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**CHOICE OF EXOGENOUS VARIABLES, STOCK MARKET DYNAMICS,
FINANCIAL SECTOR: THREE ESSAYS ON MACROECONOMIC THEORY**

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SUMMARY

The choice of exogenous variables is a fundamental element for the logical structure of economic models, leading to different positive and normative implications about growth, distribution and economic policies. In this dissertation a comparative approach is used both to study different models from a theoretical point of view and to analyze the link between the financial and the real sector of the economy.

In the first chapter we present a comparison between the neoclassical model and the alternative approach, drawn from the classical and post-keynesian literature, within a common mathematical framework based on the Solow growth model. Several variations in the canonical models are considered. We shall show in a convenient analytical framework how the fundamental differences between the two paradigms ultimately lie in the choice of the exogenous variables: factors endowments in the neoclassical approach or effective demand and, in some cases, income distribution in the alternative approach.

In the second chapter, we adopt a comparative approach to interpret stock market dynamics, pursuing two objectives. First, we shall show how the prevailing interpretation of Shiller tests on stock price volatility can all be traced back to the neoclassical model, which makes them exposed to several criticisms. Second, we shall present an alternative macroeconomic model drawn from Sraffian and Keynesian literature which suggests a different interpretation of the empirical evidence on stock market volatility.

In the third chapter we propose an integration between the classical-Keynesian model and the monetary circuit framework, evaluating its consistency and its policy implications. In particular, we shall verify whether the Keynesian multiplier can be consistently introduced in the monetary circuit framework, how monetary authorities can affect economic dynamics, how monetary circuits are intertemporally linked to each other and how the problem of interest repayments can be solved.

CHAPTER 1

CHOICE OF EXOGENOUS VARIABLES AND COMPARISON BETWEEN MODELS

1. Introduction

The choice of exogenous variables is a fundamental element for the logical structure of economic models, leading to different positive and normative implications about growth, distribution and economic policies. In this chapter we propose a comparison between models based on the neoclassical approach on the one hand and alternative models drawn from the classical and post-keynesian literature on the other hand, within a common mathematical framework based on the Solow growth model¹. We shall show in a convenient analytical framework how the fundamental differences between the two approaches lie in whether factors' endowments on the one hand or effective demand and, in some cases, income distribution on the other hand are considered as exogenous variables.

The idea to classify the models according to the variables to be considered as exogenous is not new. As Maurice Dobb (1973) recognized:

“the equational system must be made to tell us something more; and this “something more” almost inevitably has a causal form [...] an order of determination being implied as soon as

¹ For the neoclassical model, the principal references are Solow 1956, Ramsey 1928, Koopmans 1975, Blanchard 2017, Barro & Martin 1995, Romer 2001; for the alternative model, the principal references are Harrod 1939, Sraffa 1960, Keynes 1937, Kaldor 1956, Garegnani 1984,1992, Garegnani & Palumbo 1998, Ciccone 1986, Trezzini 1995, Serrano 1995, Rowthorn 1981, Badhuri & Marglin 1990, Hein 2014, Lavoie 2014, Brancaccio 2005, 2010, 2012, 2013. For an analysis of economic theories from an historical point of view, see Screpanti, Zamagni 2005.

some of the variables are treated as exogenously determined from outside the system, or else treated as constants, and hence specified as data (implicitly or explicitly) and the others as being dependent on the internal relations of the system or as the “unknowns” awaiting a solution.”²

Of course, the choice between exogenous and endogenous variables doesn't exhaust all the relevant differences between models. Suffice it to say that the choice of exogenous variables itself refers to the more general choice between two alternative paradigms: the approach based on neoclassical scarcity and the approach based on classical reproducibility (see for instance Pasinetti 1989, Brancaccio 2012), which can be evaluated not only from an ideological and theoretical point of view, but also from a logical, methodological and historical perspective (see below and the conclusions for major details). Nonetheless, these two alternative ways to represent the economic system owe their irreducible incompatibility also to the different choices about exogenous variables and the resulting formal structures adopted. In particular, neoclassical models include the endowments of factors among exogenous variables, whereas alternative models assume the level of demand and, in some cases, a distributive variable as exogenous.

As we shall see, the essential analytical feature of neoclassical models is the determination of growth, prices and distribution starting from the following exogenous variables: endowments, households' preferences and production technology. In this framework, prices

² Dobb 1973, pp. 8-9. For a detailed discussion about the choice of exogenous and endogenous variables see Brancaccio 2010, 2012. “[...] The choice of exogenous variables may be conceived as the decisive moment in which the model builder, by defining her point of view, focuses on one or the other type of representation of the economic system” (Brancaccio 2012, p. 122). See also, for instance, Hahn, Matthews 1964; see Hein 2017 for a comparison between models in an alternative framework.

are to be considered as indexes of scarcity. Alternative models, on the contrary, are not constrained by scarce resources, since all factors are considered abundant or reproducible³; the scarcity of resources affects neither prices nor quantities produced⁴. Even if alternative models present differences between them, they share the same critical view about neoclassical determination of growth, distribution and prices. Prices cannot be considered as indexes of scarcity; instead, they must be such as to make it possible the reproduction of the economic system. On the one hand, distribution, in many of these alternative models, is thought to be determined from outside the model, by social, economic, historical and sociological factors that ultimately reflect the class struggle between workers and firms. On the other hand, growth is determined by effective demand, without any constraint imposed by scarce resources.

There are several reasons why this comparative approach may be interesting. First, we can see in the clearest way which are the fundamental differences between the two approaches. Of course, it would be reductive to reduce the differences between models to a choice of exogenous and endogenous variables. Models can be compared not only from an ideological point of view, but also from a historical (Pasinetti 1979), methodological (Garegnani 1981) and logical (see for instance Petri 2004) perspective (Brancaccio 2012, p. 122). In particular, it is well known that neoclassical models suffer from a logical inconsistency relative to the theory of capital (for a survey see, for instance, Harcourt 1972). Here, these logical problems

³ Labour is considered abundant, and capital is considered flexible in the short period and reproducible in the long period.

⁴ In the post-Keynesian models inspired by Kaldor (1956), the possible constraint imposed by the non-reproducible factor (labour) is considered. However, as we shall see, this constraint assumes a total different meaning with respect to the neoclassical model, concerning the relation between supply and demand, with no reference to scarcity issues.

will not be considered, since we deal with a production system with only one commodity; nonetheless, this comparative procedure shall be useful to single out the essential features of the two approaches in a simple one-commodity framework.

Second, starting from a common mathematical and graphical framework (this procedure has been used, for instance, by Kurz, Salvadori 1995, Hahn, Matthews 1964, Hein 2017, Brancaccio 2012, 2018), we can show in a clearer way how the models differ depending on the choice between exogenous and endogenous variables. In particular, we shall show how, starting from a similar system of equations, the two models have radically different implications about growth, prices and distribution.

Third, we shall see how the distinction between the two alternative approaches doesn't depend on production technology, differently from a widespread opinion in the literature⁵; in this sense, classical-keynesian models preserve all their fundamental features even assuming an implausible continuous and differentiable function. Even if the equality between capital marginal productivity and the rate of profit is not part of the theoretical tradition of alternative models and has been criticized by critical economists, this feature is less important than the relation between exogenous and endogenous variables for determining the essential structure of a theory; moreover, as we shall see, that equality assumes a total different meaning in the alternative models⁶.

⁵ see for instance Barro, Martin (1995) on the comparison between Solow model and Harrod model based on the type of production function.

⁶ As we shall see, in the alternative models, the normal rate of profit is given and the optimal technique must adapt to it. There's no references to scarce resources, and in no way prices can be considered as scarcity indexes.

Fourth, as noted before, this type of comparison can be thought of as a first step toward a deepen criticism concerning logical, methodological and historical issues; in particular, once the one-commodity framework is replaced by a multi-sectoral analysis, it's possible to show how neoclassical theory suffers from serious logical contradictions.

Finally, the comparison between models within a common framework may be useful for stimulating the debate between alternative schools of thought⁷.

The chapter is structured as follows. First, we shall derive the analytical framework common to all models. Second, we shall present the canonical versions of both neoclassical and alternative models. Third, we shall see how variations in the canonical models don't change their essential features. For the sake of clarity, in the appendixes all graphs are reported and all models are summarized.

2. The common framework

We assume a closed economy in which one perishable good is produced by means of itself and labour through a constant return to scale technology that respects the standard Inada conditions⁸. We start by deriving the analytical structure common to all models, drawn from Solow 1956, composed by three equations relative to income distribution, the optimum technical choice and macroeconomic equilibrium.

Let Y be the normal output, that is the quantity of good produced with a normal level of capacity utilization (see below), $F(K,L)$ the production function relating input to output, K and

⁷ See, for instance, Amighini et al. 2012.

⁸ $f'(k) > 0$, $f''(k) < 0$, $\lim_{k \rightarrow \infty} f'(k) = 0$, $\lim_{k \rightarrow 0} f'(k) = \infty$.

L the quantities of capital and homogenous labour employed in the production process, w the real wage, r the rate of profit, C the demand for consumption goods, I the demand for investment goods, Z the autonomous expenditure not generating productive capacity (e.g. public expenditure, autonomous consumption and so on). All variables are expressed in real terms.

Income is entirely distributed between wages and profits:

$$Y = F(K, L) = wL + (1 + r)K$$

We can reformulate the equation in per-capita terms dividing both sides by L , taking into account the constant return to scale assumption (lowercases denote per-capita variables):

$$1) y = \frac{1}{L}F(K, L) = F\left(\frac{K}{L}, \frac{L}{L}\right) = f(k) = w + (1 + r)k$$

Optimum condition is given by firms' extra-profit maximization program, and gives the standard result for which factors are remunerated in relation to their respective marginal productivities:

$$2) \max_k \pi = f(k) - (1 + r)k - w \quad \rightarrow \quad f'(k) = 1 + r$$

Finally, the macroeconomic equilibrium between supply and aggregate demand is given by:

$$Y = C + I + Z = C + (1 + g)K + Z$$

Where g is the growth rate of capital⁹. Let S be actual savings, s the marginal propensity to save ($s = S / Y$) and z the ratio between autonomous expenditure and normal output ($z = Z / Y$)¹⁰. Thus, we can reformulate the equilibrium in per-capita terms dividing both the sides for L :

$$3) Y - C = S = sY = (1 + g)K + Z \quad \rightarrow \quad sf(k) = (1 + g)k + zf(k)$$

The analytical structure common to all models is then given by the following three equations:

$$1) f(k) = w + (1 + r)k$$

$$2) f'(k) = 1 + r$$

$$3) sf(k) = (1 + g)k + zf(k)$$

The solution of the model, the temporal sequence, the determination of distribution, production and employment, the role of prices and the policy implications vary depending on which variables are chosen as exogenous. In particular, neoclassical models include factor endowments among exogenous variables, whereas alternative models allow for an

⁹ Note that without fixed capital gross investments are precisely given by $I_t = (1 + g) K_t = K_{t+1}$. Note also that investments are considered at the end of each period, whereas capital is considered at the beginning of each period: thus, end-of-period realized investments coincide with capital at the beginning of the following period.

¹⁰ Note that both s and z are expressed in terms of normal output. Note also that the average propensity to save ($s-z$) differs from marginal propensity to save (s) when considering autonomous demand (see below).

exogenous determination of the effective demand and, in some cases, of a distributive variable.

From a graphical point of view, we can represent each model using two graphs: on the one hand, we can analyse proportions between variables in the standard, well known, Solow diagram, with capital / labour ratio on the horizontal axis and output / labour ratio on the vertical axis; on the other hand, we can see the relation between proportions and absolute levels of the variables through the isoquant plane, where the quantity of labour and capital are represented, respectively, on the horizontal axis and the vertical axis. This graphical comparison can help to better appreciate the differences between the two approaches.

3. The canonical neoclassical model

As is known, neoclassical model is based on the following exogenous variables: factor endowments, households' preferences¹¹ and production technology. Given these variables, the model determines the equilibrium levels of income distribution and production. The (discrete-time) temporal sequence can be described as follows¹².

At the beginning of each period households supply all factor endowments (labour and capital) whereas firms demand all factor endowments and start the production process. Factor prices change until all the factors are all absorbed by firms. At the end of each period firms remunerate factors according to their relative marginal productivity whereas

¹¹ In this framework, households' preferences are only represented by saving propensity, since choices among different goods are precluded in a one-commodity framework.

¹² In this section we abstract from autonomous demand.

households consume a (fixed) part of income and supply the remaining part (savings) to firms; savings will turn into investments, that is capital supplied at the beginning of the following period. If the growth rate of capital is higher than the growth rate of population, the capital / labour ratio increases but at a decreasing rate, due to the decreasing marginal productivity of capital (the opposite is true if the growth rate of capital is lesser than the growth rate of population). Steady-state equilibrium will be reached when the growth rate of capital coincides with the growth rate of population¹³.

Solution

There are two solution procedures, depending on whether we consider the steady-state position or not.

In the non-steady-state case, the model can be represented as a system of three equations in the following unknowns: r , w , and g . The solution is as follows:

- Given K and L we find k , and then $f(k)$;
- from (2), given k we find r ;
- from (1) we obtain w ;
- from (3), given s we find g .

¹³ If, say, $g > g_n$, capital-labour ratio at the beginning of the next period will increase, but at a decreasing rate: indeed, rearranging equation 2 we have $g = \frac{sf(k)-k}{k}$ and $\frac{\partial g}{\partial k} = \frac{[sf'(k)-1]k-sf(k)+k}{k^2} = \frac{s}{k} \left[f'(k) - \frac{f(k)}{k} \right] < 0$, since marginal product is always lesser than average product when considering a concave production function that respects the standard Inada conditions. Then, when capital-labour ratio increases, the growth rate will tend to decrease, until it reaches the natural growth rate of population g_n . A similar and opposite reasoning is true when $g < g_n$.

Note that this sequence cannot be interpreted as a residual determination of wages, since prices move together towards the equilibrium: firms, in their extra-profit maximization program, take the rate of profit and wages as given, but, since endowments are given, prices adjust until demand fully absorbs the endowments, and extra-profits become zero. Note also that equation 3 must be read “from left to right”: savings turn into investments, and then into capital supplied in the next period. As we shall see, these two features are not present in the alternative model.

The neoclassical steady-state is defined as the situation in which all variables grow at the natural rate g_n , that is the growth rate of population (the non-reproducible factor). The model can be represented by a system of three equations in the following unknowns: r , w , and k . The solution is as follows (ss means steady state, n means natural):

- from (3), given $g = g_n$, we find k_{ss} and then $f(k_{ss})$;
- from (2), given k_{ss} we find r_n ;
- from (1) we obtain w_n .

Note that although k is endogenous in steady-state position, the fundamental feature of neoclassical approach doesn't change: in each period, whether the steady state has been reached or not, relative prices always stem from the comparison between endowments and relative demands, reaching the value such that demand fully absorbs endowments.

This is the simpler version of the neoclassical model, but, as we shall see in the next sections, adding further elements doesn't change its essential features. For instance, we can

introduce intertemporal optimization¹⁴, public expenditures, adjustment costs, imperfections, a reaction function such the Taylor rule, the classical hypothesis of savings and so on, but the conclusions don't change: on the one hand, growth and distribution are always endogenously determined on the basis of scarce factor endowments and consumers' preferences; on the other hand, prices are to be considered as indexes of relative scarcity.

Graphical interpretation

Consider the "Solow model" graphs in appendix B. Given an initial level of factor endowments equal to K_0 and L_0 , in the isoquant plane we can find the slope of the expansion path k_0 and the output level $F(K_0, L_0)$ (point a). Then, given k_0 , in the Solow diagram we can find the per-capita level of production $f(k_0)$, the rate of profit r_0 and the level of (real) wages w_0 consistent with the full employment of labour and capital and the growth rate of capital g_0 determined by savings (point a). Since the growth rate of capital g_0 is higher than the growth rate of population g_n , capital/labour ratio k - and then output/labour ratio $f(k)$ - increases over time (from a to b); but, given decreasing marginal productivity, the growth rate of capital will tend to decrease, until it reaches the natural rate of growth (g_n). In the isoquant plane, the expansion path will rotate counter-clockwise until the slope is equal to the steady-state capital/labour ratio k_{ss} . When reaching the steady-state position, output will grow following the expansion path with a slope equal to k_{ss} .

¹⁴ In this case the steady-state rate of profit will coincide with the intertemporal rate of preference.

4. The canonical alternative model

In the alternative model we propose here (principally based on Garegnani 1984, 1990, 1992¹⁵, and partly on Kaldor 1956¹⁶), (normal) distribution and growth are taken as given.

Normal distribution is determined by economic, social and institutional forces more persistent than the other variables, regardless of any reference to given factor endowments; here we assume an exogenous rate of profit, which may be considered as a long-period rate of profit, determined by the aforementioned forces external to the model. We assume that capital and labour can adapt to any technical choice optimally chosen by firms on the basis of the given rate of profit. This adjustment is based on the hypothesis that labour is abundant and capital is flexible in the short period and reproducible in the long period¹⁷.

Deviations from normal positions - that is the positions consistent with normal distribution - are allowed: here we assume that they can be reached either by deviations from normal capacity utilization - when supply can adapt to effective demand and firms follow this adjustment process - or by deviations from normal prices (that is the prices corresponding to normal distribution) - when supply cannot adjust to effective demand or when firms follow this adjustment process - or by a mix of the two adjustment processes. For instance, we may

¹⁵ See for instance Levrero (2014) for a more general reconstruction of Garegnani's thought on the theory of distribution.

¹⁶ Other references are Ciccone 1986, Trezzini 1995, Brancaccio 2005.

¹⁷ This means that, given the normal rate of profit, firms can possibly choose the absolute (normal) quantities of labour and capital consistent with the optimal technical choice. Alternatively, given the normal rate of profit and the quantity of capital inherited from the past, firms can possibly choose the quantity of labour to be employed consistent with the optimal technical choice. However, the quantities of capital and labour actually used coincide with normal quantities just for chance; indeed, the actual quantities depend on the investment choices of firms and on the way firms choose to reach macroeconomic equilibrium.

assume that firms only choose to change actual prices when deviations from normal capacity utilization become too high: in this case prices deviate from their normal value¹⁸. We also assume that workers consume all their wages, whereas capitalists save a constant part of profits (this assumption, known as classic hypothesis of savings, makes it clearer the analysis but it's not decisive for the model).

Growth is determined by autonomous investment decisions of firms. In this section we shall take investment demand as exogenously given, without explicating an investment function. Macroeconomic equilibrium is obtained through variations in savings; the latter, in turn, are obtained through deviations of profits from their normal value, achieved either by deviations of capacity utilization from its normal level or by deviations of actual prices from their normal value (see above). Note that just for chance the (exogenous) growth rate is compatible with (exogenous) normal distribution. This is the case when capital grows at the *warranted rate* (Harrod 1939), that is the rate consistent with normal distribution. As we shall see, some authors argue that if we also consider autonomous demand not generating productive capacity and we assume that investments are driven by it (Serrano 1995; see also Lavoie 2015, pp. 405 and following), changes in autonomous demand / capacity ratio can lead the system toward the normal position¹⁹. However, in this section we shall abstract from autonomous expenditure not generating productive capacity.

¹⁸ Taking into account both of these adjustment processes in a common structure may lead to overlook the important differences arising from adopting one type of adjustment or the other; but, here we want to show how these differences are of a second order with respect to their common essential feature: the absence of any reference to scarce resources when determining relative prices.

¹⁹ For a critical discussion see Trezzini 1995. See below.

In order to take into account the preceding observations, we introduce in the common system of equations (see below) the classical hypothesis of savings (4) and the possible deviations from normal distribution (1*). Denoting with an asterisk the equation of actual distribution and with an apostrophe the equations reformulated by taking into account other equations of the model, we have the following system:

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$1^*) uf(k) = u \frac{w}{\delta} + (1 + \gamma r)k \rightarrow u \text{ (or } \delta)$$

$$2) f'(k) = 1 + r \rightarrow k$$

$$3) sf(k) = (1 + g)k$$

$$3'[3 + 4]) s_c(1 + \gamma r) = 1 + g \rightarrow \gamma$$

$$4) s = s_c(1 + \gamma r) \frac{k}{f(k)} \rightarrow s$$

Equation (1) represents income distribution according to normal position: once normal rate of profit and the technique of production are known, (1) determines normal real wages. We can also express variables in monetary terms: given monetary wages (W), (1) determines normal prices (P), that is the prices consistent with normal distribution²⁰. Equation (2)

²⁰ Real wages can be defined as the ratio between monetary wages and monetary prices: $w = \frac{W}{P}$. If W is given, (1) determines P, that is the normal prices consistent with normal rate of profit. In what follows we express variables in real terms.

determines the optimum technique consistent with the given rate of profit²¹. Since we are assuming that inputs are not scarce, they can adjust to satisfy the optimum decisions of the firms. Note that this relation does not imply a return to neoclassical approach; differently from the latter, in the alternative model the relation must be read “from right to left”: the exogenous variable is the normal rate of profit, which in turn determines the optimal capital/labour ratio. Note also that the full employment of resources is not guaranteed, since the actual amount of capital and labour to be employed is determined by firms’ choices about production and investments. What equation (2) determines is the optimal ratio between capital and labour in normal position; (2) determines neither the absolute nor the actual amount of resources employed (see below). In no way factor prices can be considered as the adjustment variables capable to lead the system toward the full employment of resources; nor they can be considered as scarcity indexes. Instead, normal prices are determined in such a way as to ensure the reproduction of the system, for a given rate of

²¹ Note that for the alternative model, as opposed to neoclassical approach, we don’t need to assume any particular hypothesis about production function. We only maintain the hypothesis of a continuous and differentiable function for the sake of comparison. As is known, neoclassical models suffer from logical inconsistency when considering an economy with more than one commodity (see for instance Petri 2004): neither capital can be considered as an exogenous data, nor an univocal (inverse) relationship exists between capital and the rate of profit. The optimum condition considered here (the equality between rate of profit and capital marginal productivity) is only logically consistent in a one-good economy. The alternative model, on the contrary, doesn’t suffer from logical inconsistency, since neither it considers capital as an exogenous variable needed to compute prices, nor it needs an inverse relation between capital and the rate of profit (for instance, the optimal capital/labour ratio can be taken as given).

profit and a given technology, without any reference to the equilibrium between scarce resources and their relative demand²².

(1*) represents actual income distribution according to the possible deviations from normal position: γ represents the deviation from the normal rate of profit ($\gamma = r_t / r$)²³, u is the actual degree of capacity utilization, that is the deviation from the normal level of capacity ($u = Y_t / Y$), δ is the deviation from normal prices ($\delta = P_t / P$)^{24 25}. When $\gamma = 1$, the actual rate of profit coincides with the normal one; when $u = 1$, actual production equals normal production; when $\delta = 1$, actual prices equal normal prices, or, that is the same, actual wage level equals normal wage level²⁶. For simplicity, we don't explicit monetary prices: all variables are expressed in real terms. We also assume that capital, but not labour, is elastic with respect to output: firms, say, can obtain a higher quantity of output employing more workers on the

²² This is true even if we consider the quantity of capital as given; in this case, the quantity of labour actually employed depends on effective demand and on the way firms choose to reach macroeconomic equilibrium (see below).

²³ The relationship between the real and the monetary actual rate of profit is the following: $(1 + \gamma r) = \frac{1 + \gamma r_m}{1 + \pi}$, where $1 + \pi$ is the inflation rate, that is the ratio between the actual monetary prices in this period and the actual monetary prices in the preceding period: $1 + \pi = \frac{\delta P}{\delta_{-1} P}$.

²⁴ Explicating monetary wages and monetary prices, equation (1*) becomes: $uf(k) = u \frac{w}{\delta P} + (1 + \gamma r)k$. In this case, (1*) determines u and δ , whereas (1) determines P .

²⁵ In monetary terms, δ represents the deviation of actual monetary prices from normal monetary prices, defined as the prices consistent with normal distribution; in this case the numeraire is expressed in terms of money. In real terms, δ represents the value of one unit of good in actual position in terms of units of good in normal position; in this case the numeraire is expressed in terms of good in normal position.

²⁶ Note that the term $\frac{1}{\delta}$ corresponds to the ratio between actual real wages and normal real wages.

same quantity of capital, given the optimal production technique; then, given the technique k , from a given K inherited from the past firms can obtain uY output employing uL workers²⁷.

(4) represents the classical hypothesis of savings: workers don't save, whereas firms save a part of the actual profits; s_c represents the saving propensity of firms. Plugging (4) into (3) we obtain macroeconomic equilibrium condition (3'): given the investment decisions of the firms, the growth rate g is also given; thus, the actual savings $s_c(1+\gamma r)$, and then the actual rate of profit γr (since s_c is a given parameter), are endogenously determined. Given the actual rate of profit needed for the macroeconomic equilibrium, 1^* determines the appropriate combination between actual capacity utilization and actual prices consistent with it. The adjustment based on deviations from normal capacity utilization leads to a quantitative effect, whereas the adjustment based on deviations from normal prices leads to a distributive effect: for instance, if economic equilibrium is reached through an increase in capacity utilization, both (actual) profits and total wages increase²⁸; instead, if equilibrium is reached through an increase in actual prices, actual profits increase and the actual wage level decreases²⁹.

²⁷ For instance, if firms want to increase production 5% beyond its normal level, they must employ 5% more workers, using the same quantity of capital more intensively, for the given optimal technique. We only consider circulating capital in order to simplify the analysis, even if it may sound a bit artificial; however, the essential features of the model don't change if we consider fixed capital.

²⁸ the rate of profit increases because a higher quantity of good is sold; wages increase because a higher quantity of workers is employed, for a given wage level.

²⁹ the actual rate of profit increases because goods are sold at a higher price; real wages (and workers' consumption) decrease because goods are purchased at a higher price.

