price. As mentioned

previously, this is implemented by setting q^T equals to q^C and by varying the portion of fundamentalists in the market (from 0 to 1).

Fig. 4.6a presents the volatility of the returns¹¹ from the asset price bounded rationality evolution for differing portions of fundamentalists (q^F). Fig. 4.6b instead shows the excess volatility of the returns¹² for differing portions of fundamentalists (q^F). Simulations clearly reveal that the volatility of the returns from the simulated time series, as well as the extent of excess volatility, is increasing in the portion of fundamentalists in the market, *ceteris paribus*. The larger the portion of fundamentalists in the market, the more volatile the returns are.

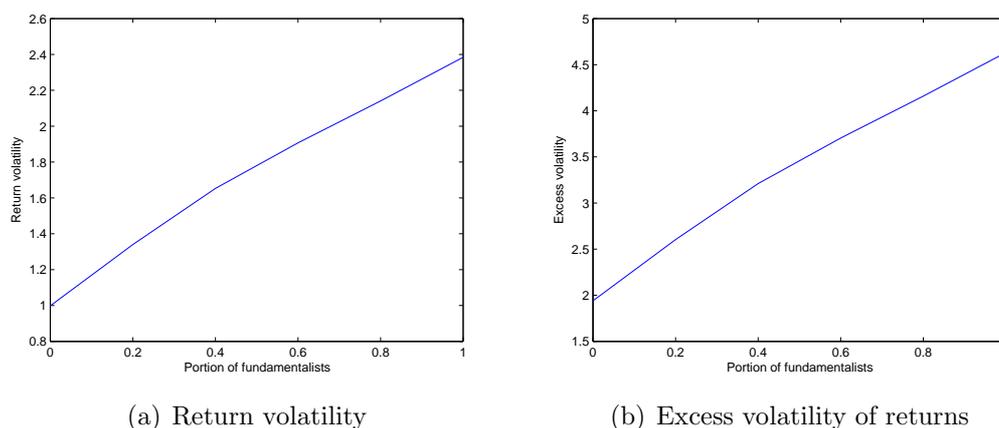


Figure 4.6: Market volatility as a function of the portion of fundamentalists (q^F) when α equals 0.5.

In consequence, when the chartist population is split between trend followers and contrarian traders, the presence of chartists tends to stabilize the market.

However, from Fig. 4.7, indicating the relative excess volatility of the returns¹³

¹¹This is measured through the standard deviation of the returns of the simulated time series.

¹²This is measured through the ratio of the volatility of the returns from the simulated time series and the volatility of the dividend returns.

¹³This is measured through the ratio of excess volatility of the returns from the simulated time

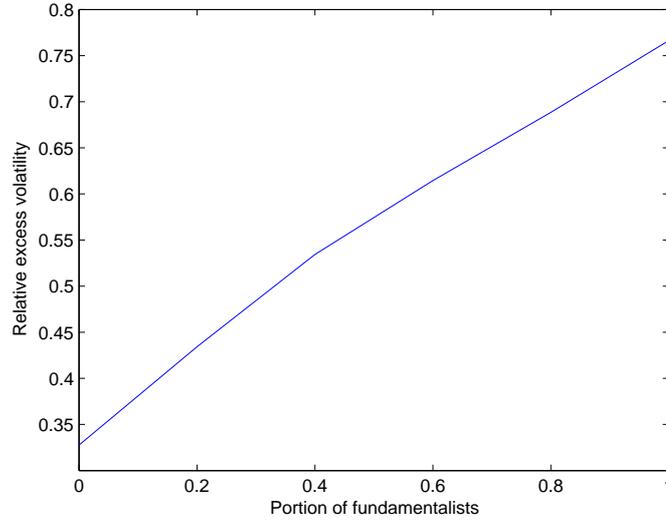


Figure 4.7: Relative excess volatility as a function of the portion of fundamentalists (q^F) when α equals 0.5.

for differing portions of fundamentalists, it can be observed that even when there are only fundamentalists in the market ($q^F = 1$), the simulated time series tend to be less volatile than the rational expectation (RE) price evolution.¹⁴ Indeed in this case, the relative excess volatility is lower than 1. Furthermore, Fig. 4.7 suggests that when chartists are present in the market ($q^F < 1$), the relative excess volatility is even lower than when there are only fundamentalists in the market. As a result, the foregoing composition of the population does not actually explain the emergence of excess volatility of returns. Rather the presence of chartists - both trend followers and contrarian traders - seems to further stabilize the price dynamics.

series and excess volatility of the RE time series.

¹⁴It is worth noting that the excess volatility of the RE price can be derived from eq. (4.3) *i.e.*, it is measured by $(\frac{\gamma}{r_f - \gamma})$. More precisely, according to the parameter values used in the simulations, in this work $(\frac{\gamma}{r_f - \gamma}) = 6$.

In order to further study the effect of chartist strategies, we now examine the price dynamics that emerge in the market for differing portions of trend followers versus contrarian traders. The first two columns of Table 4.2 show the relative excess volatility of the returns when the learning coefficient is 0.5 and for differing portions of fundamentalists.¹⁵ This table clearly reveals that the price dynamics induced by such a composition of the population does not exhibit excess volatility of returns. Indeed when α equals 0.5, the values for the relative excess volatility are lower than 1 for any portion of fundamentalists (q^F).

	$\alpha = 0.5$		$\alpha = 0.9$	
	$q^F = 0.2$	$q^F = 0.9$	$q^F = 0.2$	$q^F = 0.9$
only trend followers	0.8839	0.7413	0.9749	1.0278
only contrarian traders	0.8067	0.7343	0.8365	1.0141

Table 4.2: Relative excess volatility of the returns for differing values of α and q^F .

While, as explained in the previous subsections, our setting (with α held constant) could reproduce serial correlations in returns, as suggested by empirical evidence, and could provide a way to go in order to explain these price anomalies, it does not enable us to explain excess volatility of returns.

However, it is worth carefully examining the effect of trend followers and contrarian traders on the price dynamics. Fig. 4.8, for instance, presents the volatility of the returns from the simulated time series for differing portions of trend followers (when $q^F = 0.2$).

¹⁵In this table, we concentrate on comparing the values when the portion of fundamentalists is *high* ($q^F = 0.9$) with the case when the portion of fundamentalists is *low* ($q^F = 0.2$).

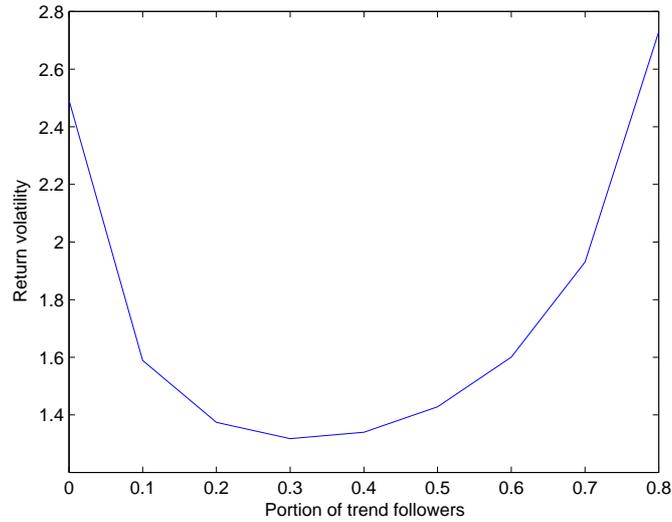


Figure 4.8: Market volatility as a function of the portion of trend followers (q^T) when α equals 0.5 and q^F equals 0.2.

Fig. 4.8 clearly reveals that, when the chartist population is split between trend followers and contrarian traders, chartists may have a stabilizing effect on the price dynamics.¹⁶ Indeed, the volatility of returns is first decreasing in the portion of trend followers, then increasing. This is actually explained by the fact that as long as contrarian traders dominate trend followers in the market, the former help to reduce return volatility. In contrast, when the portion of trend followers outweighs the portion of contrarian traders, the volatility of returns tend to be amplified. This finding suggests that, on the one hand, contrarian strategies mainly tend to dampen market volatility, hence stabilize the market. On the other hand, trend followers alone - or when they dominate the chartist population - tend to generate return volatility.