Substituting w from (1) in (1*) we obtain an explicit expression for the deviation from normal rate of profit (determined in equation 3') in terms of deviations from normal capacity utilization and from normal prices:

$$1^{*'}) \gamma = \frac{1}{r} \left\{ u \frac{f(k)}{k} \left[1 - \frac{1 - (1+r) \frac{k}{f(k)}}{\delta} \right] - 1 \right\}$$

The punctual values of u and δ needed to satisfy the equation depend on firms' behavior: assuming an exogenous δ (either at its normal value or not) the equation endogenously determines u , and viceversa. The choice between one or another option reflects the different keynesian ways to reach macroeconomic equilibrium. On the one hand, according to the Classical-keynesian school (see for instance Garegnani 1992, Garegnani, Palumbo 1998, Ciccone 1986) equilibrium is reached through variations in capacity utilization, both in the short and in the long period: in this case, $\delta = 1$ and (1*') determines u . Substituting γ from 3' into 1*' and letting δ be equal to one, we obtain an explicit solution for u :

$$1^{*''}) u = \frac{(1+g)}{s_c(1+r)}$$

If δ is given but it's different from one, then:

$$1^{*''') u = \frac{\frac{(1+g)}{s_c} \frac{k}{f(k)}}{1 - \frac{1 - (1+r) \frac{k}{f(k)}}{\delta}}$$

On the other hand, in line with the Cambridge school (see for instance Kaldor 1956), equilibrium can be reached through variations in prices: in this case, $u = 1$ and $(1^*)'$ determines δ ; if price deviations persist over time, they can influence normal distribution itself³⁰. Substituting γ from 3' into 1*' and letting u be equal to one, we obtain an explicit solution for δ :

$$1^{*''''}) \delta = \frac{1 - (1 + r) \frac{k}{f(k)}}{1 - \frac{(1 + g) k}{s_c f(k)}}$$

If u is given but it's different from one, then:

$$1^{*''''''}) \delta = \frac{1 - (1 + r) \frac{k}{f(k)}}{1 - \frac{(1 + g) k}{s_c u f(k)}}$$

Even if the two adjustment mechanisms have different analytical and policy implications, they share the common critics to the neoclassical determination of prices.

³⁰ In this case, a persistent increase (decrease) in actual prices leads to an increase (decrease) in the normal rate of profit and to a decrease (increase) in the normal wage level; in the new normal position actual prices equal normal prices and we have $w_{\text{new}} = w / \delta$, $r_{\text{new}} = \gamma r$ and $\delta_{\text{new}} = 1$. According to Cambridge school, deviations from normal capacity utilization are not excluded, but they are only allowed in the short period, since, in the long period, equilibrium is thought to be reached through variations in normal distribution. For a critical discussion see for instance Ciccone 1986, 1987. In the next chapters we shall assume that macroeconomic equilibrium can be always reached through variations in capacity utilization, without assuming a causal relation between growth and normal distribution; in particular, we don't require a positive relation between growth and the normal rate of profit, typical of Cambridge school.

Solution

The model can be represented as a system of five equations (1, 1*', 2, 3', 4) in the following unknowns: w , γ , k , δ (or u), s . The solution is as follows:

- given r , (2) determines k and (1) determines w (or P , if monetary wages W are explicated);
- given g , (3') determines γ ;
- given γ , (1') determines u (or δ) and 4 determines s .

Graphical interpretation

Consider the graphs of the alternative model in appendix B ("adjustment through deviations from normal capacity utilization" and "adjustment through deviations from normal prices"). Let the black lines correspond to the initial position, assumed to be normal, and the orange lines to the final position after an increase in investments. We can consider two cases: in the first case, we assume that macroeconomic equilibrium is only reached through deviations in capacity utilization; in the second (extreme) case, we assume that macroeconomic equilibrium is only reached through deviations in prices. We shall see that, after an increase in investment demand (capital growth), in the first case final position is characterized by a higher level of per-capita production, higher employment, higher (given) growth rate, a higher actual rate of profit, the same level of wages: this follows from the adaptation of supply to effective demand through variations in capacity utilization. In the second case, we describe two positions: on the one hand, a (temporary) situation in which actual prices have no influence on normal distribution (and then on optimal technique); on the other hand, a

fully adjusted situation in which the persistence of actual price deviations has a permanent influence on normal distribution (and then on optimal technique, given the hypotheses about production function)³¹. The temporary (final) position is characterized by the same (lower) level of per-capita production, a higher (given) growth rate, a higher rate of profit, lower (less lower) wages: this follows from the adaptation of demand to supply through variations in distribution.

The starting point is common to both models. Given r and the $f(k)$ curve (the production technology), in the Solow diagram we find k_0 (from equation 2), $f(k_0)$ and w_0 (from equation 1). For simplicity, we assume that in the starting position (point a) investments grow at the warranted rate g_0 (that is, they are consistent with capacity savings³²). Absolute quantities in the isoquant plane are determined by investment decisions of firm. Note that this economic position doesn't involve full employment if not for chance (see above). Assume firms decide to rise the growth rate of capital from g_0 to g_1 . It follows that actual savings $s(\gamma)f(k)$ must increase to intersect the capital growth line in point b³³. There are two ways for production to meet effective demand and for actual savings to equal investments.

According to the first position, in line with the classic-Keynesian tradition, given the elasticity of capital with respect to output, firms hire more workers (paying more wages for unit of *normal* labour, from w_0 to uw_0) and increase production per unit of *normal* labour (from $f(k_0)$

³¹ Generally, technique is considered as given in the alternative models, and then changes in prices can influence normal distribution without affecting the optimal capital / labour ratio.

³² capacity savings corresponds to the level of savings consistent with normal distribution. Just for chance the level of g is consistent with capacity savings, since g is autonomously determined by firms; in this example we assume g is consistent with the level of capacity savings just for simplicity.

³³ we denote actual savings with $s(\gamma)$ to highlight their dependency from the actual rate of profit.

to $uf(k_0)$) using more intensively the same quantity of capital per unit of *normal* labour k ³⁴: we move from a' to b' . In the isoquant plane we move from a to b , where a higher quantity of labour uL is employed on the same quantity of capital K ; it follows that capital for unit of *actual* labour decreases from k_0 to k_0/u : the expansion path rotates clockwise³⁵³⁶. The higher quantity of goods sold increases the actual rate of profit (from $1+r$ to $1+\gamma r$). The rise in the actual rate of profit appropriately increases actual savings, until they reach point b in Solow diagram. If we assume that g_1 remains the same in all successive periods, b is an equilibrium point, capacity utilization is persistently higher than normal level, and the actual quantities of capital, labour and production move according to the new expansion path (with a slope equal to k_0/u). In this new path, per-capita production, investments, employment, growth, total wages and the actual rate of profit are higher with respect to the initial position, whereas the wage level remains the same³⁷.

³⁴ In the Solow diagram all variables are expressed in terms of L ; this means that we consider the variables in terms of the quantity of labour consistent with normal distribution (*normal* labour, from here on). This is the reason why k_0 remains the same, whereas w and $f(k)$ change. Note that total wages increase to $w_0 u L_0$, whereas the wage level remains the same. In other words, wage for unit of *normal* labour increases to $u w_0$, whereas wage for unit of *actual* labour remains w_0 . A similar reasoning is true for production: actual production for unit of normal labour increases to $uf(k_0)$. Conversely, the quantity of capital for unit of normal labour remains the same, since capital is assumed to be inelastic with respect to production.

³⁵ With normal quantities of capital K and labour L we have $k = \frac{K}{L}$; if firms hire more workers (uL) in order to increase production, the capital / actual labour ratio becomes $k_0 = \frac{K}{uL} = \frac{K/L}{u} = \frac{k}{u}$.

³⁶ Note that in Solow diagram we consider capital / *normal* labour ratio, whereas in the isoquant plane we consider capital / *actual* labour ratio.

³⁷ In the next sections we shall remove the hypothesis of an exogenous growth rate of capital in all periods. This model may be interpreted as a situation in which average over-utilization of capacity, even if it can meet demand peaks, remains sufficiently low to avoid a persistent pressure on the maximum level of capacity utilization. Here we don't explicit the effects of capacity utilization on investments. See for instance Trezzini (2017); "The condition capable of triggering the cumulative tendency toward expansion

According to the second position, in line with the Cambridge tradition, macroeconomic equilibrium is reached through changes in actual or normal distribution without changes in capacity utilization. In particular, the inflation pressure due to the excess of demand pushes down the actual real wages (from w_0 to w_0/δ) and increases the actual rate of profit (from $1+r$ to $1+\gamma r$), since goods are sold at a higher price (and monetary wages are assumed to be constant). The rise in the actual rate of profit appropriately increases actual savings, leaving the initial production level unchanged³⁸. In the Solow diagram we move from a to b, whereas in the isoquant plane the expansion path remains the same. If the growth rate remains stable at its new higher level (and if capacity utilization remains at its normal level) actual prices tend to be considered as *new normal prices* since extra-profits are positive³⁹. Once firms realize that the profit rate has permanently changed (and has become the new normal one), competition will push firms toward the technique that maximizes real wages (k_1), pushing down actual prices. Price deviations - and then deviations of actual profits from the new normal rate of profit - become lower, disappearing in the end (point c'), and savings adjust to the new normal position in point c'⁴⁰. In the isoquant plane, production moves

is [...] not just any overutilization but the *persistence* of the actual degree of utilization at levels close to the maximum" (Trezzini, 2017, p. 13).

³⁸ The increase in actual savings and capitalists' consumption is perfectly compensated by the decrease in workers' consumption.

³⁹ we have $1+\gamma r > f'(k)$. Note the difference with respect to the previous adjustment process, where capacity utilization is let to vary: in that case we have $1+\gamma r = u f'(k)$.

⁴⁰ If we assume a not instantaneous adjustment process (i.e. no jumps from point b to point c) actual savings tends to capacity savings through variations in normal distribution: indeed, the *deviation* component of the actual rate of profit gradually loses importance in favour of variations in the normal rate of profit; that is, in the $1+\gamma r$ term, γ tends to decrease and r to increase when technique changes. Moving to the fully adjusted position, γ tends to one and r tends to the (new) normal rate; then, also actual savings tend to capacity savings. If we allow for an instantaneous adjustment process, we would have a

towards the new expansion path, characterized by a lower capital / labour ratio, in accordance with the higher new normal rate of profit. In the final position, per-capita production is lower, the normal rate of profit is higher, normal real wages are lower, (given) growth is higher relative to initial position.

The alternative model as proposed here presents some important differences with standard alternative literature. The main novelty - introduced for the sake of comparison with the neoclassical model - is the inverse relation between the capital/labour ratio and the rate of profit. As noted before, this feature, essential for the consistency of neoclassical model, is not necessary for the alternative model and it's not a crucial element to distinguish between neoclassical and alternative models. However, this hypothesis leads to some results not found in the alternative literature, such as variations in optimal capital/labour ratio and per-capita production when equilibrium is reached through price adjustments (see above) or when normal distribution changes. However, this simplification have allowed us to identify the essential differences between neoclassical and alternative model, linked to the choice of exogenous variables: on the one hand, in the neoclassical model - which includes factor endowments among exogenous variables - prices represent indexes of scarcity which, if left flexible, can lead to the full employment of resources (on this point see also the next paragraph); on the other hand, in the alternative model - where effective demand and possibly a distributive variable are taken as given - prices reflect the conditions of reproduction of the system, whereas output and employment depend on effective demand.

jump from point b to point c, where the new (optimal) normal rate of profit ensures the equality between the growth rate and capacity savings.

In the next sections we shall see how introducing some variations in both the models doesn't change their essential features.

5. Variations in the neoclassical model

Until now we have considered the most simple formulations of both the neoclassical and the alternative model. In this section and the following one, we shall see that variations in these canonical models don't change the essential features of the two approaches, related to determination of prices, growth and distribution. We begin from neoclassical models, introducing variations such as the introduction on an autonomous component of demand, the classical hypothesis of savings, endogenous savings and market imperfections.

Consider first the introduction of autonomous demand not generating productive capacity (e.g. non-productive public expenditures). Let Z be the level of autonomous demand and $z = Z / Y$ the ratio between autonomous demand and output. Then, the condition of macroeconomic equilibrium becomes:

$$3') sf(k) = (1 + g)k + zf(k)$$

or, reformulating:

$$3') (s - z) \frac{f(k)}{k} = (1 + g)$$

In the short period, with a given marginal propensity to save s , a given ratio between autonomous demand and income z and a given capital-labour ratio k (known from the given endowments K and L , assumed as exogenous) the only endogenous variable is g ; then, average savings $(s-z)$ determines investments. Prices are still determined by the optimum condition. Then, again, both growth and distribution are determined by the standard neoclassical exogenous variables. Note that variations in z influence the growth rate of capital in an opposite direction, leaving marginal savings at the same level and lowering average savings available for investments⁴¹; then, for instance, an increase in z “crowds out” investments, without influencing marginal savings. This is a consequence of the neoclassical assumption of given endowments: since resources are given - and then savings are given - the ways in which they can be allocated are alternative to each other: the more is devoted to autonomous demand, the less is devoted to investments, and vice versa. This is shown in the graph in appendix B (“Solow model with autonomous demand”): In the Solow diagram, starting from the steady-state position in point a , an increase in z leads to a decrease in average savings and then in investments; capital-labour ratio will tend to decrease, until it reaches the point where the growth rate of capital equals the growth rate of population. In the new steady-state position, capital-labour ratio and per-capita output are lower, whereas the growth rate is the same as in the preceding steady-state. In the isoquant plane the increase in autonomous demand rotates the k line clockwise. At first, the new equilibrium is

⁴¹ Note that, when considering autonomous demand, marginal propensity to save is different from average propensity to save (Serrano 1995): marginal propensity is given by s , average propensity is given by $s-z$. This name is due to the fact that term z contains, among its components, autonomous consumption, that is consumption not linked to income. Then, average propensity to save can be defined as the marginal propensity to save net of the part of income devoted to autonomous demand.

in point a' , with a lower capital/labour ratio (green line); then, the expansion path moves toward the blue line, whose slope is the (lower) capital/labour ratio of the new steady-state position k'_{ss} (blue line).

A variation in the canonical model that results particularly useful for comparison purposes is the introduction of the classical hypothesis of savings:

$$s = s_c(1 + r) \frac{k}{f(k)}$$

Substituting this expression in the macroeconomic equilibrium condition we obtain:

$$s_c(1 + r) = 1 + g$$

This equation is formally equivalent to the equation present in the alternative model, but its interpretation radically changes. Indeed, the direction of causality is reversed: equation must be read “from left to right”. Given the endowments K and L , the maximization program of firms determines the equilibrium rate of profit, and thus savings; the latter, in turn, determines the growth rate of capital. Again, if $g > g_n$, the capital in the next period will be relatively less scarce; this will lead to a lower marginal productivity, a lower rate of profit, lower savings and a lower growth rate of capital: the converging process will end when g equals g_n . Prices are determined by the intersection between demanded and supplied endowments also in this version of the model: the difference with the alternative approach remains substantial.

The essential features of the neoclassical model remain the same even allowing for an endogenous choice between consumption and savings, introducing an intertemporal maximization program by households (Ramsey 1926, Koopmans 1975). For simplicity, we shall assume no population growth. The model can be described as follows. At the end of each period households choose how much to consume according to the following intertemporal utility maximization program:

$$\max_{c_{+i}} U = \sum_{i=0}^{\infty} \frac{1}{(1 + \rho)^i} u[c_{+i}]$$

where ρ is the intertemporal rate of preference, c is the per-capita consumption and u is an increasing concave utility function ($u' > 0$, $u'' < 0$)⁴². The problem can be reformulating taking into account the macroeconomic economic equilibrium. By choosing consumption, households also choose the level of savings / investments and then the quantity of capital to supply at the beginning of the following period. Indeed, since:

$$s = 1 - \frac{c}{f(k)}$$

substituting s into economic equilibrium equation we have:

$$c = f(k) - k_{+1}$$

⁴² This means that utility increases with consumption, but at a decreasing rate. Concavity determines the willingness of households to accept deviations from a uniform path of consumption over time: the more concave is the function, the larger is the decrease in utility from more consumption, the less willing households are to deviate from a uniform path.

Then, the maximization program can be rewritten as follows:

$$\max_{k_{+1}} U = \sum_{i=0}^{\infty} \frac{1}{(1+\rho)^i} u[f(k_{+i}) - k_{+i+1}]$$

The solution is given by the following Euler equation:

$$1 + r_{+1} = \frac{u'(c)}{u'(c_{+1})} (1 + \rho)$$

Assuming a standard power utility function of the following form:

$$u(c) = \frac{c^{1-\theta} - 1}{1-\theta}$$

we obtain:

$$\frac{c_{+1}}{c} = \left(\frac{1+r}{1+\rho} \right)^{\frac{1}{\theta}}$$

which can be approximated in the following way (g_c is the growth rate of consumption):

$$5) g_c \approx \frac{1}{\theta} (r_{+1} - \rho)$$

Reformulating the macroeconomic equilibrium condition we obtain a motion equation for capital:

$$3) gk = f(k) - c - k$$

As is known, equations 3 and 5, together with transversality condition (see for instance Barro & Martin 1995), determine the level of consumption and the quantity of capital supplied in the next period and in all future periods following the saddle path⁴³; g converges to g_n and r to ρ . In the steady state, the growth rate of capital equals the growth rate of population (since we have assumed no population growth, g converges to zero) and the rate of profit - and then the marginal productivity of capital - equals the rate of intertemporal preference (this can be verified setting c equal to c_{+1} in equation 5). So, in steady state, we have:

$$5') r_n = \rho$$

$$2') f'(k_{SS}) = 1 + r_n$$

$$3') s_{SS} = (1 + g_n) \frac{k_{SS}}{f(k_{SS})}$$

The rate of intertemporal preference determines the natural rate of profit, which in turn determines the steady-state value of k ; finally, given the latter and the growth rate of

⁴³ c_{+i} and r_{+i} , $i=1\dots\infty$, are determined by the maximization program, taking into account the transversality condition $\lim_{i \rightarrow \infty} \left(\frac{1}{1+\rho}\right)^i u'(c_i)k_{i+1} = 0$. In the case of no population growth, transversality condition simply imposes a positive value to rate of intertemporal preference.

population, the economic equilibrium condition determines the steady-state savings rate⁴⁴. Then, the only differences between the standard neoclassical approach and this model concern the dynamic path towards the steady state and the endogenous determination of saving rate, both in the saddle path and in steady state. But, the essential features of neoclassical model remain unchanged: at the end of each period, the product determined by the endowments available at the beginning of the relative period is divided by households between consumption and savings according to an intertemporal maximization program; again, prices, growth and distribution are determined starting from the presence of scarce resources, which remain an essential feature also in a model with endogenous savings⁴⁵.

An interesting way to highlight the differences between the two approaches is to consider a distributive variable as given in the neoclassical model. This is a feature typical of the models based on “market imperfections”. Here, we assume the presence of a real wage higher than the “natural” one, imposed, for instance, by the unions. The model can be represented by the following equations, where the subscript “i” denotes the values assumed by the variables under the “imperfectionist” regime:

$$1) f(k_i) = w_i + (1 + r_i)k_i$$

$$1'[1 + 2]) w_i = f(k_i) - f'(k_i)k_i \rightarrow k$$

$$2) f'(k_i) = 1 + r_i \rightarrow r$$

$$3) sf(k_i) = (1 + g_i)k_i \rightarrow g$$

⁴⁴ Note that in the Solow model g_n determines k_{ss} for a given saving rate, and k_{ss} determines r_n .

⁴⁵ A more detailed analysis of this model is presented in the second chapter. See appendix A for a complete representation of the model.

Given w , substituting (2) in (1) (equation 1') we find k ; from (2) we find r ; from 3 we find g . The intuition is the following: at the beginning of each period firms demand L and K according to the extra-profit maximization program, for given w and r ; in a model without imperfections, since L and K are given, flexible prices change until the demand for productive factors conforms to the given endowments; but, in an imperfection market where a distributive variable is given, prices are not flexible, and then firms cannot absorb all the endowments, leaving partially unemployed the factor with the relatively highest price (labour, in this example). If this "imperfection" remains over time, the unemployment of labour tends to increase; indeed, the converging process of g to g_n is precluded: both capital, fully employed, and employed workers increase at the rate g , but, since population increases at the rate g_n - higher than g - unemployment tends to increase.

Consider the graphs in appendix B ("Solow model with market imperfections"). Starting from an equilibrium position in point a , where K and L are fully employed, assume that wages remain fixed at that level. In the Solow diagram, given w and the production function $f(k)$, we can find the optimal technique k_0 and the profit rate r consistent with it⁴⁶; k_0 , r and g will remain the same until w remains not flexible: no converging process starts and the economy remains at the point a . In the isoquant plane, we can see that, since population grows at a rate higher than the growth rate of capital, but the capital/labour ratio remains the same, the proportion of unemployed workers, represented by the black segments, tends to increase. Instead of moving toward the steady-state expansion path (blue line), the economy remains "anchored" to the orange line (with the slope equal to k_0).

⁴⁶ Graphically, given w and the $f(k)$ curve, k is found at the intersection point between the $f(k)$ curve and the line starting from w and tangent to the $f(k)$ curve.

Thus, in the neoclassical model, fixing a distributive variable creates distortions that prevent the growth rate to converge to the natural one, leading, in our example, to unemployment and to a lower rate of growth. Thus, the model suggests that an exogenous increase in wages (or, which is the same, a decrease in the rate of profit) creates unemployment and a lower rate of growth. Again, this conclusion stems from the hypothesis of given endowments and from the assumption that growth is endogenously determined by savings. Analytically, we can illustrate this point starting from a steady-state position and assuming an exogenous decrease in the rate of profit. From equation (2), a decrease of r , given the hypothesis of decreasing marginal productivity, leads to an increase of k , obtained through a decrease of workers employed. From (3), given the rate of savings, an increase in k leads to a decrease of g . This is even clearer assuming the classical hypothesis of savings:

$$s_c(1 + r) = (1 + g)$$

From this relation it is clear that a reduction of r , given s , leads to a permanent reduction of g . In turn, a growth rate of capital and labour force lower than growth rate of population leads to an increasing level of unemployment.

Thus, the crucial features of the neoclassical approach are also preserved in the “imperfecionist” model: the growth rate is always constrained by savings - and then by scarce resources - whether prices are flexible or sticky; in the first case prices carry out their role of indexes of scarce resources, firms absorb all endowments, and investments are determined by savings out of the maximum possible income; in the second case, price

stickiness preserves firms to absorb all endowments, and relative lower investments are determined by savings out of a lower-than-maximum income.

The relation between the canonical and the “imperfectionist” model can be summarized as follows. From a normative point of view, the neoclassical model prescribes that prices must be flexible in order to play their fundamental role of indexes of scarce resources, letting the system to converge to full employment of resources and optimal growth. From a descriptive point of view, the neoclassical approach contemplates the possibility that prices can be sticky, but it also maintains that this “imperfection” leads to distortionary effects on employment and growth. From an analytical point, the only difference between the canonical and the “imperfectionist” version of the neoclassical model is that while in the first case the given endowments are all absorbed, in the second case they just constitute an upper bound which is not reached due to price stickiness.

The differences with respect to the alternative model are clear. In the latter, neither the given distributive variable imposes a constraint on growth, nor growth is constrained by scarce resources and savings. It follows that the role of prices and distribution, also when a distributive variable is taken as given in the neoclassical model, takes different analytical meanings depending on the model chosen, and in no way its alternative role can be confused.

6. Variations in the alternative model

In this section we shall see how the introduction of further elements in the alternative model - such as autonomous demand and endogenous investments - doesn't change its

essential features. The development of these models in a Solowian mathematical framework should make it clearer the differences between these models and the neoclassical ones.

First, consider the introduction of endogenous investments. Until now, we have assumed an exogenous growth rate of capital determined by firms' decisions, but we have not mentioned the possible determinants of those decisions. In the literature we can find several approaches about this issue (see for instance Lavoie 2015 or Hein 2014 for a survey). For now, we assume that investment decisions depend on the actual degree of capacity utilization⁴⁷:

$$1 + g = f(u)$$

In implicit form, macroeconomic equilibrium becomes:

$$s[\gamma(u)]f(k) = f(u)k$$

⁴⁷ This approach, known as neo-Kaleckian model and originally developed by Rowthorn (1981) and Dutt (1984), is presented here partially following Lavoie (2014). In the original formulation, investments also depend on the actual rate of profit (which in turn depends on capacity utilization itself), but qualitative results remain the same. Other differences that don't change the principal results of the model are the following: u represents deviations from normal capacity, not from full capacity; in the short period investments depend on expected capacity utilization, whereas in long period expected and actual capacity utilization coincide; we consider the normal rate of profit without explicating profit share; we take the normal rate of profit as given from outside the model, without considering a mark-up equation; we assume no overhead costs. As noted in preceding sections, the principal difference with respect to the standard alternative literature is the introduction of the optimum condition, represented by the equality between marginal productivity of capital and the normal rate of profit. Even if this condition leads to some consequences not found in alternative literature (in particular when considering changes in normal distribution), it is less important for understanding the essential structure of economic models than the general relation between exogenous and endogenous variables (see for instance Dobb 1973, Brancaccio 2010).

Assuming that in the short period the growth rate depends on the discrepancy between the *expected* level of capacity utilization (assumed as given in the current period) and the normal capacity utilization, in each period g is given and savings must conform to it, through appropriate changes in the actual rate of profit; the latter, as we know, depend on the *current* level of capacity utilization, which is the crucial adjusting variable (in this section we shall not consider price deviations).

We can find a solution for the equilibrium level of capacity utilization explicating the investment function and the link between the actual rate of profit and capacity utilization. Assume the growth rate of capital depends on the level of capacity utilization according to the following formula:

$$1 + g = \alpha + \beta(u^e - 1)$$

α can be interpreted as the autonomous component of firms' growth decisions (*animal spirits*); β represents the sensitivity of the rate of accumulation to the divergence between the expected and the normal level of capacity utilization: the idea is that, if the expected level of capacity utilization is higher than the normal level, firms will increase the rate of accumulation in order to bring back the actual rate towards the normal rate (see for instance Lavoie 2014, p. 361). With adaptive expectations, the expected rate of capacity utilization coincides with the level of capacity utilization in the preceding period: $u^e = u_{-1}$ (see for instance Hein 2014, pp. 250 and following).

Substituting the normal real wage as determined by the normal distribution equation into the actual distribution equation, we can express the actual rate of profit in terms of capacity utilization and the normal rate of profit⁴⁸:

$$(1 + \gamma r) = u(1 + r)$$

Substituting these relations in the macroeconomic equilibrium condition and assuming that in the long period the expected level of capacity utilization equals the actual one, we can find the equilibrium rate of capacity utilization:

$$u^* = \frac{\alpha - \beta}{s_c(1 + r) - \beta}$$

For this solution to make sense, u^* must be positive, and then both the numerator and the denominator must be positive. In particular, the required positive denominator gives what is known as Keynesian stability condition (see for instance Pasinetti 1974):

$$s_c(1 + r) > \beta$$

As is known, this means that, in order to have a stable path toward the equilibrium level of capacity utilization (which in this model doesn't coincide with the normal one), savings must

⁴⁸ From the normal distribution equation we have $w = f(k) - (1 + r)k$. Substituting this expression in the actual distribution equation $uf(k) = uw + (1 + \gamma r)k$ we obtain $(1 + \gamma r) = u(1 + r)$.

react more than investments to variations in capacity utilization. The complete model is presented in appendix A.