¹⁶This pattern actually holds for q^F varying from 0.1 to 0.8.

The role of fundamentalists' memory

While with $\alpha = 0.5$, we have seen that the interplay of fundamentalists and chartists does not seem to explain the emergence of excess volatility of returns, in order to complete this work, it is worth trying to assess the effect of the memory in the learning mechanism of fundamentalists on the simulated price dynamics. This enables us to test whether excess volatility of returns may depend on the memory of fundamentalists. For this purpose, we study the price dynamics that emerge for differing values of α .

As a first step in this investigation, we focus on the price dynamics that emerge when there are only fundamentalists in the market. Fig. 4.9 illustrates the effect of the learning coefficient (α) on the volatility and the excess volatility of the simulated price dynamics when there are only fundamentalists in the market.¹⁷

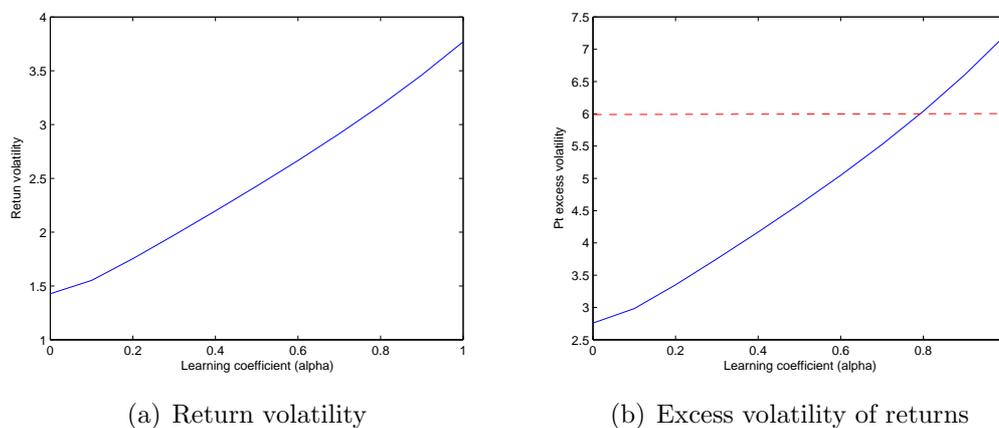


Figure 4.9: Market volatility as a function of the learning coefficient (α) when q^F equals 1.

¹⁷The *dash* line in Fig. 4.9b represents the threshold where the simulated time series are as volatile as the RE price dynamics.

Simulations clearly reveal that the memory of the learning mechanism plays a crucial role and may contribute to explaining the emergence of excess volatility of returns.

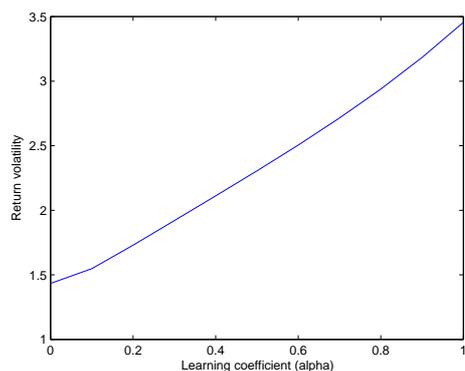
Indeed first, as shown in Fig. 4.9a, short memory in the learning mechanism of fundamentalists (*i.e.*, as $\alpha \rightarrow 1$) favors volatility of returns. The simulated time series tend to be more volatile when fundamentalists have short memory. Fundamentalists with long memory (*i.e.*, $\alpha \rightarrow 0$) rather tend to stabilize the market. Indeed when α tends to 0, the simulated time series tend to be less volatile. Second, from Fig. 4.9b, it can be observed that the effect of the memory in the learning mechanism of fundamentalists is enough pronounced to markedly generate excess volatility of returns.¹⁸ Indeed, when fundamentalists have short memory (*i.e.*, as $\alpha \rightarrow 1$), the simulated time series exhibit greater excess volatility than the theoretical RE one.¹⁹ When fundamentalists have long memory (*i.e.*, $\alpha \rightarrow 0$), excess volatility of returns from the simulated time series is rather lower than the RE one. More precisely in this case, the relative excess volatility is lower than 1. Consequently, excess volatility of returns may be explained by short memory in the learning mechanism of fundamentalists.