Now, consider the graphs in appendix B (“Neo-Kaleckian model”). In the Solow diagram, starting from normal position (black lines) assume an increase in α , and then in g . At first, the required higher savings are only reached through variations in the level of capacity utilization (blue lines). Then, an accumulation process starts, caused by the influence of capacity utilization on firms’ investment decisions. If Keynesian stability condition is satisfied, accumulation process is convergent (orange lines). During the adjustment process, on the one hand, in each period g is given and u must appropriately change in order to make savings consistent with investments; on the other hand, given stability condition, investments will grow at a progressively lower rate, until they equal savings at the equilibrium level of capacity utilization. In the isoquant plane, after the shock, employment immediately increases due to increase in capacity utilization; during the adjustment process, the line rotates clockwise, since the actual capital / labour ratio tends to decrease, due to the increase in the rate of capacity utilization. The expansion path in the equilibrium position has a slope equal to k/u^* . The equilibrium position is characterized by a level of capacity utilization higher than the normal one and a higher level of growth rate, employment, actual profits and total wages (and the same wage level for unit of actual labour).

The point to be highlighted is that, even if investments are endogenously determined by the degree of capacity utilization, in each period they are given, and they are neither constrained by scarce resources nor determined by savings.

Some authors, unsatisfied with the fact that the equilibrium position is characterized by a not normal degree of capacity utilization and that the model may suffer from Harrodian instability⁴⁹, have tried to modify the model in order to make it convergent toward an equilibrium characterized by normal capacity utilization. In this regard, a recent line of research has focused on the presence of an autonomous component of demand not generating productive capacity, capable, under some conditions, to bring the economy toward a full adjusted position (Serrano 1995). The idea is that investments are fully induced by the expected growth rate of demand, and since the latter is thought to be driven by the exogenous rate of its autonomous components, it can be proved that the system converges to this rate. The presence of autonomous demand - such as autonomous consumption, government expenditures, exports and so on - allows to distinguish between marginal propensity to save s - concerning savings out of income - and average propensity to save ($s - z$), where z is the ratio between autonomous demand and normal output; then, the macroeconomic equilibrium can be reached not only by deviations from normal capacity, but also through changes in average savings due to variations in the ratio between autonomous demand and normal output. Here we expose the model introducing an autonomous component of demand within a neokaleckian framework, partly following Lavoie (2014, pp. 405 and following) and Allain (2015).

⁴⁹ Intuitively, Harrodian instability implies that at the not normal equilibrium position, firms may tend to revise expectations about the term α , which may represent the “assessed trend growth rate of sales, or [...] the expected secular rate of growth of the economy” (Lavoie 2015, p. 378); then, an increase in α may lead to a new adjustment process which will end at an even higher rate of capacity utilization, and so on. For a different position, see, for instance, Trezzini (2017).

With an autonomous component of demand, fully adjusted equilibrium position becomes:

$$sf(k) = (1 + g_z)k + zf(k)$$

or:

$$s_c(1 + r) = (1 + g_z) + z \frac{f(k)}{k}$$

where z represents the ratio between autonomous demand and normal output and g_z is the (given) growth rate of autonomous demand.

In order to see how this model works, consider the graphs in appendix B (“Serrano model”). Starting from a normal position (point a), assume a decrease in the marginal propensity to save (s_c decreases to s'_c , and then s decreases to s' , from point a to point b). In the initial period, since the rate of growth has not changed, the level of capacity utilization must rise in order for the macroeconomic equilibrium to be reached (in the Solow diagram the equilibrium point remains a; in the isoquant plane the black line rotates clockwise and we move from a to a’):

$$s'_c u(1 + r) = (1 + g_z)k + z \frac{f(k)}{k}$$

Assuming that investments react positively to u and that Keynesian stability condition holds, the (temporary) medium-run equilibrium is reached in a way similar to that of the preceding

model, with a (temporary) higher than normal equilibrium degree of capacity utilization (in the Solow diagram, the medium-run equilibrium is reached in point c; in the isoquant plane the slope of expansion path tends to k/u^* , where u^* is the medium-run equilibrium rate of capacity utilization, see the orange line):

$$s'_c u^* (1 + r) = [1 + g(u^*)]k + z \frac{f(k)}{k}$$

where $g(u^*) > g_z$.

But, now, since investments are growing at a rate higher than the (given) growth rate of autonomous demand, and since the growth rate of total demand is a weighted average between the growth rate of investments and the growth rate of autonomous demand, we have that $g > g_y > g_z$ (see Cesaratto 2015, p. 178); it follows that the autonomous demand / capacity ratio z decreases over time⁵⁰. The decrease in z , and then the increase in average savings, makes it possible, in each period (and taking g as given in that period), to reach macroeconomic equilibrium with a progressive decrease in u . Indeed, we have (for instance from c to d):

$$[s'(u_1) - z_1] \frac{f(k)}{k} = [1 + g(u^e)]$$

⁵⁰ The dynamics of capital and autonomous demand can also be seen considering the evolution of their ratio over time. Denoting with a dot over a variable its derivative with respect to time, we have: $\left(\frac{\dot{z}}{z}\right) = \frac{\dot{z}K - z\dot{K}}{K^2} = \frac{\dot{z}}{z} \frac{z}{K} - \frac{z}{K} \frac{\dot{K}}{z} = \frac{z}{K} (g_z - g_k)$. The share of autonomous demand on output - and then average savings - continues to decrease as long as g_z is smaller than g_k . Lavoie (2014, p. 407) provides a formal proof of dynamic stability.

where $u_1 < u^*$, $z_1 < z$ and $g_z < g(u^e) < g(u^*)$.

During the adjustment process, the decreasing u slows down g , and then g_y (which is the average between the decreased g and the given g_z). In the end, u tends to its normal level and g tends to g_y , which in turn tends to g_z (in the Solow diagram, the adjustment process goes from c to a ; in the isoquant plane, the expansion path rotates anti-clockwise due to the decreasing level of capacity utilization, until its slope reaches the optimal capital / labour ratio again):

$$s'_c(1+r) = (1+g_z)k + z'f(k)$$

where $z' < z$.

In the final position, capacity utilization comes back to its normal level, all variables grow at the given growth rate of autonomous demand, average savings are the same, marginal savings are lower, the ratio between autonomous demand and normal income is lower, per-capita production and distribution are the same. The complete model is presented in appendix A.

Finally, we can consider the case in which investments depend both on capacity utilization and the *normal* rate of profit (Bhaduri, Marglin 1990; Marglin, Bhaduri)⁵¹. The equation of investments becomes (see Lavoie 2014, p. 371):

$$1 + g = \alpha + \beta u + \lambda(1 + r)$$

⁵¹ Here we consider the normal rate of profit instead of the profit share as in the Marglin-Bhaduri model.

and macroeconomic equilibrium becomes:

$$s_c u(1+r) = \alpha + \beta u + \lambda(1+r)$$

The novelty is that, with the introduction of the normal rate of profit in the investment function, there is no guarantee that, after an increase in normal wages (or a decrease in normal rate of profit, which is the same), equilibrium is reached at a higher level of both growth and capacity, as it was in neo-Kaleckian model, and viceversa (see the solutions in appendix A and the graphs in appendix B “Marglin-Bhaduri model”). Indeed, we can have three cases, depending on which effect prevails. Assume a decrease in normal rate of profit and then an increase in the optimal capital / labour ratio. At first, capacity utilization must increase in order to preserve macroeconomic equilibrium. From the following period, growth dynamics depends on the relative sensitivity of investments with respect to capacity utilization and the normal rate of profit. In the final equilibrium position, we can have three situations: a stagnationist wage-led regime, if both growth and the degree of capacity utilization increase (equilibrium is reached over point b); a stagnationist profit-led regime if capacity utilization increases but growth decreases (equilibrium is reached between point b and c); an exhilarationist profit-led regime if both capacity utilization and growth decrease (equilibrium is reached under point c). The different regimes are *cooperative* if the interests of workers and firms coincide (this can be true for a wage led stagnationist regime as well as a profit led exhilarationist regime)⁵²; they are *conflictual* if the interests of the two classes

⁵² If capacity utilization increases, employment also increases; if growth rate increases, the actual rate of profit also increases, as we can see from macroeconomic equilibrium condition.

don't coincide (this can be true for a profit led stagnationist regime). It's interesting to note how the Marglin Bhaduri model can be represented in the standard Solow diagram⁵³.

7. Conclusions

In this chapter we have shown in a simple framework how the choice of exogenous variables represents an essential element for the logical structure of economic models. In particular, we have shown how scarce resources on the one hand, and demand and, in several cases, distribution on the other hand, are the crucial elements that determine the structure of a model, leading to different normative and policy implications.

We have also shown how variations in canonical models don't change their crucial features. On the one hand, the introduction of autonomous demand, the classic hypothesis of savings, intertemporal preferences or market imperfections doesn't change the essential structure of the neoclassical model. On the other hand, the introduction of autonomous demand or the presence of endogenous investments depending on capacity utilization or normal distribution doesn't change the essential elements of the alternative approach.

The principal limit of this comparison procedure is that it can give the misleading and *naïve* impression that the complex working of a capitalist economy can be reduced to a simple theoretical debate about the exogenous or endogenous nature of some variables. Of course, this is not the case. The choice of one exogenous variable or another, at least partly, can be thought of as the analytical expression of the adherence to a particular theory (an organic

⁵³ Note that the results are not the same as those obtained with a fixed technique. However, it may be interesting to represent the model in the Solow graph for comparison purposes.

system of theoretical principles) or another; this leads to relevant policy consequences: for instance, on the one hand, according to the canonical neoclassical approach - which assumes given endowments - the economy tends to the full employment of resources, the demand has no influence on output and distribution, and *natural* distribution is thought to be independent from any political or social influence; on the other hand, according to alternative approach - which assumes a given level of demand and possibly a given normal distribution - there's no tendency to full employment, factor endowments have no influence on output and distribution, and *normal* distribution is thought to be determined by political, social, historical and institutional factors, ultimately related to the class struggle between workers and firms.

Even without denying the influence of ideology on the choice of the premises of a model, the latter can be also evaluated from a logical, historical and methodological perspective (see for instance Brancaccio 2012). From a logical point of view, all the variants of neoclassical model suffer from logical inconsistencies due to the inclusion of capital among exogenous variables and to the supposed, but logically incorrect, inverse relation between the value of capital and the rate of profit⁵⁴. From a methodological perspective, the classical-keynesian model has been supported for its capability to single out the scope of deductive approach, allowing for a fruitful link between *core* analysis and the research fields which require alternative research methods (Garegnani 1981). From a historical point of view, alternative models would reflect better the essential features of modern economy, characterized, in particular, by high technical and productive flexibility.

⁵⁴ see for instance Sraffa 1960, Garegnani 1960, 1970, Petri 2003, 2004; see Harcourt 1972 for a review.

The focus of this chapter, however, was not about *which* model to choose. Instead, it was aimed to clarify that economic theory always allows for the presence of *alternative* models; and the awareness that an opportunity of choice exists is a necessary condition to grasp it (see Brancaccio 2012).

APPENDIX A - MODELS

For the sake of simplicity and consistency, we denote with an apostrophe the equations reformulated by taking into account other equations of the model, with an asterisk the equations representing deviations from normal positions and with bold the equations used for the solution. The variables to the right of the arrows are the variables determined in the relative equation.

NEOCLASSICAL MODELS

SOLOW MODEL (short period)

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$2) f'(k) = 1 + r \rightarrow r$$

$$3) sf(k) = (1 + g)k \rightarrow g$$

SOLOW MODEL (steady state)

$$1) f(k_{ss}) = w + (1 + r)k_{ss} \rightarrow w$$

$$2) f'(k_{ss}) = 1 + r_n \rightarrow r_n$$

$$3) sf(k_{ss}) = (1 + g_n)k_{ss} \rightarrow k_{ss}$$

SOLOW MODEL WITH PUBLIC EXPENDITURES

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$2) f'(k) = 1 + r \rightarrow r$$

$$3) sf(k) = (1 + g)k + zf(k) \rightarrow g$$

SOLOW MODEL WITH CLASSICAL HYPOTHESIS OF SAVINGS

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$2) f'(k) = 1 + r \rightarrow r$$

$$3) sf(k) = (1 + g)k$$

$$3') s_c(1 + r) = (1 + g) \rightarrow g$$

$$4) s = s_c(1 + r) \frac{k}{f(k)} \rightarrow s$$

"IMPERFECTIONIST" SOLOW MODEL (w given)

$$1) f(k_i) = w_i + (1 + r_i)k_i$$

$$1'[1 + 2]) w_i = f(k_i) - f'(k_i)k_i \rightarrow k$$

$$2) f'(k_i) = 1 + r_i \rightarrow r$$

$$3) sf(k_i) = (1 + g_i)k_i \rightarrow g$$

RAMSEY MODEL (short period)

1) $f(k) = w + (1 + r)k \rightarrow w$

2) $f'(k) = 1 + r \rightarrow r$

3) $sf(k) = (1 + g)k = k_{+1}$

3'[3 + 4]) $c = f(k) - k_{+1} \rightarrow k_{+1}$

4) $s = 1 - \frac{c}{f(k)} \rightarrow s$

5) $\max_{c_{+i}} U = \sum_{i=0}^{\infty} \frac{1}{(1 + \rho)^i} u[c_{+i}]$

5'[3' + 5]) $\max_{k_{+1}} U = \sum_{i=0}^{\infty} \frac{1}{(1 + \rho)^i} u[f(k_{+i}) - k_{+i+1}]$

5'sol.) $1 + r_{+1} = \frac{u'(c)}{u'(c_{+1})} (1 + \rho)$

5''sol. [5'sol. + 6]) $\frac{c_{+1}}{c} \approx \frac{1}{\theta} (r_{+1} - \rho) \rightarrow c$

6) $u(c) = \frac{c^{1-\theta} - 1}{1 - \theta}$

7) $\lim_{i \rightarrow \infty} \frac{1}{(1 + \rho)^i} u'(c_i) k_{i+1} = 0$

RAMSEY MODEL (steady state)

1) $f(k_{ss}) = w_{ss} + (1 + r_n)k_{ss} \rightarrow w_{ss}$

2) $f'(k_{ss}) = 1 + r_n \rightarrow k_{ss}$

3) $s_{ss}f(k_{ss}) = (1 + g_n)k_{ss} \rightarrow s_{ss}$

4) $s_{ss} = 1 - \frac{c_{ss}}{f(k_{ss})} \rightarrow c_{ss}$

5) $r_n = \rho \rightarrow r_n$

6) $\rho > g_n$

ALTERNATIVE MODELS

SHORT PERIOD ADJUSTMENT (either through capacity utilization or prices or a mix of both)

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$1^*) uf(k) = u \frac{w}{\delta} + (1 + \gamma r)k \rightarrow u \text{ or } \delta \text{ or a mix}$$

$$2) f'(k) = 1 + r \rightarrow k$$

$$3) sf(k) = (1 + g)k$$

$$3') s_c(1 + \gamma r) = 1 + g \rightarrow \gamma$$

$$4) s = s_c(1 + \gamma r) \frac{k}{f(k)} \rightarrow s$$

Exogenous: r, g, u or δ

Solution: 2) k - 1) w - 3') γ - 1*) u or δ or a mix of u and δ - 4') s

LONG PERIOD ADJUSTMENT THROUGH CAPACITY UTILIZATION (Classical-keynesian school)

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$1^*) uf(k) = uw + (1 + \gamma r)k \rightarrow u$$

$$2) f'(k) = 1 + r \rightarrow k$$

$$3) sf(k) = (1 + g)k$$

$$3') s_c(1 + \gamma r) = 1 + g \rightarrow \gamma$$

$$4) s = s_c(1 + \gamma r) \frac{k}{f(k)} \rightarrow s$$

Exogenous: r, g

Solution: 2) k - 1) w - 3') γ - 1*) u - 4') s

LONG PERIOD ADJUSTMENT THROUGH PRICES / NORMAL DISTRIBUTION (Cambridge school)

$$1) f(k) = w + (1+r)k \rightarrow w$$

$$2) f'(k) = 1+r \rightarrow k$$

$$3) sf(k) = (1+g)k$$

$$3') s_c(1+r) = 1+g \rightarrow r$$

$$4) s = s_c(1+r) \frac{k}{f(k)} \rightarrow s$$

Exogenous: g

Solution: 3') r - 2) k - 1) w - 4') s

NEOKALECKIAN MODEL

$$1) f(k) = w + (1+r)k \rightarrow w$$

$$1^*) uf(k) = uw + (1+\gamma r)k$$

$$1^*'[1+1^*]) (1+\gamma r) = u(1+r) \rightarrow \gamma$$

$$2) f'(k) = 1+r \rightarrow k$$

$$3) sf(k) = (1+g)k$$

$$3'[3+4'+5]) s_c u(1+r) = \alpha + \beta(u^e - 1) \rightarrow u$$

$$4) s = s_c(1+\gamma r) \frac{k}{f(k)}$$

$$4'[4+1^*'] s = s_c u(1+r) \frac{k}{f(k)} \rightarrow s$$

$$5) 1+g = \alpha + \beta(u^e - 1) \rightarrow g$$

Solutions:

Short period ($u^e \neq u$): 2) k - 1) w - 5) g - 3') u - 1*') γ - 4') s

Long period ($u^e = u$): 2) k - 1) w - 3') u^* - 5) g^* - 1*') γ^* - 4') s^*

Explicit solutions (long period):

$$u^* = \frac{\alpha - \beta}{s_c(1+r) - \beta}$$

$$1 + g^* = \alpha + \beta(u^* - 1) = \frac{\alpha - \beta}{1 - \frac{\beta}{s_c(1+r)}}$$

$$1 + \gamma^*r = u^*(1+r) = \frac{\alpha - \beta}{s_c - (1+r)}$$

Keynesian stability condition

$$s_c(1+r) > \beta$$

SERRANO MODEL (long period, $g = g_z$)

1) $f(k) = w + (1+r)k \rightarrow w$

2) $f'(k) = 1+r \rightarrow k$

3) $sf(k) = (1+g)k + zf(k)$

3') $s_c(1+r) = (1+g_z) + z \frac{f(k)}{k} \rightarrow z$

4) $s = s_c(1+r) \frac{k}{f(k)} \rightarrow s$

5) $g = g_z \rightarrow g$

6) $g_z = \bar{g}_z$

Solution: 2) k - 1) w - 6) g_z - 5) g - 3') z - 4') s

SERRANO MODEL (medium period, $g \neq g_z, u^e = u, z$ given)

$$1) f(k) = w + (1+r)k \rightarrow w$$

$$1^*) uf(k) = uw + (1+\gamma r)k$$

$$1^*'[1+1^*'] (1+\gamma r) = u(1+r) \rightarrow \gamma$$

$$2) f'(k) = 1+r \rightarrow k$$

$$3) sf(k) = (1+g)k + zf(k)$$

$$3'[3+4'+5]) s_c u(1+r) = \alpha + \beta(u-1) + z \frac{f(k)}{k} \rightarrow u$$

$$4) s = s_c(1+\gamma r) \frac{k}{f(k)}$$

$$4'[4+1^*'] s = s_c u(1+r) \frac{k}{f(k)} \rightarrow s$$

$$5) 1+g = \alpha + \beta(u-1) \rightarrow g$$

$$6) g_z = \bar{g}_z$$

Solution: 2) k - 1) w - 3') u* - 5) g* - 1^*) \gamma* - 4') s*

Explicit solutions:

$$u^* = \frac{\alpha - \beta + z \frac{f(k)}{k}}{s_c(1+r) - \beta}$$

$$1+g^* = \alpha + \beta(u^* - 1) = \frac{\alpha - \beta + \beta z \frac{f(k)}{s_c(1+r)k}}{1 - \frac{\beta}{s_c(1+r)}}$$

$$1+\gamma^*r = u^*(1+r) = \frac{\alpha - \beta + \beta z \frac{f(k)}{s_c(1+r)k}}{s_c - \frac{\beta}{(1+r)}}$$

MARGLIN BADHURI MODEL

$$1) f(k) = w + (1+r)k \rightarrow w$$

$$1^*) uf(k) = uw + (1+\gamma r)k$$

$$1^*'[1+1^*]) (1+\gamma r) = u(1+r) \rightarrow \gamma$$

$$2) f'(k) = 1+r \rightarrow k$$

$$3) sf(k) = (1+g)k$$

$$3'[3+4'+5])s_c u(1+r) = \alpha + \beta u + \lambda(1+r) \rightarrow u(u^*)$$

$$4) s = s_c(1+\gamma r) \frac{k}{f(k)}$$

$$4'[4+1^*']) s = s_c u(1+r) \frac{k}{f(k)} \rightarrow s$$

$$5) 1+g = \alpha + \beta u^e + \lambda(1+r) \rightarrow g$$

Solutions

Short period (u^e exogenous): 2) k - 1) w - 5) g - 3') u - 1^*) γ - 4') s

Long period ($u^e = u$): 2) k - 1) w - 3') u^* - 5) g^* - 1^*) γ^* - 4') s^*

Explicit solutions

$$u^* = \frac{\alpha + \lambda(1+r)}{s_c(1+r) - \beta}$$

$$1 + \gamma^* r = u^*(1+r) = \frac{1+g^*}{s_c} = \frac{[\alpha + \lambda(1+r)](1+r)}{s_c(1+r) - \beta}$$

$$1 + g^* = \alpha + \beta u^* + \lambda(1+r) = \frac{s_c[\alpha + \lambda(1+r)](1+r)}{s_c(1+r) - \beta}$$

Implications

$$\frac{\partial u}{\partial(1+r)} = \frac{-(\beta\lambda + \alpha s_c)}{[s_c(1+r) - \beta]^2}$$

$$\frac{\partial(1+\gamma^*r)}{\partial(1+r)} = \frac{-\alpha\beta + [s_c(1+r) - 2\beta]\lambda(1+r)}{[s_c(1+r) - \beta]^2}$$

Keynesian stability condition

$$s_c(1+r) > \beta$$

Regimes

Stagnationist, wage-led (and then cooperative) regime

$$\frac{\partial u}{\partial(1+r)} < 0, \quad \beta\lambda + \alpha s_c > 0$$

$$\frac{\partial(1+\gamma^*r)}{\partial(1+r)} < 0, \quad [s_c(1+r) - 2\beta]\lambda(1+r) < \alpha\beta$$

Stagnationist, profit-led (and then conflictual) regime

$$\frac{\partial u}{\partial(1+r)} < 0, \quad \beta\lambda + \alpha s_c > 0$$

$$\frac{\partial(1+\gamma^*r)}{\partial(1+r)} > 0, \quad [s_c(1+r) - 2\beta]\lambda(1+r) > \alpha\beta$$

Exhilarationist, profit-led (and then cooperative) regime

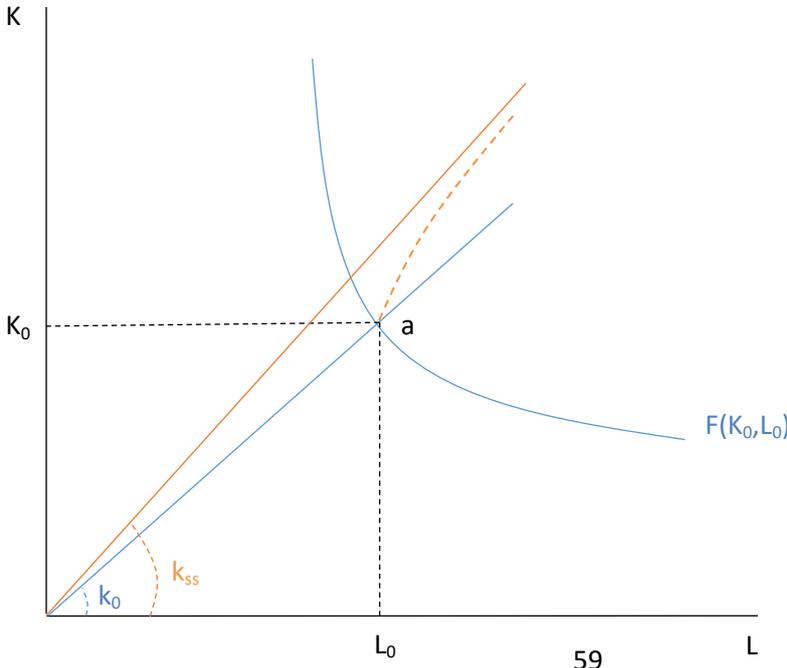
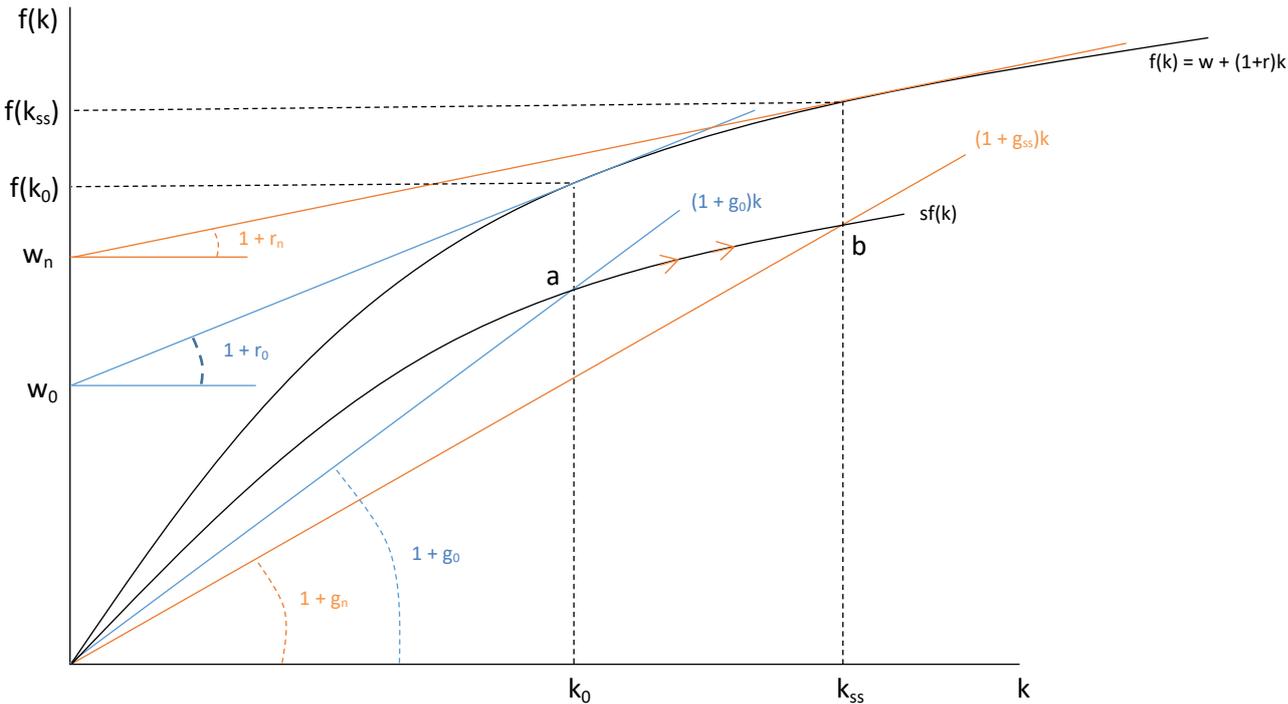
$$\frac{\partial u}{\partial(1+r)} > 0, \quad \beta\lambda + \alpha s_c < 0$$

$$\frac{\partial(1+\gamma^*r)}{\partial(1+r)} > 0, \quad [s_c(1+r) - 2\beta]\lambda(1+r) > \alpha\beta$$

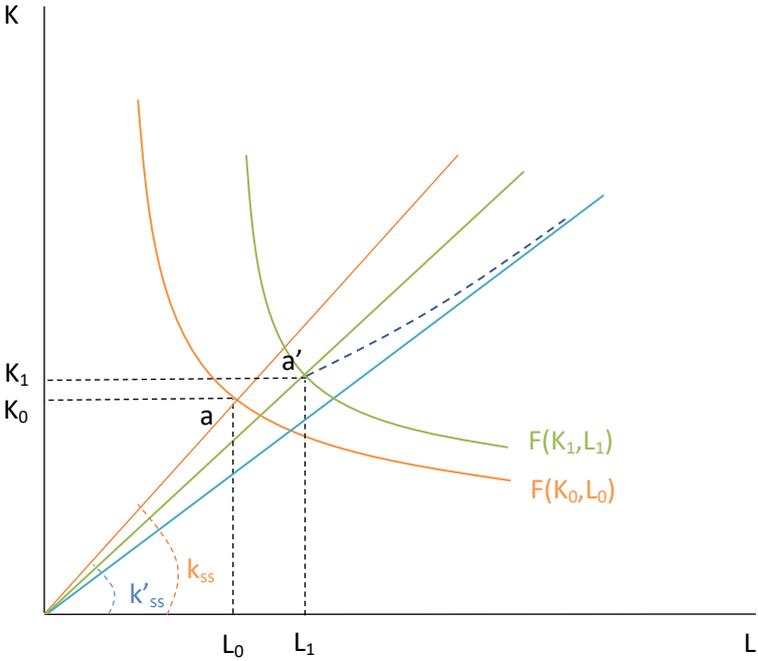
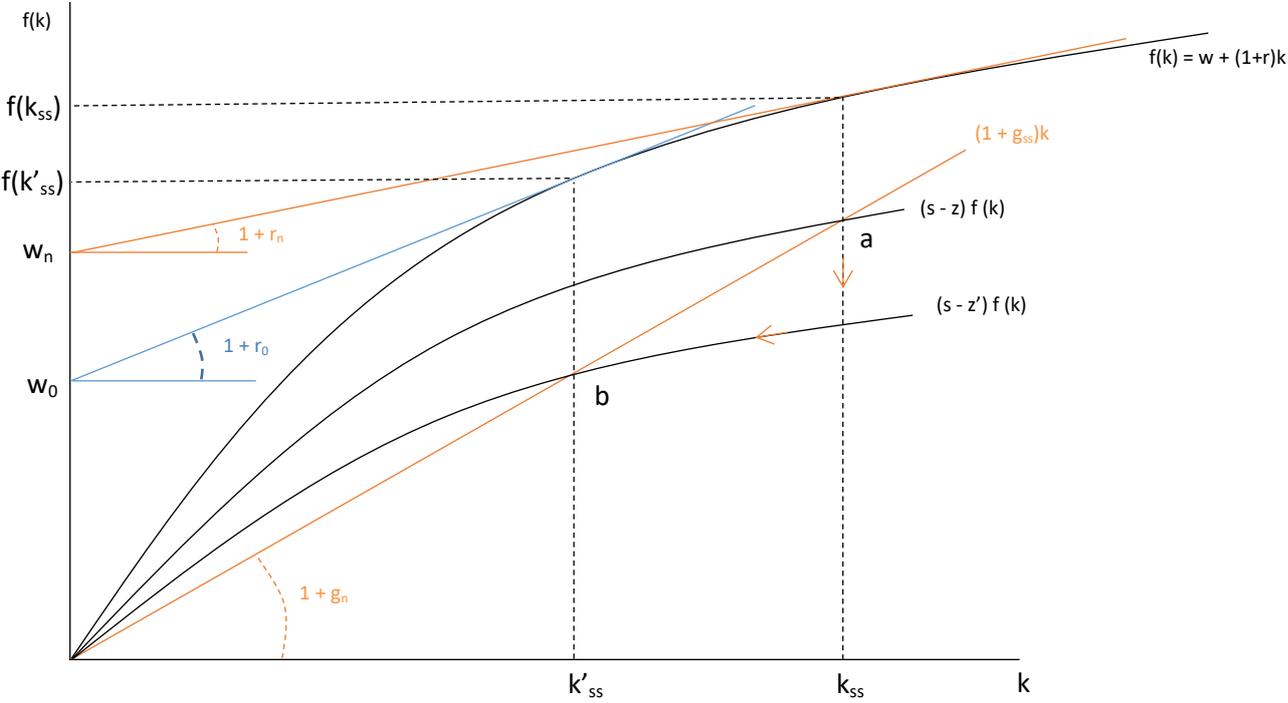
APPENDIX B - GRAPHS

NEOCLASSICAL MODELS

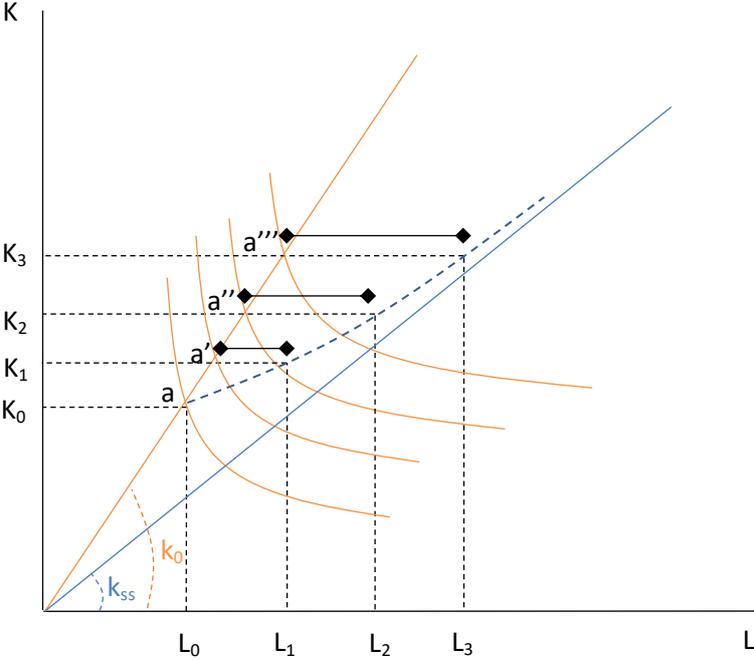
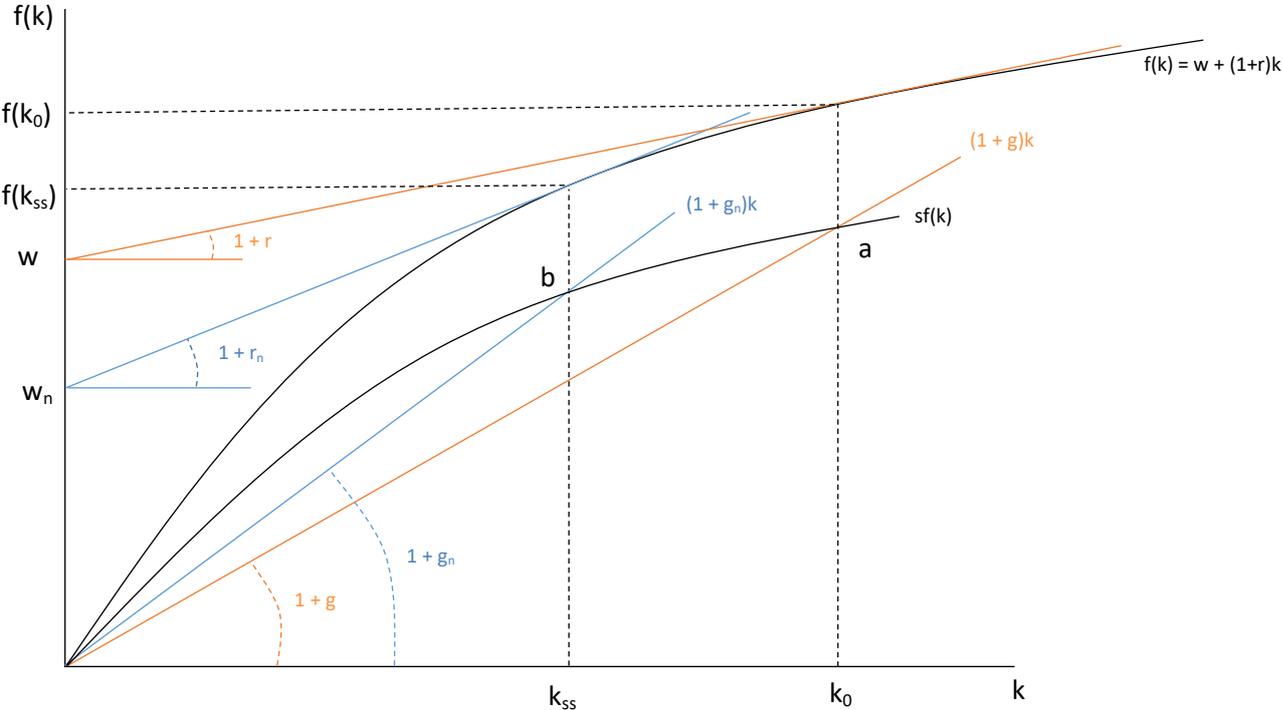
SOLOW MODEL



SOLOW MODEL WITH AUTONOMOUS DEMAND

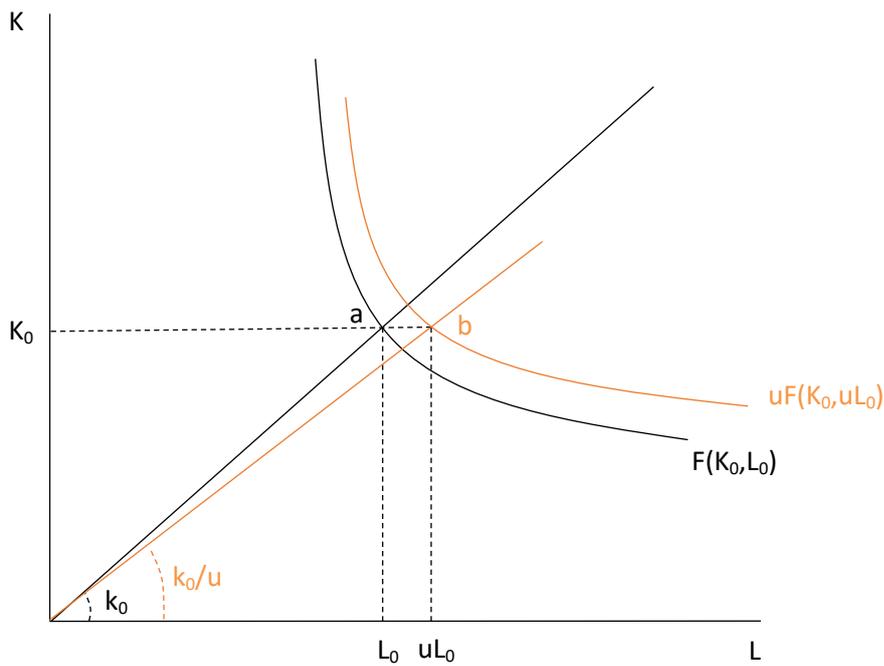
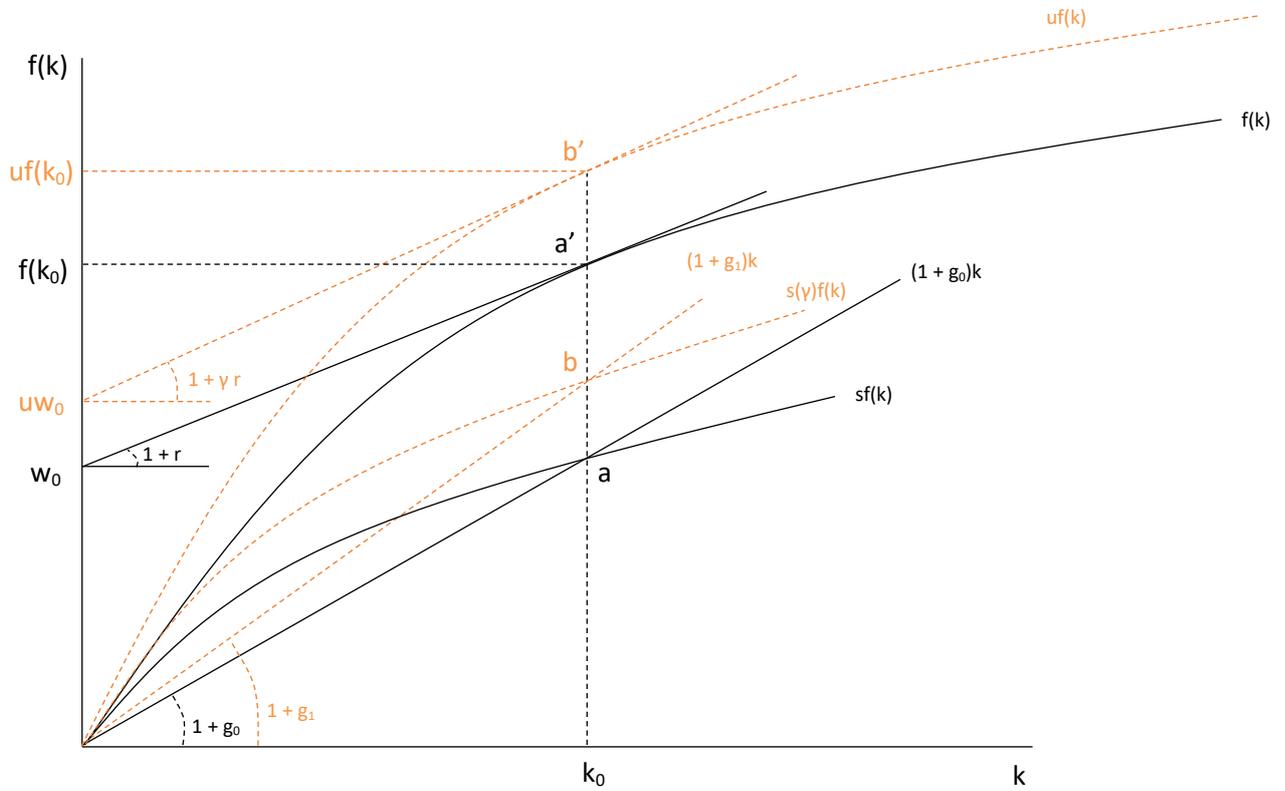


SOLOW MODEL WITH MARKET IMPERFECTIONS

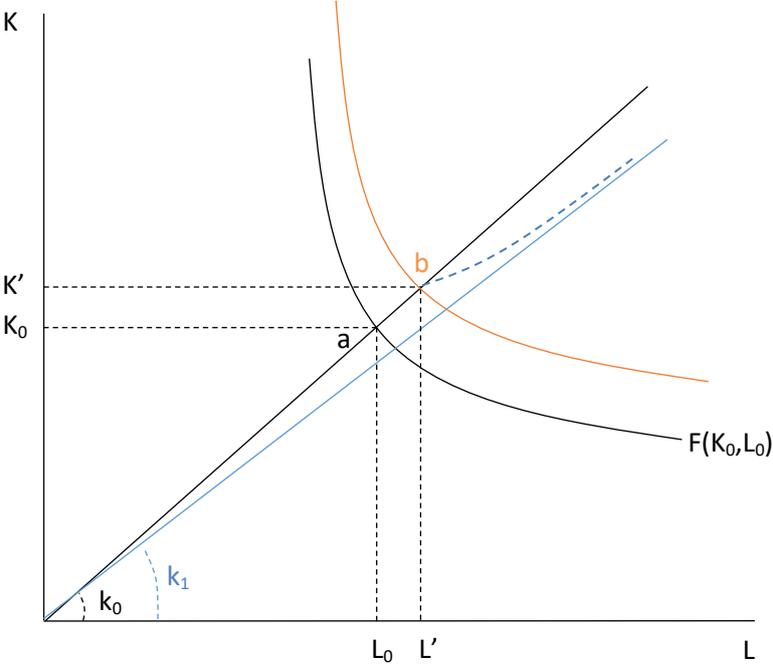
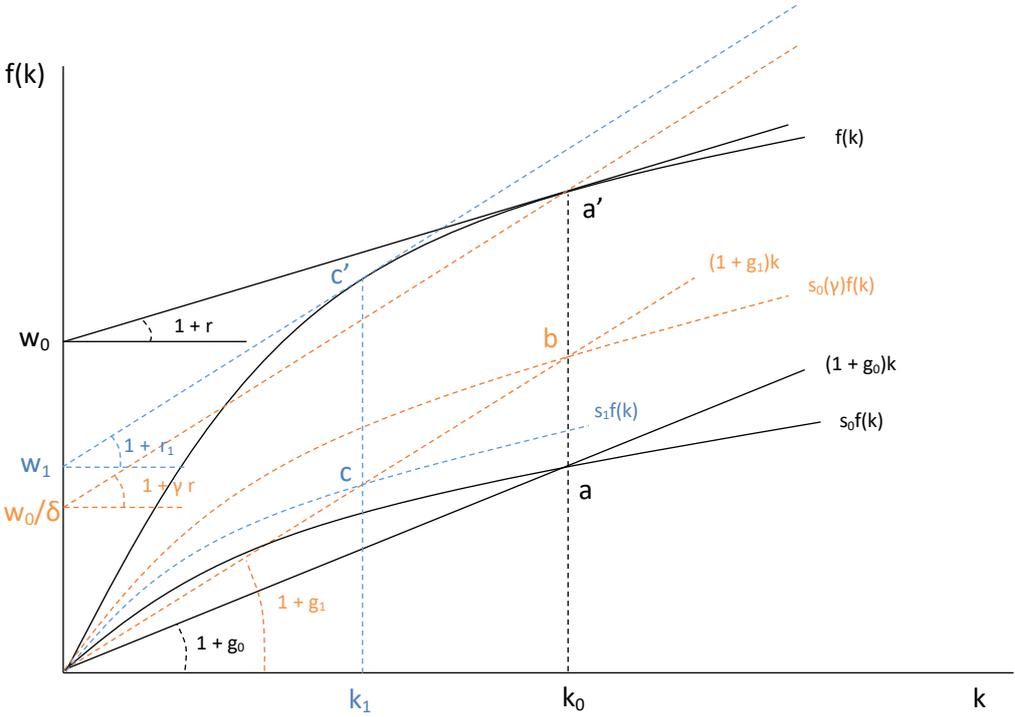


ALTERNATIVE MODELS

ADJUSTMENT THROUGH DEVIATIONS FROM NORMAL CAPACITY UTILIZATION

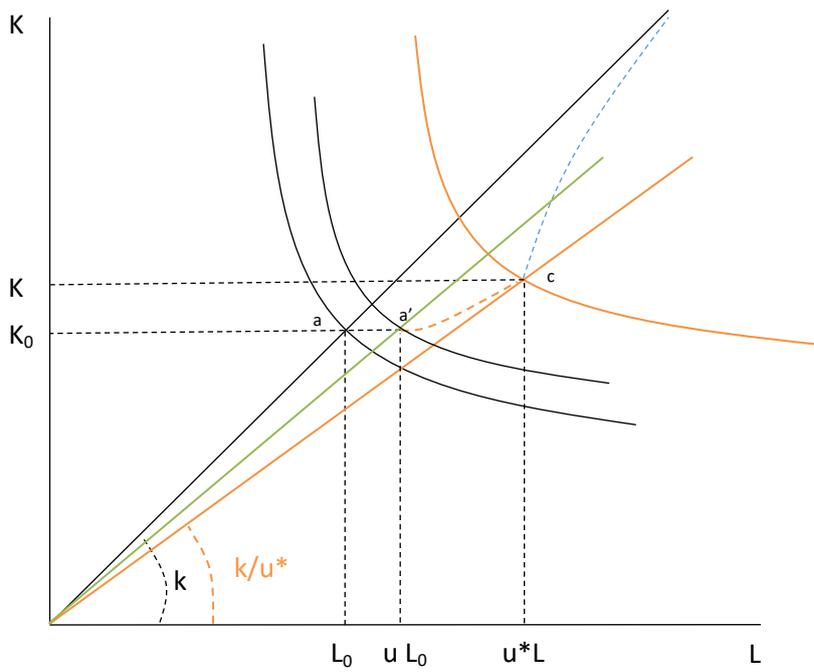
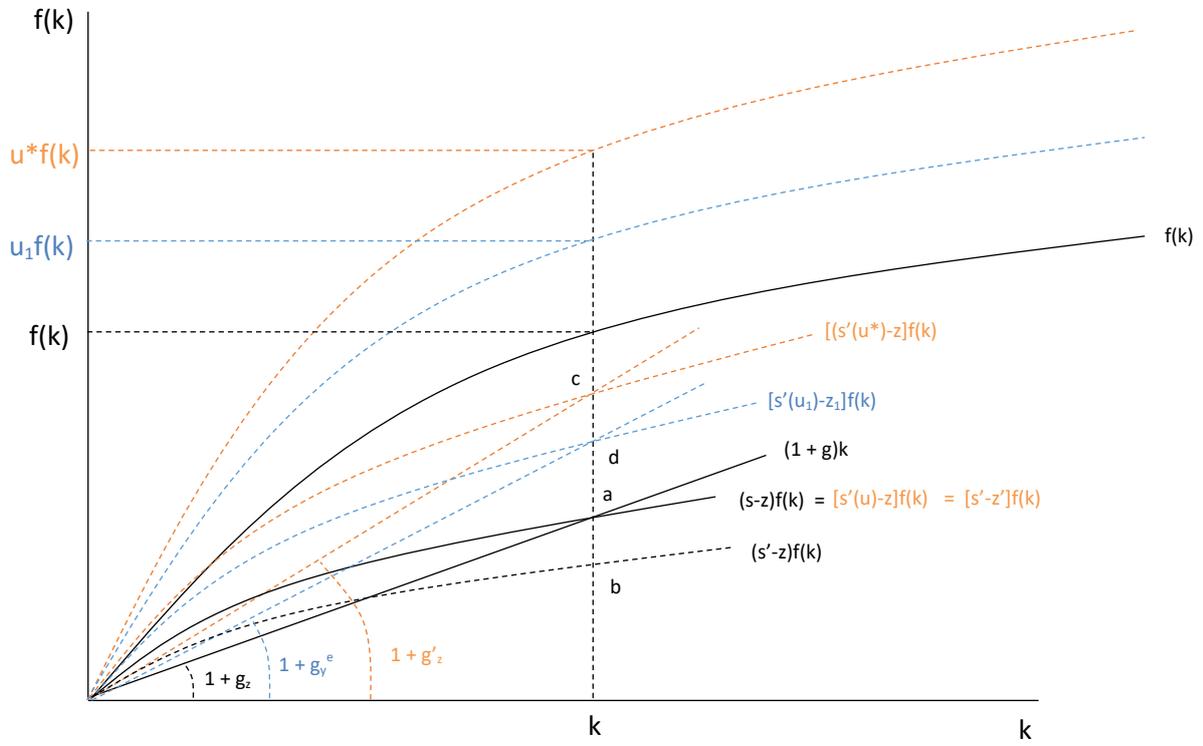


ADJUSTMENT THROUGH DEVIATIONS FROM NORMAL PRICES



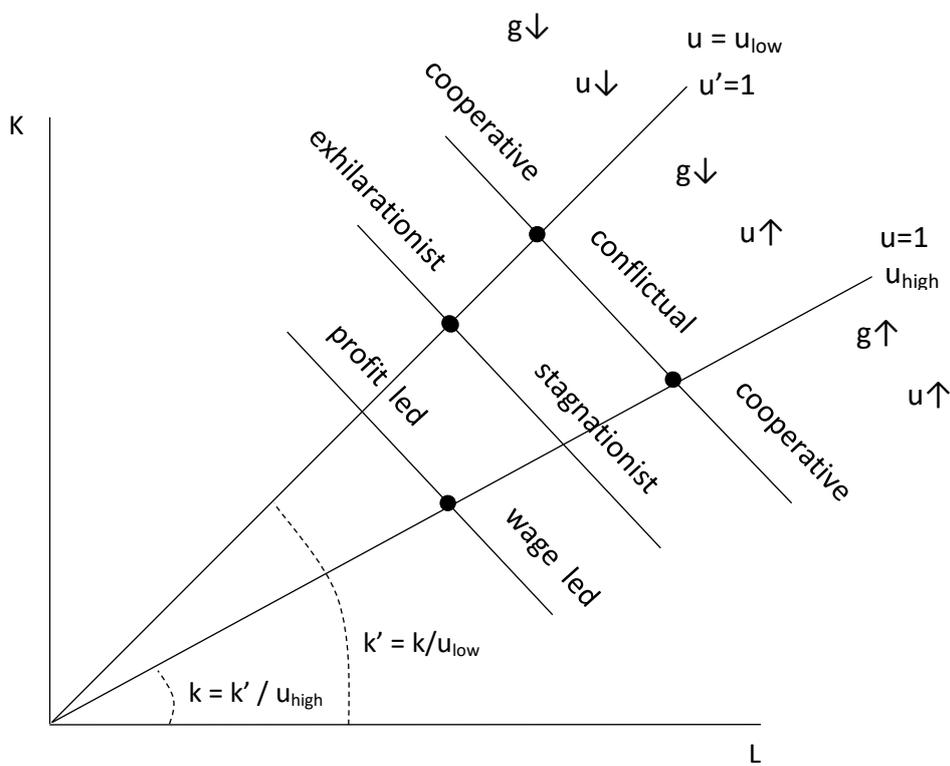
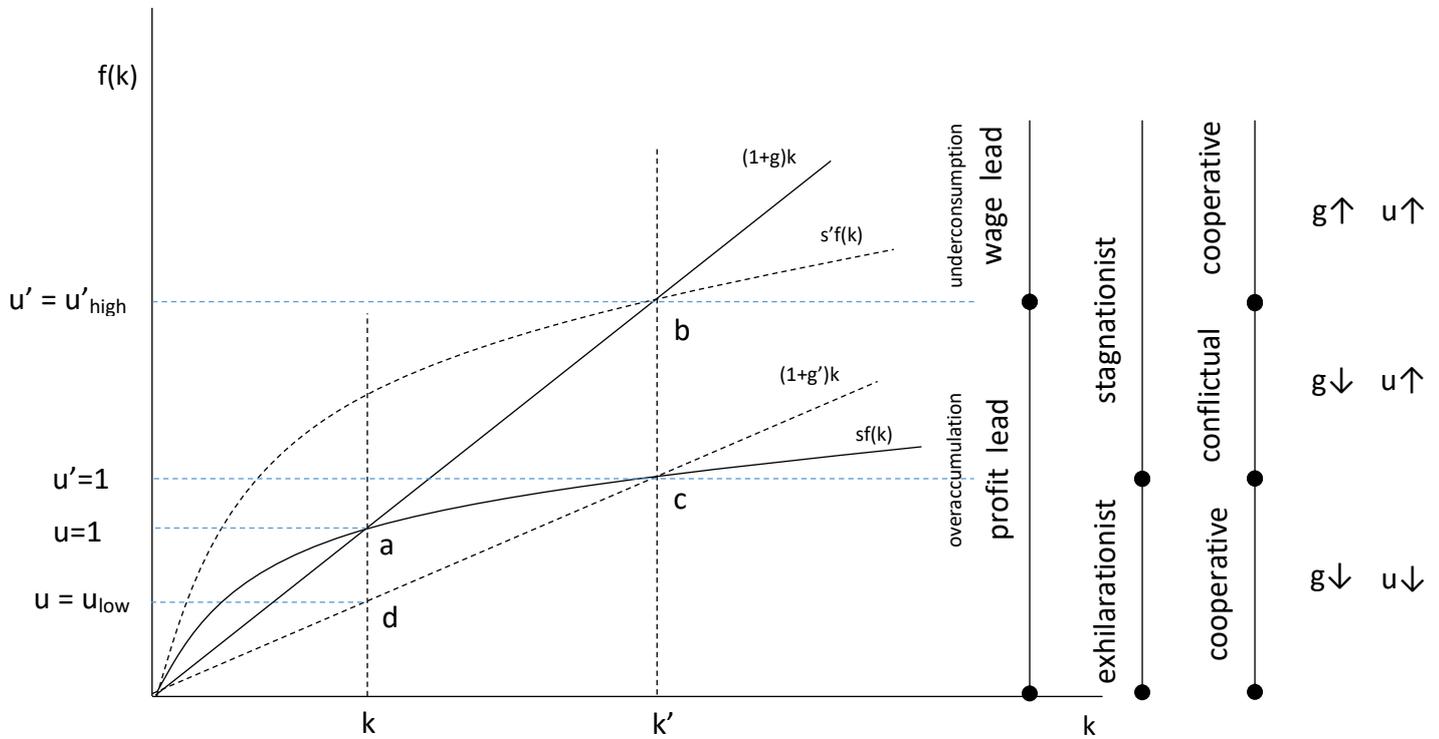
SERRANO MODEL

$s_c \downarrow$



MARGLIN BADHURI MODEL

$w \uparrow$ ($r \downarrow$)



CHAPTER 2

ALTERNATIVE INTERPRETATIONS OF STOCK MARKET DYNAMICS

1. Introduction

The mainstream interpretation of stock markets is based on the Present Value Model (PVM), according to which stock prices reflect the expected value of discounted future dividends. The model, according to its advocates, gives theoretical support to the idea that stock prices always reflect all available relevant information at all times (Fama, 1970): financial assets should always be correctly priced, and no arbitrage opportunities should arise. It follows that prices can only change in response to new information about fundamentals.