In our setting however, as mentioned in section 2, there are not only fundamentalists in the market but also chartists, both trend followers and contrarian traders. With $\alpha = 0.5$, as presented in the previous subsections, we have seen that our setting is able to simultaneously reproduce positive serial correlations of returns over short horizons and negative ones over longer horizons, though, it

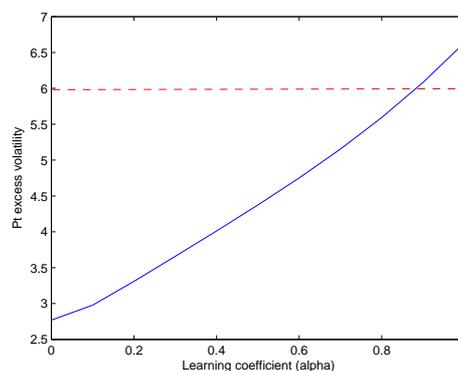
¹⁸It is worth reminding that the theoretical RE excess volatility is determined by $(\frac{\gamma}{r_f - \gamma})$, represented by the dash line in Fig. 4.9b.

¹⁹More precisely, this hold from $\alpha = 0.8$.

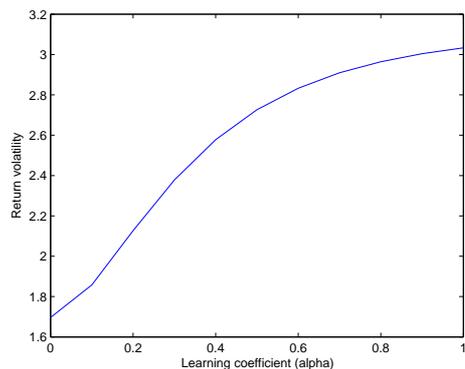
does not enable us to explain excess volatility of returns. Nonetheless, varying the value of the parameter α , simulation results support previous findings, namely, the memory of the learning coefficient plays a crucial role.



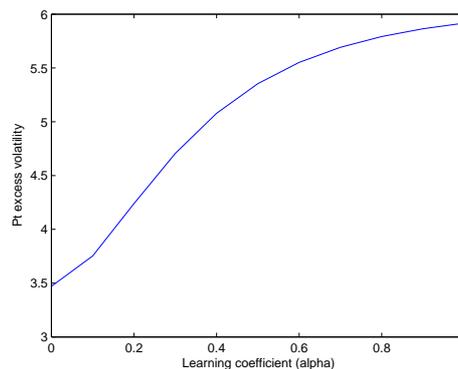
(a) Return volatility when $q^F = 0.9$ and $q^C = 0.1$



(b) Excess volatility of returns when $q^F = 0.9$ and $q^C = 0.1$.



(c) Return volatility when $q^F = 0.2$ and $q^T = 0.8$



(d) Excess volatility of returns when $q^F = 0.2$ and $q^T = 0.8$

Figure 4.10: Market volatility as a function of the learning coefficient (α) and for differing portions of fundamentalists. The *top* panel exhibits market volatility when the portion of fundamentalists is *high*. The *bottom* panel exhibits market volatility when the portion of fundamentalists is *low*.

In fact, supporting the aforementioned results - when there are only fundamentalists in the market - first, as illustrated for instance in Fig. 4.10a, the volatility

of the simulated time series appears to be much larger when fundamentalists have short memory *i.e.*, when α tends to 1. Return volatility is increasing in the value of α . Second, as presented in Fig. 4.10b, the simulated time series exhibit relative excess volatility when fundamentalists have short memory (*i.e.*, as $\alpha \rightarrow 1$).²⁰ More precisely, when $\alpha > 0.8$, the excess volatility of the simulated time series is greater than the RE one.

Nevertheless, mere comparison of Fig. 4.10b and Fig. 4.10d clearly reveals that relative excess volatility tends to emerge only when the portion of fundamentalists is large (*i.e.*, $q^F \rightarrow 1$).²¹ Indeed Fig. 4.10d, which shows the excess volatility of returns as a function of the learning coefficient, suggests that the relative excess volatility is lower than 1 when the portion of fundamentalists is low (*e.g.*, $q^F = 0.2$). Short memory in the learning mechanism of fundamentalists (*i.e.*, $\alpha \rightarrow 1$) is a necessary condition, but not a sufficient one, for the existence of relative excess volatility. This finding is actually supported by Table 4.2. Indeed, the figures presented in the last two columns confirm that relative excess volatility tends to vanish when chartists - trend followers or contrarian traders - dominate fundamentalists in the market. Relative excess volatility is greater than 1 only if the portion of fundamentalists is large. Furthermore, Table 4.2 clearly illustrates that excess volatility emerges only when fundamentalists have short memory (*e.g.*, $\alpha = 0.9$) and the portion of fundamentalists in the market is large (*e.g.*, $q^F = 0.9$). As a result, when both fundamentalists and chartists are present in the market, and the former have short memory, chartist strategies may stabilize market prices.

²⁰The *dash* line in Fig. 4.10b represents the RE excess volatility.

²¹More precisely this finding holds for $\alpha > 0.8$ and $q^F > 0.7$.

To sum up, simulations reveal that excess volatility tends to emerge when both of the following conditions are fulfilled:

- (i.) the value of the learning coefficient (α) is high ($0.8 \leq \alpha \leq 1$);
- (ii.) the portion of fundamentalists in the market is high ($0.8 \leq q^F \leq 1$).

So from the above considerations, our setting is actually able to reproduce excess volatility of returns for differing values α . Excess volatility emerges when fundamentalists have short memory and they dominate the market. Furthermore, while it has often been suggested that chartist strategies, especially trend following ones, tend to destabilize market prices (see Lux, 1995; Brock and Hommes, 1998; Lux and Marchesi, 2000; Farmer and Joshi, 2002), in our setting, chartist strategies could also, under some circumstances, stabilize market prices.

4.4 Conclusions

By assuming that boundedly rational agents follow differing trading strategies, namely, fundamentalist and chartist; and that fundamentalists update their expectations of future prices through a first-order autoregressive learning rule, we show that the price dynamics reproduce predictability of returns, namely, positive serial correlations over short horizons and negative ones over long horizons; as well as excess volatility of returns.

More precisely, we have shown that in our model, as suggested in most of the works that emphasize the importance of fundamentalist and chartist strategies, chartists tend to destabilize the market. When chartists are present in the market, short-run dependencies in time series can be explained by the presence of

trend followers.

Second, as suggested in Barucci, Monte, and Renò (2004), we found that the memory of the learning mechanism plays a crucial role. However, while Barucci, Monte, and Renò (2004) found that a longer memory induces a smaller degree of dependency when the horizon of the returns is long (*i.e.*, weaker mean-reversion effect), in our setting, when fundamentalists as well as chartists are present in the market, fundamentalist strategies may weaken predictability of returns over short horizons, when fundamentalists have short memory. In contrast, longer memory values tend to induce both trends in price over short horizons and oscillations in price over long horizons.

Third, while excess volatility may be explained by the presence of trend followers (see for instance Lux, 1995; Brock and Hommes, 1998; Lux and Marchesi, 2000; Farmer and Joshi, 2002), in our setting, when fundamentalists as well as chartists - both trend followers and contrarian traders - are present in the market, (*i.*) excess volatility of returns tends to vanish when the portion of chartists, including trend followers, is large. The presence of chartists rather stabilize market prices. (*ii.*) excess volatility of returns tends to emerge when fundamentalists have short memory (*i.e.*, $\alpha \rightarrow 1$). Longer memory values rather tend to stabilize the market. Excess volatility may thus be explained by short memory in the learning mechanism of fundamentalists.

Concluding Remarks

In this dissertation, we explored the role played by agents' heterogeneity in shaping asset prices' behaviors. Extensive empirical evidence on asset pricing anomalies suggests that traditional finance provides an inaccurate description of asset prices' behaviors. An alternative approach, based on developments in behavioral finance, consists in putting at the center of the analysis agents' behaviors as well as agents interactions.