This theory, known as Efficient Market Hypothesis (EMH), has been questioned since the publication of Shiller (1981) seminal work, in which he showed that actual stock prices vary too much with respect to their theoretical ex-post values computed using actual dividend stream: this result clashes with the fact that actual prices, being just an expectation of ex-post theoretical prices, should vary less than the latter. In other words, prices seem too volatile to just reflect future dividends. Subsequent econometric tests have confirmed this result (see, for instance, Campbell, Shiller 1988). These criticisms give support to the idea that movements in actual prices must be explained by other factors than future dividends.

On the one hand, supporters of PVM have proposed an alternative version of the model, in which the discount rate is assumed to be varying, changing with agents' intertemporal preferences or behaviour toward risk (see Campbell, Lo, McKinley (1997), Cuthbertson and

Nitzsche (2005), Cochrane (2005) for a survey). In this way, ex-ante variations of the *subjective required* discount rates give more volatility to theoretical prices, which should better fit the actual ones. The research program followed by PVM supporters has been aimed to find the “right” theoretical model for the time varying discount rate – generally linked with consumption dynamics, such as the stochastic discount factor (SDF) - so as to give theoretical prices a sufficient volatility.

On the other hand, critics of PVM have tried to explain its empirical rejections as due to the presence of economic bubbles, that is factors that influence the price letting it to deviate from its fundamental value. Bubbles can be introduced both in rational and in a non-rational framework: in the first case bubbles are correctly predicted by the agents, so that prices include a bubble term, and price variations stem from potential higher capital gain/losses not justified by future dividends (Blanchard, Watson 1982); in the second case, agents may not know the “right” model of the economy, or may adopt non-standard behaviour (Shiller 2000) or, in a more heterodox tradition, can overestimate future dividends during the “euphoric phase” of stock markets (Minsky (1982), Kindleberger (1989)).

Allowing for these changes in the original PVM, however, doesn't necessarily lead to a rejection of the neoclassical model, which is the theoretical foundation of PVM: bubbles, irrational behaviour, market anomalies and so on may just be interpreted as *imperfections* of a model that, without them, would remain right. In all of these approaches, stock prices are nonetheless assumed to be determined by the presence of scarce resources, which - together with preferences, technology and possible shocks in fundamentals - influence both current and future equilibrium path of prices. Thus, it seems that a large part of the debate

does not involve the theoretical foundations of PVM, but just the possibility that prices can deviate from their fundamental value, not questioned.

However, an alternative interpretation of stock market dynamics can be traced following a post-Keynesian and Sraffian perspective. In this framework, stock returns are more difficult to predict, since they depend on many factors, such as: the state of social relationship; the decisions of firms about production and demand, and then about the degree of capacity utilization and the deviations from normal prices; the decisions of financial authorities about credit provision; and, in general, the stability of the economic system. Moreover, the alternative model doesn't provide a natural equilibrium similar to the neoclassical one, and then "steady pivots" analogous to the neoclassical fundamentals are absent, making hard to discern between rational and irrational behaviour. The very concept of bubble can be misleading, since no natural equilibrium, nor deviations from it, exist.

Generally, the introduction of equities in a model is justified by the fact that stock returns differ from the interest rate, due to the presence of uncertainty or adjustment costs. For our purposes, it will be sufficient to assume a deterministic framework with possible exogenous shocks, since the principal aim is to single out the main differences between the two models in the simplest way. In this way, it is possible to show which elements affect stock prices in the two models, and then to make some conjecture about their predictability and volatility.

The chapter is structured as follows: first, we shall make explicit the logical nexus between the PVM and the neoclassical model of growth and distribution, introducing equities in the latter: we shall see how stock prices are determined by the standard neoclassical fundamentals. Second, we shall show how the theoretical and analytical problems of

neoclassical model cast doubts on the consistency of PVM. Third, we shall determine equity prices in the alternative model. Finally, we shall compare the two models, verifying how the alternative approach can provide a more consistent explanation of bubbles.

2. The present value model

According to PVM, stock prices should reflect the rational expectation of discounted future dividends, also known as the *fundamental value* (Fama 1970):

$$P_t = E_t[P_t^*]$$

$$P_t^* = \sum_{i=1}^k \delta^i D_{t+i}$$

Where P is the actual price in period t; P* is the *ex-post* price or perfect foresight price, that is the price we would have been in t if agents had *exactly* forecast dividends from t onwards; E_t is the expectation at time t using all the relevant information in t; δ is the discount factor; D are the dividends.

This formula can be derived from the definition of stock return, i.e. the sum of end-of-period capital gains ($P_{t+1} - P_t$) and end-of-period dividends (D_{t+1}) divided by the current price (P_t):

$$1) R_{t+1} = \frac{P_{t+1} - P_t + D_{t+1}}{P_t}$$

The standard PVM is obtained once we make three assumptions (for simplicity we define $\delta = 1 / (1 + R)$) (see for instance Campbell, Lo, MacKinlay 1997):

- the required rate of return (i.e. the equilibrium level of the discount rate) is constant:

$R_t = R_{t+1} = \dots = R$; this stems from the assumption that agents have constant intertemporal preferences and behaviour toward risk.

- individuals cannot improve their expectations about future dividends: differences between expected and actual dividends are ex ante unpredictable and can only be due to random errors not included in the current information set. This result follows from two assumptions.

First, prices incorporate all relevant information, so that changes in prices can only be caused by new no-forecastable information; i.e. prices always reflect the “fundamentals” (future discounted dividends) and immediately change after new information becomes available, so that no arbitrage opportunities can arise (Efficient Market Hypothesis). Second, all the agents form their expectations on the basis of the same best “true” model of the economy: the dividend path is rationally forecasted by individuals according to the “right” theory (Rational Expectations Hypothesis). Given REH we can derive the law of iterated expectations, for which the current expectations of next period expectations of next period price equal the current expectations of next period prices, i.e.: $E_t [E_{t+1} (P_{t+2})] = E_t [P_{t+2}]$; in other words, agents cannot know how they’ll alter their expectations in the future. This means that futures changes in the rate of return can only be considered random.

- the present value of stock prices shrinks to zero as time goes to infinite:

$\lim_{k \rightarrow \infty} E_t(\delta^k P_{t+k}) = 0$; this condition will be satisfied unless stock price is expected to

grow forever at a rate higher or equal to R . This assumption, known as *transversality condition*, allows to remove speculative bubbles.

Taking expectations of (1) and considering the first two assumptions (for which $E_t[R_{t+1}] = R$) we obtain:

$$2) P_t = E_t \left[\frac{P_{t+1} + D_{t+1}}{1 + R} \right] = E_t[\delta(P_{t+1} + D_{t+1})]$$

Solving forward (2) and applying the Law of Iterative Expectations we obtain:

$$3) P_t = E_t \left[\sum_{i=1}^k \delta^i D_{t+i} \right] + E_t[\delta^k P_{t+k}]$$

If we impose transversality condition we obtain the final PVM:

$$4) P_t = E_t \left[\sum_{i=1}^k \delta^i D_{t+i} \right]$$

Current stock prices can be considered as the present value of expected future dividends discounted at a constant rate, also known as the *fundamental value*. In an efficient market, prices reflect all public information available in the current period; they can only change if news about future dividends change; moreover, prices change immediately as new information becomes available.

3. The Shiller volatility test

Since Shiller (1981) seminal work, many empirical tests have rejected PVM. Intuitively, this kind of tests involves a comparison between the variance of perfect forecasted prices - i.e. the present value of *actual* future dividends - and of actual prices - which should be well described by PVM: if the variance of the latter results greater than the variance of the former, we can conclude that actual prices contain more than just the *fundamentals* predicted by PVM; in other words, if actual prices are too *volatile* with respect to future dividends, PVM is rejected. This can be seen from equation 4: if the variance of the present value of *actual* future dividends (the expression between the brackets in the RHS) is lesser than the variance of the present value of *expected* future dividends (RHS, theoretically coinciding with actual prices), then it turns out that stock prices must be driven by other variables besides future dividends.

Since actual prices P are just an expectation of ex-post theoretical price P^* , they should vary less than the latter, that is they cannot incorporate *surprises*. It is then possible to impose volatility bounds on stock prices in the following way. The actual price, in an efficient market, is the optimal forecast of ex-post price:

$$P_t^* = P_t + u_t$$

where u_t is the forecast error in t ⁵⁵. Taking the variance from both sides, and noting that forecast error must be uncorrelated with the effective price⁵⁶, we have:

$$\text{Var}(P_t^*) = \text{Var}(P_t) + \text{Var}(u_t) \rightarrow \text{Var}(P_t^*) \geq \text{Var}(P_t) \rightarrow \sigma(P_t) \leq \sigma(P_t^*)$$

Then, the standard deviation of actual prices should be lesser than the standard deviation of ex-post prices.

Shiller (1981), using a data set including S&P stock prices and dividends between 1871 and 1980⁵⁷, finds that stock prices are excessively volatile: the standard deviation of P_t is 5.69 times the s.d. of P_t^* . Then volatility bound is grossly violated, and (this version of) PVM is to be rejected. In other words, prices seem too volatile to simply reflect future dividends, and thus they must be explained by other factors.

Here we replicate Shiller test using data updated to now. We use real S&P 500 prices and dividends series detrended by dividing by an exponential growth factor. The trend factor is obtained by regressing the logarithm of prices on a constant and time:

$$\ln(P_t) = \alpha + \beta t + u \rightarrow b \rightarrow \lambda = e^b$$

⁵⁵ The forecast error can be computed as the sum of all the forecast errors in dividends from t onwards.

⁵⁶ If P and u were not independent, that is if the forecast error were correlated with the forecast itself, this would mean that the forecast could be improved; but this is in contrast with the idea that prices are the best forecasts of the fundamental value.

⁵⁷ More specifically, Shiller (1981) uses real S&P 500 prices and dividends, detrended by dividing by a long-run exponential growth factor.

Then, we can obtain the detrended series by dividing the real price (dividend) in one year by the trend factor in that same year, taking into account the base year T:

$$p_t = \frac{P_t}{\lambda^{t-T}} = \frac{P_t}{e^{b(t-T)}}$$

$$d_t = \frac{D_t}{\lambda^{t+1-T}} = \frac{D_t}{e^{b(t+1-T)}}$$

Now, we can rearrange PVM in terms of detrending variables dividing both sides of PVM equation by λ^{T-t} and multiplying and dividing RHS by λ^{k+1} ⁵⁸:

$$\frac{P_t}{\lambda^{t-T}} = p_t = E_t \sum_{k=0}^{\infty} \gamma^{k+1} \frac{D_{t+k}}{\lambda^{t-T}} \frac{\lambda^{k+1}}{\lambda^{k+1}} = E_t \sum_{k=0}^{\infty} (\gamma\lambda)^{k+1} \frac{D_{t+k}}{\lambda^{t+k+1-T}} = E_t \sum_{k=0}^{\infty} (\bar{\gamma})^{k+1} d_{t+k}$$

where $\bar{\gamma}$ is the discount factor for detrended variables, defined as $\bar{\gamma} = \gamma\lambda$. Then, the discount

factor for detrended variables is such that $\bar{\gamma} = \frac{1}{1+r}$.

Then, we have⁵⁹:

$$p_t = E_t(p_t^*) = E_t \sum_{k=0}^{\infty} (\bar{\gamma})^{k+1} d_{t+k}$$

⁵⁸ Note in period k we have $d_{t+k} = \frac{D_{t+k}}{\lambda^{t+k+1-T}} = \frac{D_{t+k}}{e^{b(t+k+1-T)}}$.

⁵⁹ Note this is just an algebraic transformation of PVM.

Taking unconditional expectation, we find the discount rate for detrended series:

$$E(p) = \sum_{k=0}^{\infty} (\bar{\gamma})^{k+1} E(d) = E(d) \sum_{k=0}^{\infty} (\bar{\gamma})^{k+1} = E(d) \bar{\gamma} \sum_{k=0}^{\infty} (\bar{\gamma})^k = E(d) \bar{\gamma} \frac{1}{1 - \bar{\gamma}}$$

$$\frac{1 - \bar{\gamma}}{\bar{\gamma}} = \frac{E(d)}{E(p)} \rightarrow \frac{1 - \frac{1}{1 + \bar{r}}}{\frac{1}{1 + \bar{r}}} = \frac{E(d)}{E(p)} \rightarrow \bar{r} = \frac{E(d)}{E(p)}$$

Since PVM is an equation with infinite terms, we cannot directly compute the price; then, we must choose a terminal price, which we equal, following Shiller, to the average value of detrended real prices. Now, PVM can be approximated by:

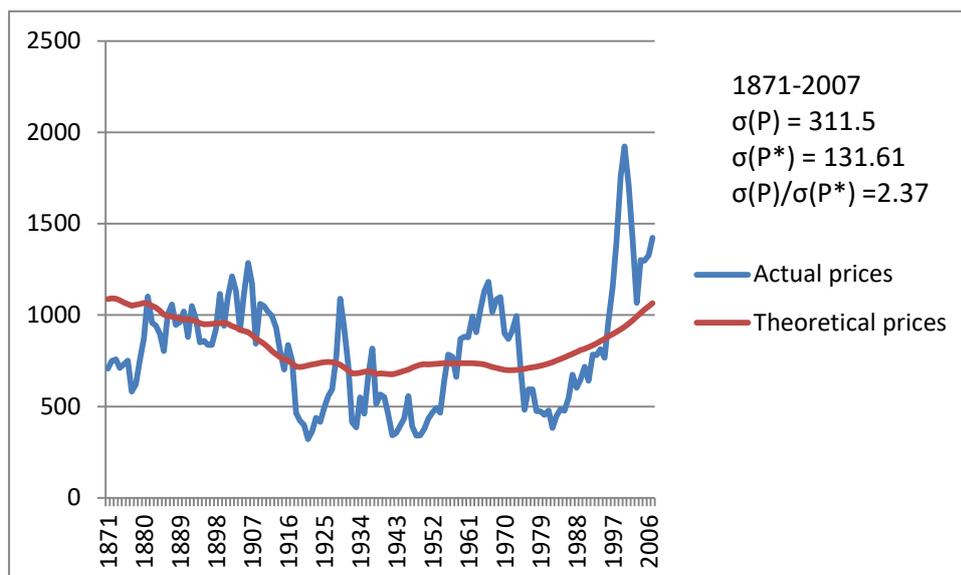
$$5) p_t = E_t(p_t^*) = E_t \left[\sum_{k=0}^{T-1} (\bar{\gamma})^{k+1} d_{t+k} + (\bar{\gamma})^{T-t} p_T \right]$$

This formula is consistent with the following equation:

$$p_t^* = \delta(p_{t+1}^* + d_{t+1})$$

Then, assuming as terminal value $p_T = \frac{1}{t} \sum_{i=1}^T p_{t+i}$, we can use this formula to compute p_T and then proceeding by backward recursion⁶⁰. The results are illustrated in the figure.

⁶⁰ The results continue to hold if, for instance, we consider as terminal value the average of prices in the last five or twenty years.



It turns out that the standard deviation of actual prices is about 2.4 times the standard deviation of theoretical prices: then, even if volatility is lesser than that found with the 1871-1979 series, it remains relatively high.

4. The replies to Shiller volatility test

Shiller volatility bounds have raised some critical issues (Flavin 1983, Kleidon 1986), but subsequent empirical works have confirmed the result that prices cannot be explained just by future dividend path (see for instance Campbell, Shiller 1988, Cochrane 1991): for instance Campbell and Shiller (1988), using a VAR methodology and an approximated log-linearized dividend-price ratio model to overcome stationary problems, find that VAR forecast of future dividends can explain just about a half of dividend-price ratio.

These empirical results have given rise to a still-ongoing debate. The principal replies to Shiller tests can be divided in three categories: those who have tried to deny stock price volatility proposing appropriate assumptions on households' preference structure (see for instance Cochrane 2005); those who have accepted price volatility explaining it with the presence of rational bubbles (see for instance Blanchard, Watson 1982); those who have accepted price volatility assuming the presence of not-fully rational behaviour (see for instance Shiller 2000).

PVM supporters have criticized the assumption of a constant discount rate. PVM supporters' research has been aimed to find the "right" model for the *subjective* time varying stochastic discount factor (SDF), in order to make theoretical prices to fit the actual ones (Cochrane 2005). SDF (i.e. the intertemporal marginal rate of substitution) is defined as the subjective discount factor times the discounted ratio between marginal utilities of consumption in two successive periods⁶¹. The required rate of return implicit in the SDF is supposed to vary according to expectations about future consumptions, so influencing the current theoretical prices, giving them more volatility. The underlying idea is that consumption smoothing behaviour and risk aversion imply large stock price movements during business cycles, linked to expected changes in marginal utility: on the one hand, households try to smooth consumption increasing it in bad times, when it is expected to rise, requiring a greater return (a smaller stock price) to save, given the smaller value they attach to less scarce future consumption; on the other hand, households require a larger risk premium for assets whose

⁶¹ $M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)}$. The intuition is the following: the higher is the consumption in t+1 with respect to t, the less is the utility obtained in t+1 with respect to t (due to decreasing marginal utility), the less is the value attached to future consumption with respect to the current consumption, the less is the SDF, the higher is the rate of return required by household to invest.

returns have a large negative covariance with the SDF, since they give low returns precisely when consumption grows less (marginal utility grows more) and is more valuable.

It's not clear if consumption based models (CBM) can provide support for price volatility. The traditional version of CBM, based on power utility function, doesn't fit well the data, especially since the early '50 (Grossman, Shiller 1981). Moreover, it faces an important puzzle (Mehra, Prescott 1985; Mera 2006): since the historical consumption path is quite smooth, its covariance with asset returns is quite low, and then also the risk premium is relatively low, unless we assume such a high risk aversion that even a small covariance between returns and consumption growth leads to a high risk premium. This is the equity premium puzzle: a very high risk aversion is needed to make consistent high returns found in the data with smooth consumption growth. But, with the power utility function, a high risk aversion implies a high aversion to intertemporal substitution of consumption, and then a large smoothness propensity, which in turn leads to a relatively high risk-free rate, not observed in the data. This is the risk-free rate puzzle: the high risk premium needed to solve the equity premium puzzle implies a too high risk-free rate. Several models have been proposed to overcome these puzzles. Some of these models are aimed to find the functional form of the utility function that best fit the data: for instance, we can separate the risk aversion coefficient and the elasticity of intertemporal substitution, or we can introduce habit persistence. Anyway, we should point out that all these models try to explain price variations through ex-ante variations in the subjective required rate of return; then, modifications of rates of returns are a cause, and not a consequence, of changes in prices.

Critics of PVM have interpreted its rejection as evidence in support of the existence of financial bubbles. There are two different lines of research about this issue. On the one hand, removing rational expectation hypothesis, prices can follow a path different from that predicted by the dividend path, either because agents don't know the right model of the economy or because they follow fads or because they are influenced by an euphoric phase (Minsky 1981, Kindleberger 1989). Behavioural finance is concerned with this kind of problems (see for example Shiller 1984, 2000; Cuthbertson, Nitzsche 2005 for a survey). The formation of *irrational* bubbles is the logical consequence of these models.

On the other hand, removing transversality condition allows us to add a *bubble* term to the fundamental value given by PVM (Blanchard, Watson 1982)⁶²; indeed, once we drop the convergence assumption represented by transversality condition, there is an infinite number of solutions to equation 3. Any solution can be expressed in the following form:

$$P_t^* = \sum_{i=1}^k \delta^i D_{t+i} + B_t = P_t^f + B_t$$

$$B_t = \delta E_t[B_{t+1}]$$

Prices can deviate from their fundamental value by an amount equal to the term B, which appears just because it is expected to appear the following period: in other words, the bubble is a self-fulfilling expectation. Even if individuals are able to distinguish between the fundamental value and the bubble term, they nonetheless *rationally* include B in the

⁶² The second relation is a restriction on B_t in order for it to be a solution for eq. 2 (see Blanchard, Fisher 1989, p. 221): the bubble must follow a martingale process.

determination of price, since it is expected to persist in the future. For this reason we can call it *rational* bubble. It follows that prices are not more uniquely determined by the expected dividend path; instead, they depend also on the bubble path, and, since B_t is arbitrary, the stock price is not unique: given infinite possible paths for the bubble term, we have infinite paths for the prices. For example, if we assume constant dividends and an exponential path for the bubble, the stock price will increase even if the fundamental value is constant: individuals purchase stocks just to sell them in the following period earning the capital gain, not justified by future dividends.

What is common to these approaches critical toward PVM is the acceptance of its neoclassical foundations: if agents were not irrational, or if bubbles didn't exist, prices would continue to reflect their fundamental value, given by the discounted expected future dividends, which in turn are supposed to be based on the neoclassical fundamentals. But, an *imperfektionist* approach is not a sufficient condition for a paradigm change.

5. The present value model and the value of a firm

Before to see how PVM can be derived from the neoclassical model, it can be useful to explicit the link between PVM and the value of a firm, highlighting the relation between financial and real variables. Following Miller-Modigliani (1961), according to PVM the value of a stock is the expected discounted value of the future dividends the stock ensures; in turn, the dividends are determined by the future profits and investment decisions of the firm issuing the relative stocks. Stock price must be such that the rate of return on each share is

the same throughout the market. If we assume perfect certainty we must have (we drop the firm subscript for simplicity):

$$\rho_t = \frac{p_t + d_t - p_{t-1}}{p_{t-1}} \quad \text{for each firm}$$

Or equivalently (imposing the transversality condition for the second equality):

$$p_{t-1} = \frac{d_t + p_t}{(1 + \rho)_t} = \sum_{\tau=0}^{\infty} \frac{d_{t+\tau}}{(1 + \rho)_{t,t+\tau}}$$

where $(1 + \rho)_{t,t+\tau} = \prod_{k=t}^{t+\tau} (1 + \rho)_k = (1 + \rho_t)(1 + \rho_{t+1}) \dots (1 + \rho_{t+\tau})$.

We can restate this equation in terms of the total value of a firm if we multiply both sides for the number of firm shares present at the start of period t . Let n_t (n_{t+1}) be the outstanding shares at the start of t ($t+1$), m_{t+1} the new shares issued at the start of $t+1$ (note $n_{t+1} = n_t + m_{t+1}$), X_t the gross profits at the end of t , I_t the *decided* investments at the end of t , $V_t = n_t p_{t-1}$ the value of the firm at the start of t ⁶³, $D_t = n_t d_t$ the total dividends at the end of t ; then, we have:

⁶³ note that p_t is the ex-dividend stock price at the end of t , whereas V_t is the value of the firm at the beginning of t .

$$\begin{aligned}
V_t \equiv n_t p_{t-1} &= \frac{n_t d_t + n_t p_t}{(1 + \rho)_t} = \frac{D_t + (n_{t+1} - m_{t+1}) p_t}{(1 + \rho)_t} = \frac{\mathbf{D_t + V_{t+1} - m_{t+1} p_t}}{(1 + \rho)_t} \\
&= \frac{D_t + V_{t+1} - [I_t - (X_t - D_t)]}{(1 + \rho)_t} = \frac{V_{t+1} + X_t - I_t}{(1 + \rho)_t} = \sum_{\tau=0}^{\infty} \frac{\mathbf{X_{t+\tau} - I_{t+\tau}}}{(1 + \rho)_{t,t+\tau}}
\end{aligned}$$

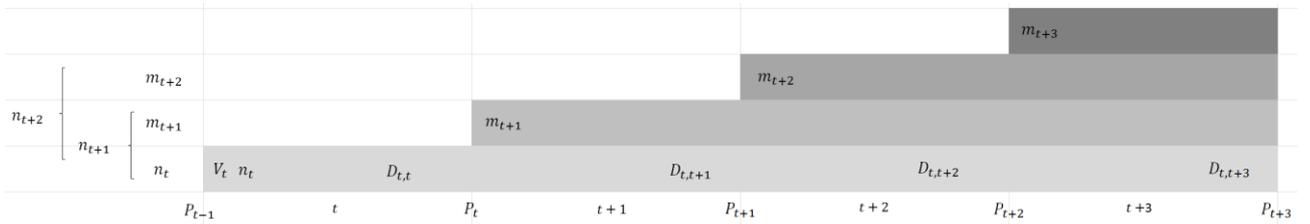
The fourth passage (after the expression in bold) follows from the fact that the decided investments in excess to the available reinvested profits (i.e. net-of-dividends profits) must be raised issuing new shares; the final passage follows from imposing a transversality condition on the value of the firm.

Thus, the value of the firm can be expressed in two ways (the two expressions in bold). First, it can be expressed as the discounted value of the end of period dividends plus the next period value of the firm net of the value of new shares; again, the latter are issued at the beginning of the next period in order to raise capital to satisfy the desired investment, taking into account the net-of-dividend reinvested profits. Second, it can be expressed as the discounted value of the future net-of-investment profits, i.e. the future not reinvested profits.

The same conclusion holds if we start from the perfect forecasted PVM:

$$\begin{aligned}
V_t \equiv n_t p_{t-1} &= \sum_{\tau=0}^{\infty} \frac{n_t d_{t+\tau}}{(1+\rho)_{t,t+\tau}} = \sum_{\tau=0}^{\infty} \frac{\mathbf{D}_{t,t+\tau}}{(1+\rho)_{t,t+\tau}} = \frac{1}{(1+\rho)_t} \left[D_t + \sum_{\tau=1}^{\infty} \frac{D_{t,t+\tau}}{(1+\rho)_{t+1,t+\tau}} \right] = \\
&= \frac{1}{(1+\rho)_t} \left[D_t + \sum_{\tau=0}^{\infty} \frac{D_{t,t+\tau+1}}{(1+\rho)_{t+1,t+\tau+1}} \right] = \frac{1}{(1+\rho)_t} \left[D_t + \frac{n_{t+1}}{n_{t+1}} \sum_{\tau=0}^{\infty} \frac{n_t d_{t+\tau+1}}{(1+\rho)_{t+1,t+\tau+1}} \right] \\
&= \frac{1}{(1+\rho)_t} \left[D_t + \frac{n_t}{n_{t+1}} \sum_{\tau=0}^{\infty} \frac{n_{t+1} d_{t+\tau+1}}{(1+\rho)_{t+1,t+\tau+1}} \right] \\
&= \frac{1}{(1+\rho)_t} \left[D_t + \left(1 - \frac{m_{t+1}}{n_{t+1}}\right) \sum_{\tau=0}^{\infty} \frac{D_{t+1,t+\tau+1}}{(1+\rho)_{t+1,t+\tau+1}} \right] \\
&= \frac{1}{(1+\rho)_t} \left[D_t + \left(1 - \frac{m_{t+1} p_t}{n_{t+1} p_t}\right) V_{t+1} \right] = \frac{\mathbf{D}_t + \mathbf{V}_{t+1} - \mathbf{m}_{t+1} \mathbf{p}_{t+1}}{(1+\rho)_t} \\
&= \sum_{\tau=0}^{\infty} \frac{\mathbf{X}_{t+\tau} - \mathbf{I}_{t+\tau}}{(1+\rho)_{t,t+\tau}}
\end{aligned}$$

where $D_{t,t+\tau} = n_t D_{t+\tau}$ is the portion of total dividends $D_{t+\tau}$ that accrues to the holders of shares at the start of t . The following figure can help to understand the first expression in bold:



The model determines the price of a share for a given stream of future profits and investments, possibly based on the market interest rate itself; moreover, the firm is free to decide about the dividend policy, which does not influence its present value.

Now, in a standard neoclassical framework, future profits, investments and market interest rate are all determined by *fundamentals*, that is factor endowments, households' preferences and production technology. It follows that stock prices are determined on the basis of current and expected future fundamentals; moreover, nothing but possible shocks or wrong expectations on future fundamentals can influence current prices. This also means that rational bubbles cannot persist if transversality condition holds.

6. The neoclassical foundations of present value model

Even if PVM is widely used in literature, only few works relate it to the neoclassical model of growth and distribution (see for instance Balvers et al. 1990, Rouwenhorst 1996, Jermann 1998), without, however, explicating its neoclassical foundation. Here we try to make explicit the link between PVM and the neoclassical model in the most simple way. This link can be showed starting from a very simple model, which may be heuristically interpreted either as the introduction of shares in the Ramsey (1928) model or the introduction of production in the Lucas (1978) Tree model.

We assume a closed economy in which just one perishable commodity is produced, by means of itself and labour, through a constant return to scale technology that respects the

standard Inada conditions⁶⁴. We assume a deterministic framework with possible exogenous shocks. Population is constant. Households are endowed with (homogenous) labour and shares of the representative firm, that give a claim on all future dividends - or, that is the same, on end-of-period cum-dividend price, i.e. gross rate of return. For simplicity, we divide households in two groups: workers and share-holders. We also assume a pseudo “classical hypothesis of savings”, which simplifies the analysis without influencing the essential features of the model: workers supply, at the beginning of each period, all labour for end-of-period wages they consume entirely; share-holders decide, at the end of each period, how much to consume and how much to invest on shares on the basis of an intertemporal maximization program. At the beginning of each period firms demand all the factor endowments remunerating them, at the end of period, their respective marginal productivity, following the dividend maximization program. We abstract from price variations stemming from variations in the number of shares: we assume neither buybacks nor issue of new shares, and then no external finance. Finally, we introduce a further simplifying and not essential assumption: firms distribute dividends after observing the possible future shock, according to shareholders’ preferences; this means that shareholders’ savings equal firms’ retained profits. To further clarify the analysis, in this section we make a strong simplification: we consider a logarithmic utility function. In the next section we shall remove this hypotheses, explicating the link between the neoclassical model and the approaches to stock market dynamics we have seen in the preceding paragraph in a more complete framework.

⁶⁴ $f'(k) > 0, f''(k) < 0, \lim_{k \rightarrow \infty} f'(k) = 0, \lim_{k \rightarrow 0} f'(k) = \infty$.

Let P be the real share price, D the real dividends, A the number of shares, C the shareholders' consumption, R the gross rate of return, β the discount factor, W the real wages, L the quantity of labour, K the quantity of capital, $F(K, L)$ the production function, M the intertemporal rate of substitution, β the subjective discount factor and σ the subjective discount rate. The intertemporal rate of substitution is defined as follows:

$$M_{t+1} = \beta \frac{U'(C_{t+1})}{U'(C_t)} = \frac{1}{1+\sigma} \frac{U'(C_{t+1})}{U'(C_t)}.$$

All variables but factor endowments (K, L) and the number of shares (A) are end-of-period variables. We start by deriving both firms' and shareholders' maximization program.

In each period firms choose the amount of investments (i.e. the quantity of capital at the beginning of the next period) to maximize the value of the firm, given by the expected discounted value of all future dividends⁶⁵:

$$V_t = E_t \left[\sum_{k=0}^{\infty} \frac{1}{R_{t,t+k}} D_{t+k} \right]$$

where dividends are defined as the residual value of the product once wage payments have been made and investments have been financed:

$$D_t = F(K_t, L_t) - W_t L_t - K_{t+1}$$

⁶⁵ Actually, we should consider the expected value of the discounted future dividends; but, as said above, we consider a deterministic framework with possible exogenous shocks in order to simplify the analysis and to single out in the most simple way the differences between the neoclassical model and the alternative model.

In turn, the product is equal to the sum between wages and profits:

$$F(K_t, L_t) = W_t L_t + R_t K_t$$

The maximization program of the firms is the following:

$$\max_{K_{t+1}} \sum_{k=0}^{\infty} \frac{1}{R_{t,t+k}} [F(K_{t+k}, L_{t+k}) - W_{t+k} L_{t+k} - K_{t+k+1}]$$

This program yields the stochastic Euler condition:

$$E_t [R_{t+1}^{-1} F_K(K_{t+1}, L_{t+1})] = 1 \quad \text{for all } t$$

This means that the marginal product of capital (investment), properly discounted, must be equal to the unit of consumption good sacrificed to investments. In order to simplify the analysis and to single out in the most simple way the differences between the neoclassical model and the alternative approach, we consider a deterministic framework with possible exogenous shocks. It follows that the latter condition gives the standard result for which the rate of return must equal capital marginal productivity:

$$F_K(K_{t+k}, L_{t+k}) = R_{t+k} \quad \text{for } k = 0 \dots \infty$$

As regards to shareholders, at the end of each period they maximize:

$$\sum_{k=0}^{\infty} \beta^k U(C_{t+k})$$

subject to:

$$(P_t + D_t)A_t = C_t + P_t A_{t+1}$$

The maximization program yields the following Euler equation with respect to the choice variable A_{t+1} :

$$U'(C_t)P_t = \beta U'(C_{t+1})(P_{t+1} + D_{t+1})$$

Reminding the definition of rate of return:

$$R_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

we can reformulate the Euler equation in the following way:

$$U'(C_t) = \beta E[R_{t+1} U'(C_{t+1})]$$

This means that current marginal utility of consumption must be equal to the expected discounted return of buying a share times the marginal utility of consumption next period, when the share is sold.

Reformulating the Euler equation, solving forward and imposing the transversality condition yields an expression for the ex-dividend share price:

$$P_t = E_t \left[\sum_{k=1}^{\infty} \beta^k \frac{U'(C_{t+k})}{U'(C_t)} D_{t+k} \right]$$

To solve the model analytically we assume a logarithmic utility function:

$$U(C) = \ln(C)$$

This assumption is useful to simplify the analysis, but it doesn't allow to identify some essential features of the various approaches to stock market dynamics. In the next section we shall remove this hypothesis.

Supply of shares can be appropriately normalized to one, so that, at each time, $A_t = 1$ clears the market. Then, under this market-clearing condition, the budget constraint implies that $C_t = D_t$, that is shareholders' demand for the good equals the supply to the shareholders of the good.

Using the fact that $U'(C_t) = \frac{1}{C_t}$ and $C_t = D_t$, share price equation becomes:

$$P_t = E_t \left[\sum_{k=1}^{\infty} \beta^k D_{t+k} \right] = \frac{\beta}{1-\beta} D_t = \frac{D_t}{\sigma}$$

Then, the model can be described by the following equations (we remove the subscript t for simplicity):

$$1) F(K, L) = WL + (1+r)P_{-1} = WL + (P + D)$$

$$2) F_K(K, L) = 1 + r$$

$$3) 1 + r = \frac{P + D}{P_{-1}}$$

$$4) P = \frac{D}{\sigma}$$

$$5) D = C$$

$$6) K_{+1} = P$$

According to equation (1), income at the end of t is entirely distributed between wages and gross returns. The latter coincide with the cum-dividend price, since we have normalized shares to one. (2) is the optimum condition, obtained by the dividend maximization program of the firm. (3) defines the gross return on shares as the cum-dividend price divided by the preceding-period share price. (4) and (5) define share price and dividends as we have seen

before. (6) defines the capital at the beginning of next period (investments) as the real value of the shares purchased. This is a system of six equations in six unknowns:

R, P, D, W, C, K_{+1}

and four exogenous variables:

$K_t, L_t, P_{t-1}, \sigma$

The solution of the system is as follows: given K and L , (2) determines the rate of return r . Given K, L, P_{-1} and r , (1) determines W . Given r, P_{-1} and σ , 3 and 4 determine P and D ; given D , 5 determines C ; finally, given P , (6) determines K_{+1} . In particular, assuming a standard Cobb-Douglas production function:

$$F(K, L) = A K^\alpha L^{1-\alpha}$$

the explicit solutions for P and D are the following:

$$P = \frac{\alpha A K^\alpha L^{1-\alpha}}{1 + \sigma}$$

$$D = \frac{\sigma}{1 + \sigma} \alpha A K^\alpha L^{1-\alpha}$$

In steady-state equilibrium, P and D are given by:

$$P^* = L \left(\frac{\alpha A}{1 + \sigma} \right)^{\frac{1}{1-\alpha}}$$

$$D^* = \sigma L \left(\frac{\alpha A}{1 + \sigma} \right)^{\frac{1}{1-\alpha}}$$

In steady state, the rate of return equals the rate of time preference: From 3, imposing $P = P_{-1}$, we have $r_{SS} = \frac{D_{SS}}{P_{SS}}$; from 4 we have: $\sigma = \frac{D_{SS}}{P_{SS}}$; then, $r_{SS} = \sigma$. With no population growth and no technological progress, steady-state stock price is constant. The system is convergent, given the usual hypotheses.

Note that, with a logarithmic utility function, substitution effect and income effect cancel exactly, so that changes in future fundamentals don't influence current prices, and PVM boils down to equation 4: that is, stock price reflects the present value of an infinite series of dividends equal to the current one. However, this simplification, which will be removed below, is useful to clarify two aspects of the neoclassical model which also hold in the less simplified model: first, stock prices, which reflect the scarcity of resources, are fully determined by the standard neoclassical fundamentals: endowments, preferences and technology; then, stock price variations can only be justified by changes in fundamentals, that is by technological shocks (change in A), preference shocks (change in σ) or manna from heaven shocks (change in K and/or L). Second, wide and frequent stock price movements can only stem from wide and frequent changes in fundamentals.

7. The complete model

In order to explicate some important features of the model, we introduce a more general utility function. Assume that households' preferences can be modeled by the standard power utility function:

$$U(C) = \frac{C^{1-\gamma} - 1}{1-\gamma}$$

where γ is the risk aversion coefficient, which is also the reciprocal of the elasticity of intertemporal substitution. Ex-dividend share price is now given by:

$$4') P_t = E_t \left[\sum_{k=1}^{\infty} \beta^k \left(\frac{D_t}{D_{t+k}} \right)^\gamma D_{t+k} \right]$$

The other equations remain the same as before. This new model, composed by equations 1,2,3,4',5,6 and 7, cannot be solved analytically; however, it can be used to give an intuitive explanation of stock market dynamics according to the different approaches mentioned in the preceding sections, that is present value model, consumption-based model, irrational and rational bubbles models. In order to analyze these approaches in the light of this model, we reformulate both the Euler equation and the rate of return equation, obtaining two difference equations in P and D which can be represented in a pseudo-Ramsey graph, with P on the horizontal axis and D on the vertical axis (see the figure in the appendix):

$$3') P_t = \alpha A P_{t-1}^\alpha L^{1-\alpha} - D_t$$

$$4'') \frac{D_{t+1}}{D_t} = [\beta(\alpha A P_t^{\alpha-1} L^{1-\alpha})]^\frac{1}{\gamma}$$

Before exposing the different approaches to stock market dynamics we have seen in the preceding sections, it may be useful to summarize some results of this simple model. The steady state values of P and D, obtained equalling P_t to P_{t+1} and D_t to D_{t+1} in expressions 3' and 4'', are the same as before (see equations 10 and 11). The black curve, which corresponds to $D = rP$, shows pairs of P and D that satisfy $\Delta P = 0$ in equation 3'. For each level of P_{-1} there is only one level of D such that the (real) investment in shares is just sufficient to reproduce capital without accumulation (and without issue or buy back of shares); were the dividends smaller than this level (under the line), the real share price would increase (the opposite is true if dividends were higher than that level). Then, under (over) the curve prices tend to increase (decrease). The black vertical line, which corresponds to $\alpha A P^{\alpha-1} L^{1-\alpha} = 1 + \rho$, shows the level of P that satisfies $\Delta D = 0$ in equation 4''. There is only one level of P such that the rate of return equals the rate of time preference; were the level of P smaller than this level, the relatively low quantity of capital would yield a rate of return higher than the rate of time preference (due to decreasing marginal productivity of capital), so pushing up investment in shares and then dividend growth (the opposite is true if the level of P were higher than that level). Then, to the left (right) of the line, dividends tend to increase (decrease). If transversality condition holds, the dynamic equilibrium follows the unique saddle path toward steady state.

Now, we can explicit the link between this model and the four approaches to stock market dynamics we have mentioned before. Let's start from standard PVM, which has been the most influential approach for decades, as we have seen before. The essential feature of PVM is the dependency of stock prices from future dividends, which means that prices increase (decrease) if shareholders (rationally) forecast an increase (decrease) in future dividends. In the model presented here, this happens if $0 < \gamma < 1$ ⁶⁶: in this case, the dividend effect prevails, and prices react in the same direction of a possible productive shock. As we have seen, given the empirical finding that prices don't reflect future dividends, a more recent stream of literature has stressed the link between stock prices and the stochastic discount factor, defined as the subjective discount rate times the ratio between marginal utility of consumption in two successive periods. The idea is that prices reflect mostly the relative utility, measured in terms of relative scarcity, of the future dividend stream: a future increase in dividends, making future consumption less scarce and then less valuable than the current one, leads shareholders to increase current consumption and to decrease investment in shares, so pushing down stock prices. In other words, future dividends are discounted at a relatively high rate. In the simple model presented here, this consumption-based approach holds if $\gamma > 1$ (see note 1): in this case, the smoothness effect prevails, and prices react in the opposite direction of a possible productive shock.

⁶⁶ Reformulating equation 4' we obtain: $P_t = \beta D_t^\gamma \sum_{k=1}^{\infty} D_{t+k}^{1-\gamma}$. The first derivative of P with respect to D_{t+k} is: $\frac{\partial P_t}{\partial D_{t+k}} = \beta D_t^\gamma (1 - \gamma) \sum_{k=1}^{\infty} D_{t+k}^{-\gamma}$, which is positive, negative or equal to zero if, respectively, $0 < \gamma < 1$, $\gamma > 1$ or $\gamma = 1$.

Both PVM and consumption-based approaches assume that shareholders have rational expectations when taking investment decisions, that is they use their information in the best way, according to the “right” model of the economy. Removing this hypothesis, a new research field, known as behavioural finance, has tried to link stock price movements to the non-rational behaviour of agents, influenced by psychological biases. An example of such a behavior is when “optimistic” agents over-react to positive news about future dividends, buying a relative high quantity of shares and then pushing stock prices over their fundamental value. In the model exposed here, we can introduce this behaviour allowing for an expected post-shock steady state different from that one predicted by the theory. Finally, some authors have pointed out that prices can diverge from fundamental value even if agents are rational: this is the case of rational bubbles, that is price changes caused by the speculative behaviour of agents, who buy shares to resell them at higher price not justified by future dividends. Rational bubbles are generally presented in a partial equilibrium framework (Blanchard, Watson 1982) or in overlapping generation models (Tirole 1985). In the model exposed here, transversality condition rules out rational bubbles, since over-investment in shares would lead to an over-accumulation of capital, which, in turn, would lead consumption toward zero. In the case of no population growth and no technological progress, transversality condition simply imposes a positive value to the rate of intertemporal preference.

In order to give an intuitive explanation and a graphical representation of the approaches summarized above, we can see how stock prices react to an economic shock according to each approach. Assume a future permanent positive shock (a higher A), forecasted in the current period, which moves $\Delta d/d=0$ curve to the right and $\Delta p/p=0$ curve upward. PVM (blue

line) predicts an increase in price after shock “announcement”, reflecting the future increase in dividends. While Prices increase from the beginning, dividends first decrease, due to shareholders’ willingness to invest in shares even before the shock appears, and then, after the shock, they start to increase. Consumption based models (orange line) predict a decrease in price after shock announcement, reflecting consumption smoothness propensity: future (higher) dividends are discounted at a higher rate; shareholders require a higher return to invest in shares, since they attach a higher marginal utility to current, more scarce, consumption. While dividends increase from the beginning (following a path that ensures consumption smoothness), prices first decrease, due to shareholders’ willingness to increase consume in times where its marginal utility is relatively high, and then, after the shock, they start to increase. Irrational bubbles (purple, and then blue line) arise from wrong expectations about future shocks: in this simple example, shareholders, expecting a shock higher than the actual one (grey curves), over-invest in shares aiming to move toward the (wrong) saddle path at the time of the shock (assuming dividend effect prevails); when the (overvalued) shock appears, shareholders react selling shares in order to move toward the right saddle path. Then, after shock announcement, prices increase more than their fundamental value, reflecting a wrong over-valued dividend path; after the shock, the bubble bursts and prices suddenly decrease to their fundamental value. Finally, as said before, rational bubbles (purple and then yellow line) arise when shareholders over-invest in shares - pushing up their prices - in order to resell them at a higher price, not justified by fundamentals; but, speculative behavior is ruled out by transversality condition.

Again, the principal aspects of neoclassical model also hold in this less trivial case. Stock prices strictly depend on the given endowments and their evolution over time, influenced by

possible shocks. In this sense, both PVM and consumption-based models rest on neoclassical foundations: prices reflect the evolution of future dividends or future required rates of return, which in turn depend on neoclassical fundamentals: stock price variations can only be justified by current or future changes in fundamentals⁶⁷. Together with technology, preferences and transversality condition, factor endowments uniquely determine the efficient equilibrium path of prices and dividends toward steady-state position. High price variations can only be justified either by a high instability of fundamentals or by wide and frequent mistakes in expectations. Note that irrational bubble approaches, even if they allow for out-of-equilibrium price variations, are nonetheless founded on neoclassical model: indeed, these models just signal the *spread* between actual and theoretical prices, that is the deviations from a natural equilibrium path based on neoclassical fundamentals which is not questioned.

Here we have shown in the most simple framework how PVM and other standard approaches in literature are based on the standard neoclassical fundamentals, and how prices can only change if current or expected future fundamentals change. Of course, this is just a simplified model, which can be complicated, for instance, allowing for adjustment costs, technological progress, growing population, a stochastic process for output shocks, a saving rate different from the retention rate, a multi-asset framework, finite-living agents, partial equilibrium analysis and so on. In any case, the point to be underscored is the logical

⁶⁷ Note that while current shocks influence both gross return and current prices and dividends, future (expected) shocks only influences current prices and dividends, leaving current return unchanged, determined by factor endowment at the beginning of period. The intuition is that, while gross return is already known given the factor endowments, how the gross rate of return is distributed between dividend yield and capital gain is only known after the maximization program has been solved. This is a consequence of the simplifying assumption that saving rate be equal to retention rate.

dependency of the approaches found in literature on the neoclassical model of growth and distribution. In the next section we shall see some critical aspects of neoclassical interpretation of stock market dynamics, proposing an alternative view.

8. An alternative interpretation of stock price dynamics

As we have seen, price volatility can only be justified by volatility in fundamentals, that is variations in dividends, discount factor or expectations. Shiller (1981) volatility tests have shown how actual prices vary too much with respect to ex-post theoretical prices computed using actual future dividends; then, stock prices cannot just reflect future dividends. In the simple model presented here, this means that productive shocks are too few and too small to justify price volatility. Also, empirical literature has shown how variations in discount factor, using a power utility function, cannot justify price volatility (Grossman, Shiller 1981). In the model presented here, this means that, allowing for few and small productive shocks, smoothness effect is not sufficient to justify price volatility.

After these empirical rejections, literature has focused on two research streams, one based on extensions of the SDF (see for instance Epstein and Zin 1989, Campbell and Cochrane 1999), the other based on behavioral finance (see for instance Shiller 2000), but none of these models managed to solve volatility puzzle; the debate is still open.

The models presented here, due to their extreme simplicity, can replicate none of the econometric tests about volatility. However, it could be interesting to note the similarities between PVM presented here and PVM as tested by Shiller (1981). The only difference is that Shiller (1981) uses a constant discount factor, whereas here we use a time varying

discount factor also for the original PVM; but, this is due to the fact that the model presented here is a general equilibrium model that involves also production, whereas Shiller (1981) presents a partial equilibrium model with a given discount rate.

Besides the empirical literature, from a theoretical point of view the idea that stock prices reflect neoclassical fundamentals relies upon the existence of such fundamentals and, more generally, on the consistency of the neoclassical model. In literature we can find numerous criticisms of the neoclassical model of growth and distribution, relating both on internal consistency linked to the neoclassical theory of capital (see for instance Sraffa 1960; Garegnani 1960, 1970; Pasinetti 2000; Petri 1989, 2003, 2004) and on external criticisms linked to the utility and scarcity-based representation of the economic process (see for instance Graziani 2003)⁶⁸.

Once the neoclassical model is rejected, we can no longer accept the fundamental value equation: stock prices cannot reflect a natural equilibrium, nor an equilibrium path toward it, that doesn't exist. The very definition of bubbles is misleading, since it presumes deviations from an equilibrium path; but, if the existence of a natural equilibrium path is denied, it's not possible to define deviations from it.

Now, the question arises if it's possible to find alternative interpretations of stock market dynamics in general and of stock price volatility in particular. There is a large amount of empirical evidence about stock price volatility; we shall see if, in the alternative model, this evidence represents a problem at all.

⁶⁸ See Lavoie (2014) for an overview of recent research on alternative models.

Starting from a similar set of equations, we can give a different interpretation of stock market dynamics based on an alternative theoretical framework drawn from Keynesian and Sraffian literature (see Lavoie 2015 for a survey). We shall see how stock prices cannot be interpreted in this model as *forecasters* of future economic *fundamentals*; instead, they reflect autonomous decisions of firms about production, willingness of financial system to support and promote aggregate demand and sustainability of credit expansion. In particular, since in this framework a natural equilibrium doesn't exist, price dynamics is not *anchored* to fundamentals, and the very concept of bubbles shall be reformulated. Moreover, higher price volatility with respect to dividends can be explained in this simple model by the larger willingness of shareholders to invest in shares (i.e. the actual propensity to save) during the *euphoric phase* of financial markets (and viceversa)⁶⁹.