First, while markets are mostly efficient, asset prices may happen to be misaligned for some periods of time and asset returns happen to be, at least partially, predictable. With this in mind, the aim of Chapter 1 was twofold. We sought to understand:

(*i.*) why the traditional finance theory may be powerless to explain some asset pricing anomalies observed in financial markets (*e.g.*, financial bubbles and crashes, momentum and reversals in asset returns, implying some predictability of returns, excess volatility of returns, implying that market prices are not always consistent with fundamentals);

(*ii.*) why the behavioral finance theory can be an alternative, better approach to describe investors' as well as asset prices' behaviors.

Financial anomalies, which are hardly rationalized within a representative, rational agent framework, have seriously challenged the efficient market hypothesis. Behavioral finance, based on experimental evidence gathered by psychologists, has instead provided new light and insight on agents' and asset prices' behaviors. These developments conduced to build an alternative approach based on the bounded-rationality hypothesis and agents' heterogeneity.

The flourishing works in behavioral finance have been convincing enough to believe that investigating agents' behaviors is the right way to go in order to better understand asset prices' behaviors. Furthermore, once it has been well-documented that agents' "irrationality" can systematically and markedly affect asset prices, it is no longer satisfactory to build a framework based on agent rationality, which allows summarizing agents' behavior through a representative agent. In contrast, heterogeneous agent models have already been successful in providing alternative, better explanations of some of the remaining asset pricing anomalies. While a common opposition to the behavioral finance approach, focusing on some behavioral specificities, stresses that such an approach is unable to provide a unified theory of asset prices' behaviors, some steps have been already made in this direction and we believe that further work can still be done.

With this purpose in mind, in this dissertation, we proposed, in line with earlier heterogeneous agent models, based on the distinction between fundamentalists and chartists, three distinct models. An additional common feature of these models is that chartists are not only represented by trend followers who chase trends, we rather account for the fact that investors can also go against trends in prices

that may emerge in financial markets.

As a first step in this direction, in Chapter 2, we presented a simple heterogeneous agent model which aimed at assessing one of the predictions of the efficient market hypothesis *i.e.*, asset prices should be consistent with fundamentals (*e.g.*, Fama, Fisher, Jensen, and Roll, 1969; Fama, 1970). Our model is able to reproduce asset price misalignments *i.e.*, over-(under-)valued assets, which can be explained by the presence of trend following as well as contrarian strategies. Furthermore, despite its simplicity, our model is able to generate a wide variety of price dynamics, from stable to unstable ones. The findings of this simple model clearly indicate that (*i.*) the presence of arbitrageurs appears to be one of the backbone of price stability; (*ii.*) the composition of the population plays a crucial role in explaining financial phenomena. In particular, while earlier works based on the distinction between fundamentalists and chartists - mainly trend followers - have suggested that trend following strategies can destabilize market prices (*e.g.*, Lux, 1995; Lux and Marchesi, 1999), our model results extend these earlier findings to contrarian strategies as well. While there is extensive evidence of contrarian strategies profitability, our model suggests that this kind of strategies can also markedly affect asset prices.

In Chapter 3, we explored whether slow diffusion of information (or heterogeneous beliefs among arbitrageurs) could prevent arbitrageurs from bringing back asset prices towards fundamental values. Indeed, with respect to the model presented in Chapter 2, we relaxed the assumption according to which arbitrageurs are immediately aware of any news regarding asset fundamentals. In line with

earlier works (*e.g.*, Abreu and Brunnermeier, 2002, 2003), arbitrageurs are rather sequentially informed about any new information. We showed that this new model was able to reproduce some stylized facts observed in financial data, namely, positive serial correlations in returns over short horizons (*e.g.*, Lo and MacKinlay, 1988). Moreover, we found that slow diffusion of information and/or dispersion of opinions among arbitrageurs may influence the emergence of short-run dependencies in financial time series. Furthermore, while earlier works have established that trend following strategies tend to destabilize asset prices, especially when the portion of arbitrageurs is low, we found that this can be the case even when arbitrageurs dominate the market. Misalignments in asset prices, which persist over time, can hence be explained by trend following strategies, when information diffusion in the market is slow. However, when chartists dominate the market, we suggest that contrarian strategies can favor mispricing correction. While arbitrage strategies appear to play a key role in explaining that asset prices are consistent with fundamentals, the effect of some chartist strategies *i.e.*, contrarian ones, may be a crucial determinant as well. Moreover, while price underreaction has often been associated with slow diffusion of information, we demonstrated that this is not satisfactory enough, since the presence of contrarian traders can reduce positive serial correlations in returns over short horizons.

The main motivation of the model built in Chapter 4 rests on the inability of the models developed in previous chapters to explain excess volatility of returns as well. In Chapter 4, we turn to investigate the role of fundamentalists' memory on price dynamics. More precisely, we proposed a heterogeneous agent model wherein fundamentalists forecast future prices cum dividend through an adaptive

learning rule, in order to investigate predictability (*i.e.*, momentum and reversals in returns) as well as excess volatility of returns in financial time series.

First, in line with earlier heterogeneous agent models and similarly to the model of Chapter 3, we found that chartists can destabilize the market. Short-run dependencies in time series can be explained by the presence of trend followers. Second, while previous works have suggested that fundamentalists with longer memory induce weaker mean-reversion effect, we found that fundamentalists with short memory can reduce return predictability over short horizons. Longer memory values rather induce both trends in prices over short horizons and oscillations in prices over long horizons. Third, our model is also able to reproduce excess volatility of returns, which can be explained by the presence of fundamentalists with short memory.

Overall, through the simple heterogeneous agent models developed in this dissertation, we have been able to reproduce some of the stylized facts observed in financial data. Relaxing the representative, rational agent assumption, we provided some explanations of remaining anomalies. The model discussed in Chapter 4, for instance, explains simultaneously momentum and reversals in returns, as well as excess volatility of returns.

Of course, a lot of work is still necessary to provide a unified and complete theory of asset prices' behaviors. We now turn to discuss three possible lines for future research.

First, in the models developed in this dissertation, the market maker based method of price formation is linear. One opposition to this assumption would be that such

an assumption is too simplistic. Indeed, rather than adjusting prices according to order flows, market makers could adjust prices according to their positions as well. This would enable us to account for inventories as well as order flow signal. Westerhoff (2003), for instance, finds that inventory control may actually limit the positions of market makers and may cause markets to be less efficient. Extensive evidence has documented the key role played by market microstructures on price dynamics (*e.g.*, Bottazzi, Dosi, and Rebesco, 2005; Arifovic and Ledyard, 2007). Furthermore, the comparative statics experiments regarding the price adjustment parameter in Chapter 2 provide some elements to believe that in fact this element may be crucial in explaining price dynamics and it would be worth further investigating in this direction.

Second, in the models presented in this dissertation, agents were assumed to use a single trading strategy. One opposition to this assumption would be that over their life span in financial markets, investors tend to experiment different trading strategies. Indeed, investors tend to adjust their strategies according to, for instance, the environment or their performance. Furthermore, evidence suggests that local interactions among individuals actually have a significant effect on price dynamics (*e.g.*, Kirman, 1991, presents a model based on such an approach). The formation of investors' opinions appears to be a crucial element that could be worth accounting for when one wants to provide a description of asset prices' behaviors. From the works developed in this dissertation, this kind of improvement could, for instance, be implemented within the model discussed in Chapter 4. The memory of the learning mechanism within this model is assumed to be constant over time. However, fundamentalists' learning mechanism could be determined endogenously, based on the interactions among agents in the market. As a result, the learning

coefficient would be time varying, so as to reflect local interactions.

Lastly, while we have been able to propose simple frameworks which are successful in simultaneously explaining important anomalies observed in real financial time series, other stylized facts in financial markets have been well-documented as well (for instance, clustered volatility) (see *e.g.*, Mandelbrot, 1963; Kirman and Teyssiere, 2002). An interesting path for future research would be to investigate whether the simple frameworks presented in this dissertation would also be suitable to explain such financial phenomena.

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