The alternative model has the following features. Normal distribution is given, determined by social, economic and institutional factors; in particular, we assume the normal rate of return on shares is given. Investments and employment are autonomously determined by firms, both in the short and in the long period. In order to make it consistent these two features we assume both an endogenous degree of capacity utilization and an endogenous retention ratio, which, for simplicity, we set equal to the rate of savings⁷⁰.

⁶⁹ Shaikh (2007) uses the business retention ratio to explain gravitation of capacity utilization around normal level. Here it is used to justify price volatility with respect to dividends, without any reference about convergence to normal positions, in line with the idea that there is no need for actual growth to gravitate around the growth level consistent with normal distribution (Trezzini 1995).

⁷⁰ See the preceding section for a justification of this assumption. A more complete model with a non-trivial share market should distinguish between retention rate and savings rate. See Lavoie (1998), which develops a model starting from the neo-Pasinetti theorem (Kaldor 1966).

Let γ be the deviation of actual rate of profit from the normal rate of profit, u the deviation of actual capacity from the normal level of capacity utilization, P and D the normal prices and dividends consistent with normal distribution, P_t and D_t the actual prices and dividends consistent with macroeconomic equilibrium, s_c the savings rate of shareholders.

The mathematical framework is similar to that presented in the preceding section, but the interpretation radically changes.

Assuming we start from a normal position, the model is described by the following system:

$$1) F(K, L) = WL + (1 + r)P_{-1} = WL + (P + D)$$

$$1^*) uF(K, L) = WuL + (1 + \gamma r)P_{-1} = WuL + (P_t + D_t)$$

$$2) F_k(K, L) = 1 + r$$

$$3) (1 + \gamma r) = \frac{P_t + D_t}{P_{-1}}$$

$$4) D_t = (1 - s_c)(1 + \gamma r)P_{-1}$$

$$5) C_t = D_t$$

$$6) P_t = K_{t+1}$$

$$7) s_c = s_c(g^e)$$

Equation (1) represents normal distribution, consistent with normal rate of return. (1*) represents actual distribution, taking into account deviations from normal capacity

consistent with macroeconomic equilibrium⁷¹. (2) determines the optimal normal quantity of labour employed by firms when capital grows at the warranted rate, given normal rate of return and capital inherited from past investments⁷². (3) expresses actual rate of return in terms of actual prices and dividends. (4) defines actual dividends as the not retained part of actual profits. From 3 and 4 we obtain macroeconomic equilibrium condition⁷³:

$$3') s_c(1 + \gamma r)P_{-1} = P_t$$

According to (3'), savings out of returns must equal real share price⁷⁴. According to (5) consumption equals dividends (see the preceding model). (6) defines actual real stock price as the quantity of capital demanded by firms in period t, available as capital at the beginning of the following period (then, capitalization level is determined by firms' investment decisions). According to (7), rate of savings depends on expectation about future growth. With this assumption, we would like to formalize the possible speculative behaviour of

⁷¹ We assume a higher product can be obtained employing a higher quantity of labour on the same amount of capital.

⁷² As is known, there is no need for the models drawn from raffian and post-keynesian literature to assume such an unrealistic production function. We only maintain this function for comparison purposes. Note also that just for chance real rate of return is consistent with full employment of workers.

⁷³ Again, we normalize outstanding shares to one.

⁷⁴ This condition can also be derived from the income-demand equation, normalizing outstanding shares to one: $W_t L_t + (1 + \gamma r)P_{t-1}A_{t-1} = W_t L_t + (1 - s_c)(1 + \gamma r)P_{t-1} + P_t A_t$. Assuming workers consume all their wages, this condition says that shareholders' income is to be divided between consumption and investment in new shares. Since in the alternative model investments are autonomously decided by firms, capacity utilization must change in order to make the actual level of production - and then actual profits and savings - consistent with investment decisions of firms. Here we assume that sufficient capacity margins exist to reach macroeconomic equilibrium.

shareholders: for instance, in times of boom, shareholders might want to buy a higher amount of shares.⁷⁵

This is a system of eight equations in eight unknowns⁷⁶:

$L, W, P_t, s_c, \gamma, D, C, u$

and five exogenous variables:

K, r, P_{-1}, P, K_{+1}

Solution of the system is as follows: given r , (2) determines normal employment level L (knowing K)⁷⁷ and (1) determines normal wages W . Given P_t from (6) and s_c from (7), (3') determines the actual rate of profit consistent with macroeconomic equilibrium, 4 determines dividends and 5 determines consumption⁷⁸. Finally, (1*) determines the degree

⁷⁵ In a more consistent model, we should distinguish between saving rate and retained ratio. If the two ratios coincide - as in this model - share price is totally determined by firms and coincides with the capital owned by them. Separating the two ratios, share prices would also be determined by shareholders' savings decisions, even if investments would still be determined by firms. Some authors consider the possible influence of stock prices on investment decisions, but this would lead to counter-intuitive results from a post-keynesian perspective (see Lavoie 1998).

⁷⁶ (3) is just the definition of normal rate of return.

⁷⁷ Note that fixing an exogenous amount of capital does not imply any return to neoclassical logic of scarce resources.

⁷⁸ Since investments depend on autonomous decisions of firms, normal rate of return - and then normal prices and dividends - can be observed just for chance. Note that without population growth, normal prices are constant over time, and dividends equal net returns.

of capacity utilization u consistent with the actual rate of return needed for the macroeconomic equilibrium.

Assuming a Cobb Douglas production function, the explicit solutions for the relevant variables are the following:

$$P_t = K_{t+1}$$

$$1 + \gamma r = \frac{K_{t+1}}{s_c K}$$

$$D_t = \frac{1 - s_c}{s_c} K_{t+1}$$

$$L = \left(\frac{1 + r}{\alpha A} \right)^{\frac{1}{1-\alpha}} K$$

$$u = \frac{K_{+1}}{s_c(1 + r)K}$$

Some key-points must be stressed about this simple model. First, differently from neoclassical model, stock prices reflect the autonomous investment decisions of the firms (assuming no financial constraints) and don't reflect neither current nor future fundamentals, since a natural equilibrium does not even exist. For instance, an increase in investments allowed by flexible rate of capacity utilization increases share price / value of the firm, without this being influenced by any sort of resource constraints. In this sense, rises and booms in prices just reflect the willingness of the financial system to finance investments and autonomous demand (not considered for simplicity) and the willingness of

firms to increase demand and production⁷⁹. Of course, investment decisions, and then stock prices, are not only based on *animal spirits*; they can be influenced, for instance, by expectations about future demand or future capacity utilization. But, the essential difference with respect to the neoclassical model is that future demand is not constrained by scarce resources; rather, it's the amount of demand that can constitute a constraint on growth: in other words, the constraints on growth are given by the level of effective demand, not by the level of scarce resource. Then, stock prices don't reflect neoclassical fundamentals, and, even less, they don't forecast an economic dynamics supposed to be bounded by scarce resources or by possible shocks on fundamentals. Moreover, there is no guarantee that resources are fully employed, since no price mechanism similar to the neoclassical one is present in the critical model.

Second, in the alternative model stock price movements mostly depend on investment decisions; then, wide and frequent price movements can be justified by wide and frequent changes in investment decisions, regardless of any change in fundamentals, as in the neoclassical model. In this framework, stock price booms and crashes are easier to justify, since *over-optimistic* and *over-pessimistic* firms' behaviour is allowed, not bounded from possible changes in fundamentals. The term bubble can be misleading, since it presupposes a deviation from an equilibrium path; but, in the alternative approach, no natural equilibrium exists similar to the neoclassical one.

⁷⁹ Continuous rises in prices may also become fundamental to repay old loans and to contract new loans. In the end, if prices don't grow sufficiently to repay loans, economic dynamics may become explosive, bringing to deep crises.

Third, the alternative model can consistently justify volatility of prices with respect to dividends, letting the saving rate / retention ratio to vary along with the rate of growth, without this implying an inefficient economic path as in the neoclassical model. For instance, in an euphoric phase, when firms increase investments and then share prices, also the demand for shares may increase: this leads to a relative increase of saving rate / retention ratio and to a relative decrease of dividends. Then, in this context, the increase in price / dividend ratio, linked to speculative behaviour, easy financing and willingness to invest, can be justified, whereas in neoclassical model rational bubbles are ruled out by transversality condition. It could be interesting compare analytically the two models. In the neoclassical model, the rate of return is given by factor endowments, whereas how it is divided between price and dividends depends on an intertemporal optimization program by households. In the alternative model, price is given by investment decisions of firms, whereas the actual rate of return and the actual dividends are determined by saving decisions of shareholders: the larger is the saving rate, the lesser is the actual rate of return needed for the macroeconomic equilibrium, the lesser is the degree of capacity utilization, the lesser are the actual dividends⁸⁰. Then, price / dividend volatility can be justified in this model, even if income multiplier effect is reduced.

Finally, volatility tests must be read in a different way. Since stock prices are no longer forecasters of a natural equilibrium path (which doesn't exist) their higher volatility with respect to dividends has no consequences for the consistency of the model. The direction of causality is reversed: prices don't reflect current and future fundamentals, and then there is

⁸⁰ As noted before, the fact that saving decisions can affect dividends but not prices is a consequence of the simplicity of the model, in particular of the assumption of a saving rate equal to retention ratio.

no reason to expect they correctly forecast future dividends or discount rates; instead, their growth may be considered a symptom of an euphoric phase in which easy loans from financial sector allow firms to increase production and repay old loans, while movements of retention ratio in the same direction of price growth justify the lower volatility of dividends without incurring in analytical inconsistencies. Of course current investments depend also on expectations about future variables, in particular demand; but these variables, far from being determined by fundamentals, are strictly linked to the same propulsion factors that determine the current price dynamics. Analytically, removing the transversality condition does not create inconsistencies in the alternative model, whereas it is a vital requirement in a model based on the assumption of scarce resources. In the end, we could say that, in the alternative model, financial markets don't simply forecast the future: at least partly, they autonomously conceive a tendency and then they produce it; in other words, the presence of speculative bubbles and stock price booms and crashes is perfectly consistent with the alternative model, since prices are not *anchored* to any current or future fundamental, differently from the neoclassical model.

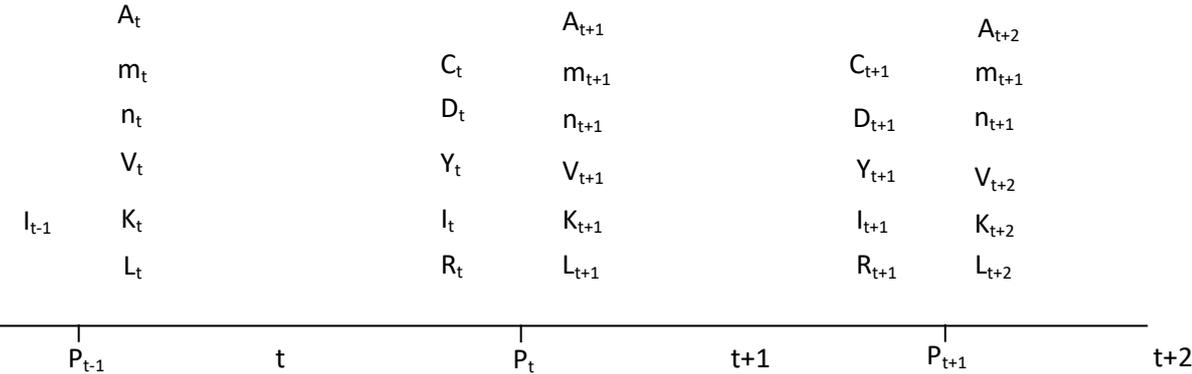
9. Conclusions

In this chapter we have shown that the interpretation of stock market dynamics based on present value model is logically dependent on neoclassical theory of growth and distribution, and then it is exposed to several criticisms advanced to that theory during the last decades; moreover, empirical tests seem suggest that present value model, both in its canonical and more advanced versions, is not able to explain stock market volatility. We have also shown

that, in a different theoretical framework, we can give an alternative interpretation of stock market dynamics, according to which stock prices reflect the autonomous investment decisions of firms, and stock price volatility depends on saving decisions of shareholders.

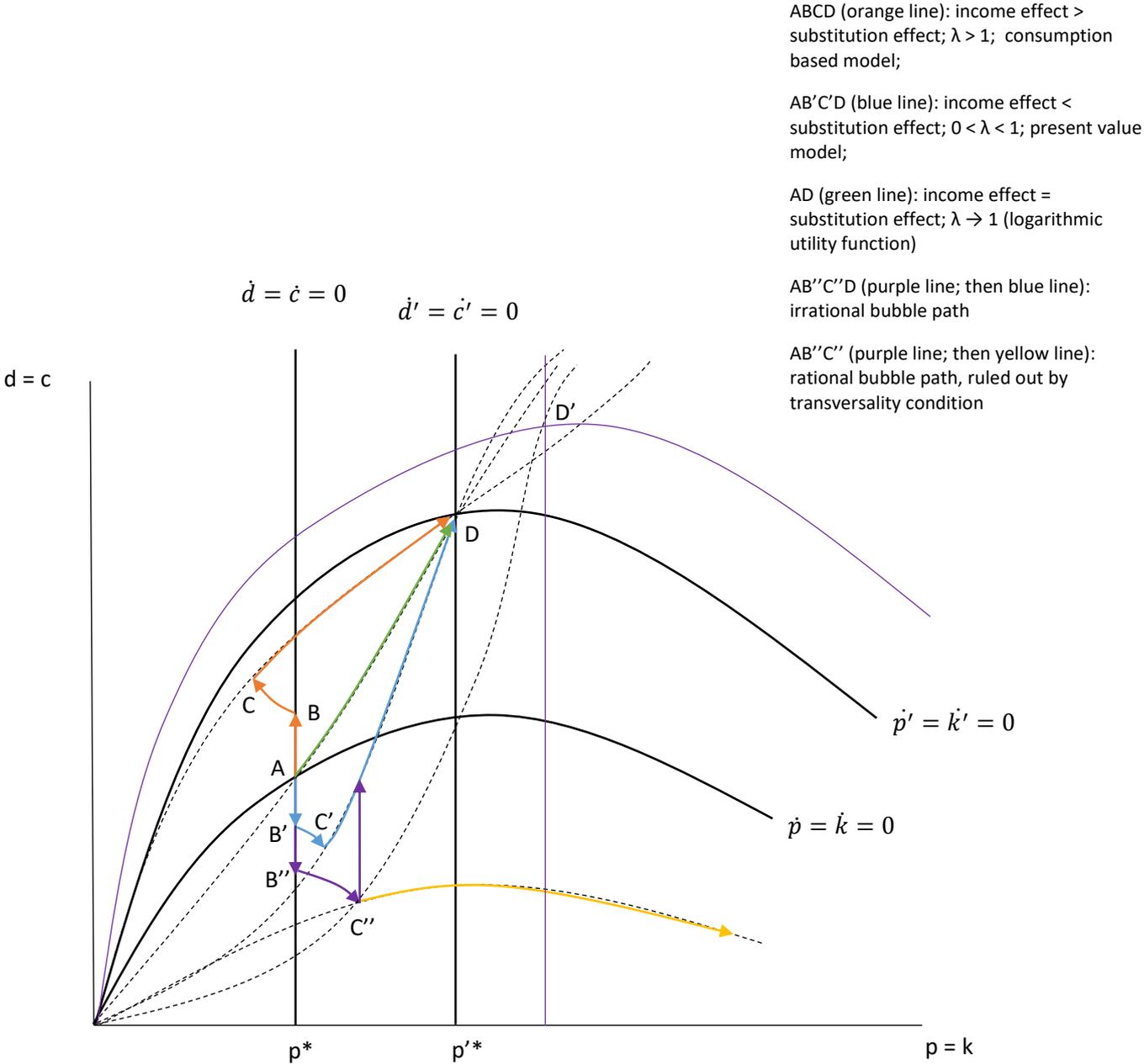
Of course, the macroeconomic model outlined in this paper presents several simplifications: it considers a simple system of production with just one commodity, workers are supposed to consume all their wages, shareholders' savings coincides with firms' retained rate, the number of shares is supposed to be constant. However, it's precisely this simplification that allows us to single out the essential logical structure of the two models and to highlight the logical dependency of the present value model on the neoclassical theory. The internal dialectic in the neoclassical field appears to boil down to verify to which extent present value model departs from neoclassical fundamentals, without however question the fundamentals themselves (this is the case of behavioural finance). However, through a logical "overturning" of the neoclassical model, we have shown how it is possible to derive an alternative interpretation of stock market dynamics. In this framework, speculative behaviour can be fully integrated in the model, without any logical contradiction, and can provide some insights into the recent crisis. This aspect can be an interesting topics for future research.

APPENDIX A - TIME LINE

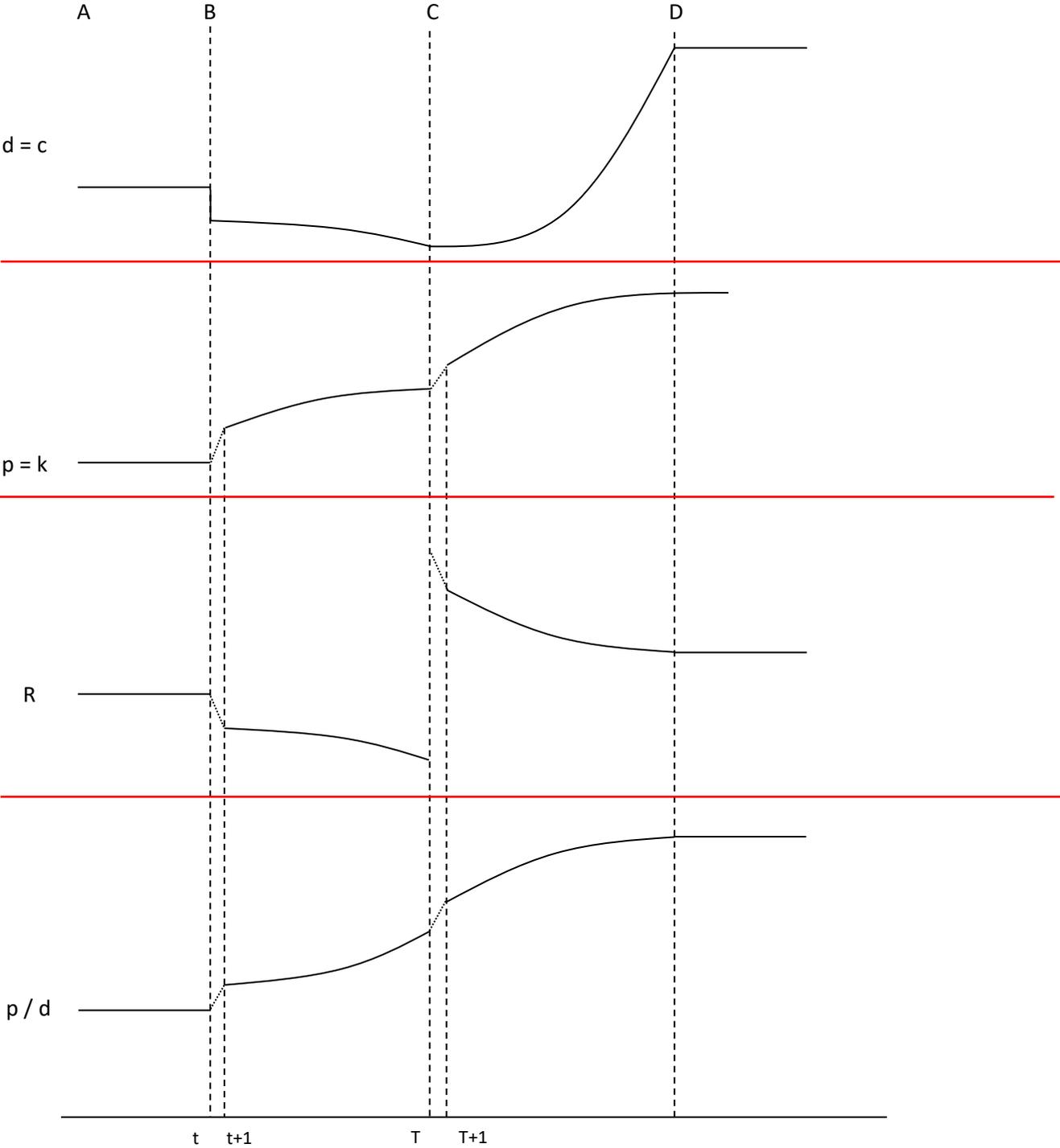


APPENDIX B - GRAPHS

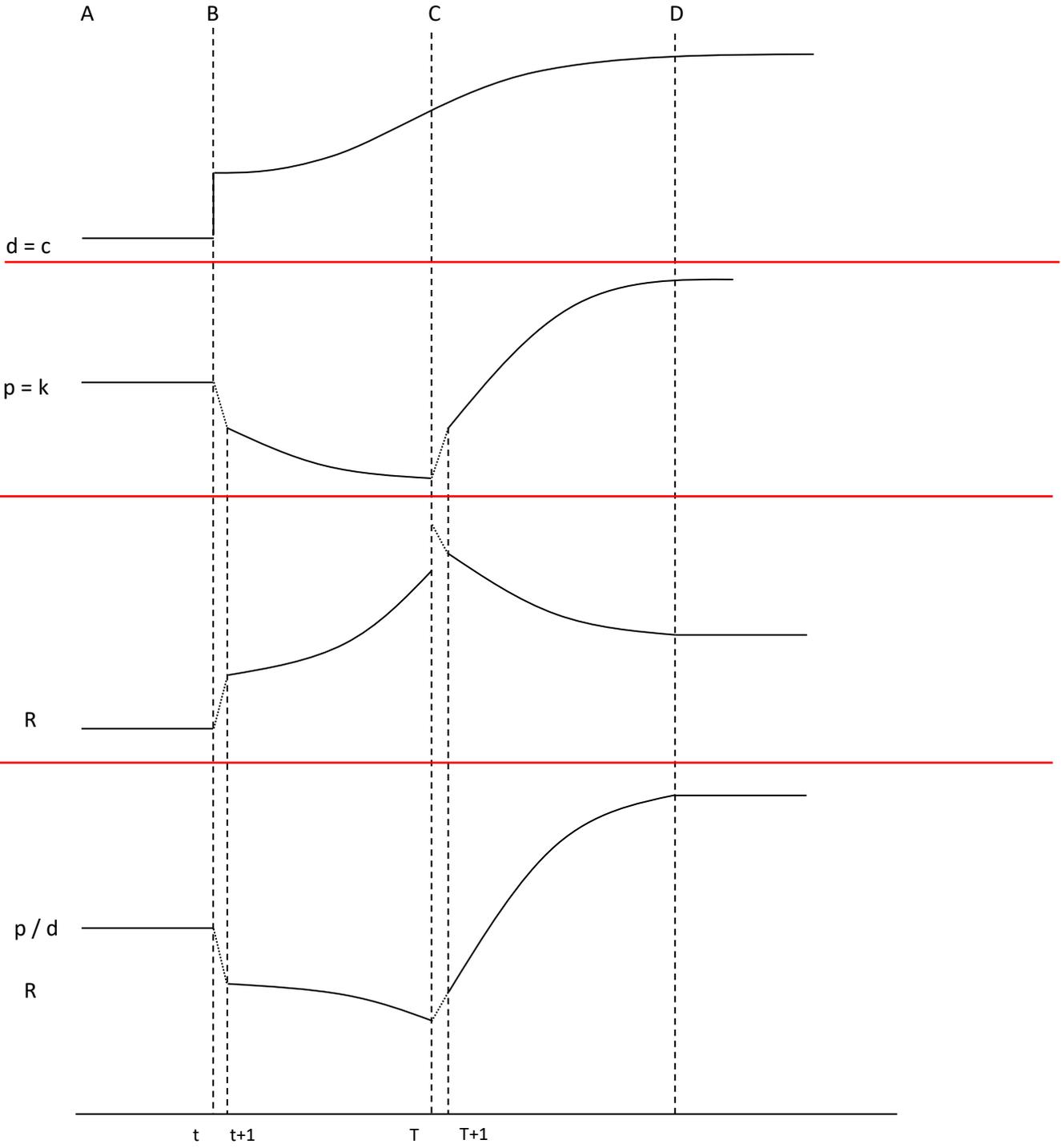
PERMANENT EXPECTED FUTURE POSITIVE PRODUCTIVE SHOCK



Case in which substitution effect is higher than income effect (blue line)



Case in which income effect is higher than substitution effect (orange line)



CHAPTER 3

CLASSICAL-KEYNESIAN MODELS AND THE MONETARY CIRCUIT

1. Introduction

The aim of this chapter is to develop a classical-keynesian model of growth and distribution integrated in a monetary circuit framework, evaluating its consistency and its policy implications⁸¹. In particular, we want to verify if keynesian multiplier can be consistently introduced in the monetary circuit framework, how monetary authorities can affect economic dynamics, how monetary circuits are linked to each other and how the problem of interest repayments can be solved.

In the recent literature we can find several attempts to integrate the monetary circuit scheme on the one hand and the sraffian price system on the other hand⁸². Partly based on this literature, the model proposed in this chapter presents the following general features. First, in contrast with the methodological individualism and the subjectivism of the neoclassical approach, the alternative model proposes an objectivist view of society, in which the principal actors are social classes with conflicting roles, functions and interests. Second, in contrast with the neoclassical one-way view of production process - represented as a linear process going from scarce resources and households' preferences to prices and

⁸¹ For the classical-keynesian model the main references are Garegnani 1984, 1990, 1992; Garegnani, Palumbo 1998; Ciccone 1986; for the monetary circuit scheme the main references are Graziani 1984, 2003.

⁸² Lunghini, Bianchi 2004; Halevi Taouil 1998; Brancaccio, Realfonzo 2008; Brancaccio 2005, 2010.

quantities produced - the alternative approach considers the economy as a circular process in which commodities are both input and output of the system: in this framework, prices cannot be considered as scarcity indexes; instead, they are determined in such a way to ensure the reproduction of the system, given the balance of power between social classes⁸³. As we shall see, the concept of reproduction is interpreted in an extensive way, involving both physical and monetary variables. Third, differently from neoclassical model, distribution is determined independently from any reference to scarce resources; instead, it depends on social, institutional and historical factors that ultimately reflect the balance of power between workers and firms, according to a *conflictual* view on distribution, as opposed to the neoclassical *compatibilist* approach⁸⁴. Fourth, analyzing the monetary flows allows to explicate the asymmetries between social actors about access to credit: the exclusive access firms have on money and bank financing allows them to have a high influence on both production and distribution.

As we shall see, this model may help to clarify the role and the behavior of monetary authorities, explicating a monetary rule alternative to the Taylor rule, which is a reaction function used in neoclassical models to identify the optimal monetary policy for central banks (Taylor 1993, 1999)⁸⁵. Within the critical approach, it is possible to derive an alternative monetary rule, according to which banks set the interest rate so as to ensure the

⁸³ See for instance Pasinetti 1989, which describes the antagonism between the neoclassical paradigm of scarcity and the classical paradigm of reproduction.

⁸⁴ On the antagonism between conflictualism and compatibilism see Graziani 1981, Brancaccio & Realfonzo 2008).

⁸⁵ According to the Taylor rule, central banks must set the interest rate on the basis of both the divergence of actual inflation from the target rate and the divergence of actual output from its level of natural equilibrium.

average conditions of solvency of firms (Brancaccio, Fontana 2013). In the next sections we shall show how the behavior of central banks is crucial in determining the level of production, income distribution and the degree of capital centralization.

The chapter is structured as follows. In the second paragraph we shall present a simple model, useful to clarify some controversial aspects such as the relationship between the keynesian multiplier and the monetary circuit scheme, the possibility for firms to repay interests on loans at the end of the circuit, the behavior of central banks when they follow the solvency rule. In the third paragraph we shall present an extension of the simple model, introducing autonomous consumption, the possibility for firms to repay part of loan interests through new loans and an arbitrary behavior of central banks. Finally, in the fourth paragraph we shall consider the several ways through which banks can influence the economic dynamics.

2. A simple starting model

We assume a closed economy in which one commodity is produced by means of itself and labour. Capital fully depreciates after one period. For the sake of simplicity, differently from the preceding chapters, technology of the system is represented by a unique technique, given by capital / labour ratio $k = K / L$, corresponding to normal capacity utilization. Given k , also per-capita product $y = f(k)$ and capital / productive capacity ratio $v = k / f(k)$ are given. In what follows, we shall express variables in real terms even when considering the monetary circuit, explicating monetary prices in the footnotes: this should simplify the analysis.

The real income is distributed between wages and profits:

$$Y = wL + (1 + r)K$$

where w is the *normal* real wage and r is the *normal* rate of profit. Dividing the expression by L we obtain per-capita income:

$$1) f(k) = w + (1 + r)k$$

Following the classical tradition (e.g. Sraffa 1960, Garegnani 1981, 1990) we assume that *normal* distribution is determined by economic, social and institutional forces more persistent than the other ones. In particular, we assume a given normal rate of profit. Then, given r and the technique k , the income equation residually determines the normal real wage. Note that we can also express equation (1) in monetary terms: $1) f(k) = \frac{W}{P} + (1 + r)k$, where W is the monetary wage and P is the *normal* monetary price, that is the monetary price consistent with normal distribution; given W from the bargaining between workers and firms, (1) determines P .

Deviations from normal distribution are allowed. We assume that actual distribution may deviate from the normal one for two reasons. First, actual prices may deviate from normal prices; we denote price deviations with $\delta = P_t / P$, where P_t is the actual price. If $\delta = 1$ there are no price deviations. Second, actual capacity utilization may deviate from the normal one; we denote deviations from normal capacity utilization with $u = y_t / y$, where y_t is actual output and y is normal output. If $u = 1$, there are no deviations from normal capacity

utilization. We assume that firms can obtain different levels of production employing different quantities of (flexible) labour on the same quantity of (not flexible) capital, given the same technique; in other words, firms can realize an actual output uY employing an actual quantity of labour uL on the same quantity of capital K , given the optimal technique $k = K / L$. Actual real wages for unit of normal labour are given by $u \frac{w}{\delta}$ ⁸⁶. Deviations from normal prices and normal capacity utilization lead the actual rate of profit to deviate from the normal one; we denote deviations from the normal rate of profit with $\gamma = r_t / r$, where r_t is the net actual rate of profit. Then, the actual per-capita distribution becomes⁸⁷:

$$1^*) \quad uf(k) = u \frac{w}{\delta} + (1 + \gamma r)k$$

Substituting w from the normal income equation in the actual income equation, we obtain an expression for the actual rate of profit in terms of u , δ and the normal rate of profit:

$$1^{*'}) \quad \gamma = \frac{1}{r} \left\{ u \frac{f(k)}{k} \left[1 - \frac{1 - (1 + r) \frac{k}{f(k)}}{\delta} \right] - 1 \right\}$$

⁸⁶ In monetary terms actual wages are given by $u \frac{w}{\delta P}$.

⁸⁷ Note that also the rate of profit is expressed in real terms. Monetary profits are expressed as follows: $(1 + \gamma r_m) \delta_{-1} P K$. The relationship between real and monetary rate of profit, for a given normal price, is the following: $(1 + \gamma r_m) \delta_{-1} P = (1 + \gamma r) \delta P$.

It's clear that, with neither capacity utilization nor price deviations ($u = \delta = 1$), the actual rate of profit coincides with the normal one ($\gamma = 1$). If $\delta = 1$, the actual rate of profit is only reached through deviations from normal capacity:

$$(1 + \gamma r) = u(1 + r)$$

The model is similar to the alternative models presented in the previous chapters; the only difference is that k is given and it doesn't depend on the normal rate of profit. Given the technique k and the rate of profit, the normal income equation determines w ; given the rate of growth autonomously decided by firms, the macroeconomic equilibrium condition determines the deviations from the normal rate of profit needed to obtain the equilibrium, γ . Given γ , the actual income equation determines the combination between price deviations and capacity utilization deviations needed to reach the actual rate of profit consistent with macroeconomic equilibrium.

All of the models presented so far are based on a simplifying assumption: we have implicitly assumed that firms can obtain from the financial sector all the loans needed for their investment and production decisions. Now, we shall introduce the financial sector, drawing from monetary circuit literature, showing how it can influence economic dynamics. Even if money plays a fundamental role in the model, we continue to use real variables, so as to simplify the analysis.

According to the canonical monetary circuit scheme, the economic process can be divided in three steps (see for instance Graziani 2003, p. 27): first, firms obtain from the bank sector the loans needed to start the production process (*initial finance*); since firms are considered

as an integrated sector, the required loans coincide with the wage bill, which, in turn, depends on monetary wage and on the number of workers firms decide to hire according to their investment plans. Second, firms sell the goods produced partly to other firms and partly to workers. Assuming that workers use their income either to consume or to buy shares, all money flows back to firms (*final finance*)⁸⁸. Finally, firms repay their initial debt: money is destroyed and monetary circuit is closed.

As it is known, this scheme presents a problem, known as profit realization problem or profit paradox (Fontana & Realfonzo 2005, Messori & Zazzaro 2005, Rochon 2005), which we shall try to overcome. The critical point is the following: if at the beginning of period a certain amount of money enters the circuit, at the end of period the same amount of money must exit the circuit, and then firms cannot repay interests on loans. In what follows we try to overcome this problem.

We assume that, at the beginning of each period, firms demand loans to finance both investments (demand for new capital goods) and the monetary wages needed to hire the labour force required for their production plans. Then, firms start the production process employing workers on the capital goods produced and purchased in the preceding period, available at the beginning of the current period. At the end of each period, on the one hand workers use all their monetary wages to buy goods they consume entirely; on the other hand, firms sell the product and reimburse wage loans, whereas investment loans are only reimbursed at the end of the following period; also, we assume that interests are only paid

⁸⁸ If workers decided to hoard part of wages, an equal amount would remain in existence, representing firms' debt and workers' credit towards banks. in the model presented here we shall assume that workers consume all of their wages.

on investment loans. While the assumption of a negligible interest rate on wage loans is not decisive for the model but it is useful to simplify the analysis (see Brancaccio, Suppa 2009), the time gap between investment loans - obtained at the beginning of current period - and investment repayments - made at the end of the following period - allows to solve the interest-repayments problem. Without this time gap, firms could just repay the loans without interests: if a certain quantity of money enters the circuit at the beginning of the period, just the same quantity can flow back to banks at the end of period, thus preventing any interest repayment. The time gap allows firms to repay gross-of-interest loans, even if the repayment occurs in the following period. In the appendix we reformulate the model explicating the interests on wage loans.

Now, given this temporal sequence, we can derive both macroeconomic equilibrium and solvency condition, that is the requirements for firms to be able to repay their loans. According to the classic hypothesis of savings, workers consume all their wages, whereas capitalists save a part of their profits. Then we have:

$$wuL + (1 + \gamma r)\delta K = wuL + (1 - s_c)(1 + \gamma r)\delta K + (1 + g)\delta K$$

Note that, differently from workers' expenditure, investments' expenditure involves the term δ : this means that investments adapt to actual price variations. This assumption can be justified with the idea that firms have a privileged access to bank credit with respect to workers, and then they can preserve the real value of their investments from price variations.

Dividing all terms by K we obtain the macroeconomic condition in per-capital terms:

$$\frac{uw}{\delta k} + (1 + \gamma r) = \frac{uw}{\delta k} + (1 - s_c)(1 + \gamma r) + (1 + g)$$

Rearranging we have:

$$sf(k) = (1 + g)k$$

Where the average propensity to save, s , is given by:

$$s = s_c(1 + \gamma r)$$

Firms are solvent if their incomes and loans are higher or equal to the expenditures and the repayments on previous loans. Assuming that capitalists' consumption is not financed by debt, we have:

$$\begin{aligned} wuL + (1 + \gamma r)\delta K + (1 + g)\delta K \\ \geq wuL + (1 - s_c)(1 + \gamma r)\delta K + (1 + g)\delta K + (1 + i)(1 + g_{-1})\delta K_{-1} \end{aligned}$$

The first two terms on LHS represent actual income, divided between actual wages and actual profits. The third term on LHS represents investment loans granted by banks at the beginning of current period. The first three terms on RHS are the expenditures: at the end of each period, workers consume all their wages, whereas capitalists consume a part of profits

and demand an amount of investments equal to the loans obtained at the beginning of the period⁸⁹. The fourth term on RHS represents the real interest repayments on the investment loans contracted at the beginning of the preceding period, which in turn coincide with the previous investments; since the previous investments coincide with the capital available at the beginning of the current period⁹⁰, the fourth term can be reformulated in the following way⁹¹:

$$(1 + i)(1 + g_{-1})\delta K_{-1} = (1 + i)\delta K$$

From the solvency condition, dividing all terms by k and δ we obtain:

$$\frac{uw}{\delta k} + (1 + \gamma r) + (1 + g) \geq \frac{uw}{\delta k} + (1 - s_c)(1 + \gamma r) + (1 + g) + (1 + i)$$

Rearranging, we obtain the solvency rule in the final form:

$$s_c(1 + \gamma r) \geq 1 + i$$

⁸⁹ Assuming no financial constraints (that is a passive behaviour of banks), investments (and then the term g) are autonomously decided by firms. As noted before, we assume that firms can preserve the real value of their investments from price variations.

⁹⁰ Assuming a depreciation rate equal to one, we have $(1+g_{-1}) k_{-1} = i_{-1} = k$, where i is per-capita investment.

⁹¹ Note that, since all rates are expressed in real terms, (real) price deviations are referred to (end of) current period.

This condition simply says that firms are only solvent (on average) if capitalists' savings are at least equal to interest repayments⁹²: were capitalists' savings lower than interest repayments, the money issued at the beginning of the period (start of the circuit) would not be sufficient to cover the repayments at the end of the period (end of the circuit). Indeed, substituting the macroeconomic equilibrium condition into the solvency condition we obtain:

$$g \geq i$$

This condition has an intuitive interpretation: firms are only solvent, and the circuit only closes without outstanding debt, if the money introduced in the circuit at the beginning of the period is at least equal to the money flowed back to banks at the end of the period.

An interesting feature of this model is that the Keynesian multiplier process is perfectly consistent with (this version of) monetary circuit scheme: the money introduced in the circuit at the beginning of the period gives rise to a multiplicative process that increases income and then profits; at the end of this process, savings out of profits will be equal to initial investments / current loans. If solvency condition holds with strict equality, savings will be just sufficient to repay previous loans. This can be illustrate with a simple example, explicating all the sub-periods between the beginning and the end of the circuit (see the table in the appendix). In order to simplify the analysis we don't explicit neither workers' consumption nor wage loans, which have no role in the multiplier process, assuming firms

⁹² we continue to assume that consumption is not financed by debt.

are always able to obtain the wages loans needed to hire the amount of labour consistent with their production plans.

Assume firms are able to obtain, at the beginning of the period, an amount of per-capita loans $(1+g)k$ equal to x . At the end of sub-period 1, firms produce the capital goods demanded, obtaining per-capita profits $(1+\gamma r)k$ equal to x , which will be partly spent in consumption in the next sub-period. At the end of sub-period 2, firms produce the consumption goods demanded by capitalists, obtaining per-capita profits equal to $(1 - s_c) x$, which will be partly spent in consumption in the next sub-period; capitalists' savings are $s_c x$. At the end of sub-period 3, firms produce the consumption goods demanded by firms, obtaining per-capita profits equal to $(1 - s_c)^2 x$, which will be partly spent in consumption in the next sub-period; firms' savings are $s_c^2 x$. The process will continue until, in the end, actual profits equal the ratio between initial investments (current loans) and s_c (the multiplier), savings equal initial investments (current loans obtained at the beginning of period), and consumption equals the not-saved part of actual total profits. In the end, savings will be equal to initial investments and, assuming solvency condition holds with strict equality, they will also be exactly sufficient to repay previous loans.

In order to clarify the temporal sequence of the process, we sum up the steps of this version of the monetary circuit (see the figures in the appendix):

1. Firms demand an amount of loans equal to investment demand and to the actual wage bill needed to hire the amount of workers required for their production plans. Note that, on the one hand, workers can only decide about monetary wages: actual employment as well as

monetary prices - and then real wages - are decided by firms; on the other hand, both monetary and real investments are decided by firms.

2. Firms start the production process employing - and paying - the required quantity of labour on the capital inherited from past investment decisions;

3a. workers consume all their monetary wages, which flow back to firms; the real quantity of per-capita consumption depends on the prices set by firms.

3b. firms demand the planned investments using the loans obtained at the beginning of the period; part of the income obtained selling investment goods is consumed and gives rise to the multiplier process;

4. At the end of the period - and of the multiplicative process - the money held by firms equals the total (paid and consumed) wages plus the saved part of total profits⁹³.

5. Firms repay bank loans: on the one hand they repay the wage loans obtained at the beginning of the current period; on the other hand they repay the (gross) interests on the investment loans obtained at the beginning of the preceding period, which, as we have seen, actually coincide with the investments at the end of the preceding period, and then with the capital available at the beginning of the current period.

⁹³ As we have seen, total profits are the profits realized at the end of the multiplicative process, and they are equal to the (infinite, at the limit) sum of the profits realized in each sub-period.

Summing up, assuming the solvency condition holds with strict equality, the model can be expressed as follows:

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$1^*) uf(k) = u \frac{w}{\delta} + (1 + \gamma r)k \rightarrow u \text{ or } \delta \text{ or a mix}$$

$$2) k = \bar{k} \rightarrow k$$

$$3) sf(k) = (1 + g)k$$

$$3'[3 + 4]) s_c(1 + \gamma r) = (1 + g) \rightarrow \gamma$$

$$4) s = s_c(1 + \gamma r) \frac{k}{f(k)} \rightarrow s$$

$$5) s_c(1 + \gamma r) = (1 + i)$$

$$5'[5 + 3']) g = i \rightarrow i$$

Given k from 2 and r from outside the model, 1 determines w . Given g , 3' determines γ and 5' determines i ; given γ and δ (or u), 1* determines u (or δ), and 4 determines s . This solution is based on the assumption that banks set the interest rate at the level consistent with the average solvency condition of firms. Then, equation 5' can be interpreted as a sort of *solvency rule* that the central bank can follow (Brancaccio, Fontana 2013): by varying the interest rate, central bank acts as a “regulator” of a social conflict between firms abundantly solvent and insolvent firms; a relatively high interest rate increases the amount of firms at risk of insolvency, stimulating the tendency toward bankruptcies, takeovers and then the “centralisation” of capital. If the central bank follows the solvency rule, it can ensure the

average solvency of the system. In the next section we shall show the several ways through which central bank can influence macroeconomic dynamics.

3. The Complete model

In this section, with respect to the preceding model, we add the following features:

- Part of the interest repayments can be financed by loans in the same period. The degree of debt refinancing, λ , depends on banks' orientation⁹⁴. Then, loans also involve the term $\lambda(1+i)$: if $\lambda = 0$, firms (and workers) repay all loans and interests on loans without debt refinancing (*hedge borrowers*); if $0 < \lambda \leq 1 / (1+i)$, part of loans is repaid through debt refinancing (*speculative borrowers*); if $1 / (1+i) \leq \lambda \leq 1$, both loans and part of interests on loans are repaid with debt refinancing (*ultraspeculative borrowers*). Then the term λ represents the degree of financial instability: the more banks are willing to refinance part of loans and interest on loans, the more probable is the tendency toward speculative positions and financial instability (see Brancaccio, Fontana 2013; Minsky 1977; Kindleberger 1978).
- Additional workers' consumption, Z , can be financed through loans granted in the same period.
- if loans exceed repayments, the outstanding credit (*surplus income*), H , can be used in the following period for consumption purposes; if repayments exceed loans, the outstanding debt (*shortcoming income*) must be deduced from consumption in the next period. This is true for both firms and workers. This happens when banks don't follow solvency condition.

⁹⁴ For the sake of simplicity, the monetary circuit linking central bank and firms is not modelled.

Now, we can reformulate both macroeconomic equilibrium and solvency conditions.

Solvency condition of firms becomes:

$$\begin{aligned} wuL + (1 + \gamma r)\delta K + \lambda(1 + i)\delta K + \hat{H}_c + (1 + g)\delta K \\ = (1 - s_c)(1 + \gamma r)\delta K + \hat{H}_c + H_c + wuL + (1 + g)\delta K + (1 + i)\delta K \end{aligned}$$

Or, in per-capital terms:

$$\begin{aligned} \frac{uw}{\delta k} + (1 + \gamma r) + (1 + g) + \hat{h}_c \frac{f(k)}{k} \\ = \frac{uw}{\delta k} + (1 - s_c)(1 + \gamma r) + \hat{h}_c \frac{f(k)}{k} + h_c \frac{f(k)}{k} + (1 + g) + (1 - \lambda)(1 + i) \end{aligned}$$

where $\hat{H}_c = H_{c,-1}$ is surplus (shortcoming) income of preceding period, H_c is surplus (shortcoming) income of current period, $h_c = \frac{H_c}{Y}$ is the proportion of current surplus (shortcoming) income to current capacity and $\hat{h}_c = \frac{\hat{H}_c}{Y}$ is the proportion of preceding surplus (shortcoming) income to current capacity. Reformulating we obtain:

$$s_c(1 + \gamma r) - (1 - \lambda)(1 + i) = h_c \frac{f(k)}{k}$$

If central banks set the interest rate according to firms' solvency condition, surplus (shortcoming) income is zero and the condition becomes:

$$s_c(1 + \gamma r) = (1 - \lambda)(1 + i)$$

This condition - less stringent than the condition formulated in the preceding section - takes into account the possible presence of debt refinancing: capitalists' savings must be equal to firms' repayments net of the part of repayments financed through loans. If banks don't follow this solvency rule, capitalists remain with an outstanding credit (debt) to be spent (not to be spent) in the following period equal to H_c , or $h_c \frac{f(k)}{k}$ in per-capita terms.

Solvency condition of workers is the following:

$$wuL + Z + \hat{H}_w + \lambda(1 + ai)Z_{-1} = wuL + \hat{H}_w + H_w + (1 + ai)\hat{Z}$$

Or, in per-capita terms:

$$\begin{aligned} uw + (1 + b)\hat{z}f(k) + \hat{h}_w f(k) + \lambda(1 + ai)\hat{z}f(k) \\ = uw + \hat{h}_w f(k) + h_w f(k) + (1 + ai)\hat{z}f(k) \end{aligned}$$

where $\hat{H}_w = H_{w,-1}$ is workers' surplus (shortcoming) income of preceding period, H_w is workers' surplus (shortcoming) income of current period, $h_w = \frac{H_w}{Y}$ is the proportion of workers' current surplus (shortcoming) income to current capacity, $\hat{h}_w = \frac{\hat{H}_w}{Y}$ is the proportion of workers' preceding surplus (shortcoming) income to current capacity, Z is the autonomous consumption of workers (that is the additional workers' consumption financed by loans), $\hat{Z} = Z_{-1}$ is the autonomous consumption of preceding period, $z = \frac{Z}{Y}$ is the proportion of current autonomous consumption to current capacity, $\hat{z} = \frac{Z_{-1}}{Y}$ is the proportion of preceding autonomous consumption to current capacity, $(1 + b) = \frac{Z}{Z_{-1}}$ is the

growth rate of autonomous consumption, α is the ratio between the interest rate applied on firms' loans and the interest rate applied on workers' loans. Reformulating we obtain:

$$[(1 + b) - (1 - \lambda)(1 + \alpha i)]\hat{z} = h_w$$

If banks set the interest rate according to workers' solvency condition, surplus (shortcoming) income is zero and the condition becomes:

$$(1 + b) = (1 - \lambda)(1 + \alpha i)$$

This condition says that workers are exactly solvent if consumption loans grow at a rate equal to workers' repayments net of the part of repayments financed through loans. If banks don't follow this solvency rule, workers remain with an outstanding credit (debt) to be spent (not to be spent) in the following period equal to H_w , or $h_w \frac{f(k)}{k}$ in per-capital terms.

Macroeconomic equilibrium can be expressed in the following way:

$$wuL + (1 + \gamma r)\delta K = wuL + (1 - s_c)(1 + \gamma r)\delta K + \hat{H} + (1 + g)\delta K + Z$$

Or, in per-capital terms:

$$\frac{uw}{\delta k} + (1 + \gamma r) = \frac{uw}{\delta k} + (1 - s_c)(1 + \gamma r) + \hat{h} \frac{f(k)}{k} + (1 + g) + z \frac{f(k)}{k}$$

where $\hat{H} = \hat{H}_c + \hat{H}_w$ is the total surplus (shortcoming) income of the preceding period and $\hat{h} = \hat{h}_c + \hat{h}_w$ is the proportion of total preceding surplus (shortcoming) income to current capacity. Reformulating we obtain

$$s_c(1 + \gamma r) - (\hat{h} + z) \frac{f(k)}{k} = (1 + g)$$

which can be expressed in the standard way:

$$sf(k) = (1 + g)k + (\hat{h} + z)f(k)$$

$$s = [s_c(1 + \gamma r)] \frac{k}{f(k)}$$

Now, macroeconomic equilibrium takes into account both the current autonomous consumption and the total amount of outstanding credit (debt) of the preceding period, which is assumed to be spent (not to be spent) for consumption purposes in the current period.

Substituting macroeconomic equilibrium equation in firms' solvency equation we obtain:

$$(1 + g) + (\hat{h} + z) \frac{f(k)}{k} - (1 - \lambda)(1 + i) = h_c \frac{f(k)}{k}$$

This conditions can be interpreted in the following way: if the sum between investment loans, consumption loans and preceding surplus (shortcoming) total income is higher (lower)

than repayments net of the part of repayments financed through loans, a surplus (shortcoming) income arises in the current period, available (not available) for consumption purposes in the next period.

If banks set the interest rate according to solvency condition of both firms and workers, surplus (shortcoming) income is zero and this condition becomes:

$$(1 + g) + z \frac{f(k)}{k} = (1 - \lambda)(1 + i)$$

This means that current loans (used for investments and autonomous consumption) are equal to net repayments.

In order to derive the solution of the model, we re-state the equations for ease of reference⁹⁵:

⁹⁵ Note that $z = (1+b) z_{-1}$; then, given z_{-1} , once b is known also z is known (and vice versa). for the sake of clarity, In equation 4 (and then 3' and 5') we maintain the term z , whereas in equation 6 we use the term b .

$$1) f(k) = w + (1 + r)k \rightarrow w$$

$$1^*) uf(k) = u \frac{w}{\delta} + (1 + \gamma r)k \rightarrow u \text{ or } \delta \text{ or a mix}$$

$$2) k = \bar{k} \rightarrow k$$

$$3) sf(k) = (1 + g)k + (\hat{h} + z)f(k)$$

$$3'[3 + 4]) s_c(1 + \gamma r) - (\hat{h} + z) \frac{f(k)}{k} = (1 + g) \rightarrow \gamma$$

$$4) s = [s_c(1 + \gamma r)] \frac{k}{f(k)} \rightarrow s$$

$$5) s_c(1 + \gamma r) - (1 - \lambda)(1 + i) = h_c \frac{f(k)}{k}$$

$$5'[5 + 3']) (1 + g) + (\hat{h} + z) \frac{f(k)}{k} - (1 - \lambda)(1 + i) = h_c \frac{f(k)}{k} \rightarrow h_c$$

$$6) [(1 + b) - (1 - \lambda)(1 + \alpha i)] \hat{z} = h_w \rightarrow h_w$$

This is a system of seven equation with ten exogenous variables:

$$r, g, \hat{h}, z, \hat{z}, i, \lambda, \alpha, \delta, b$$

and seven endogenous variables:

$$k, w, \gamma, u, s, h_c, h_w$$

The solution is the following: given k from 2 and r from outside the model, 1 determines w ; given g , \hat{h} and z , 3' determines γ , and 4 determines s ; given δ , 1* determines u (or viceversa); given i , λ , g , \hat{h} and z , 5' determines h_c ; given i , λ , g , \hat{h} , \hat{z} and b , 6 determines h_w .

Note that, differently from the model of preceding section, the central bank can autonomously decide the level of interest rate; solvency rule only becomes a policy behaviour among others. As we shall see in the next section, the two versions of the model can be used to understand different phases of economic cycles.

4. Policy implications

From the previous analysis, it should be clear that both central banks and the bank sector can have a decisive role in influencing economic dynamics and all of the economic variables considered. In particular, they can control the following variables: the interest rate, and then the solvency degree of firms; the degree of debt refinancing, and then the degree of financial instability; the amount of investment loans, wage loans, workers' consumption loans. Controlling these variables, they can influence growth, distribution, the degree of capital centralization and the amount of outstanding credit (debt) to be spent (not to be spent) in the following periods. Then, they can also influence economic dynamics with pro-cyclical or counter-cyclical policies, for instance supporting a *bubble* or easing its *burst*.

In what follows, we shall analyse the economic consequences resulting from the decisions that the monetary authorities and the bank sector can take about all the variables they can control.

1. Interest rate. Controlling the interest rate, monetary authorities can influence several variables. First, they can affect the solvency of both firms and workers. When firms' solvency rule is always exactly respected⁹⁶, consumption loans are zero and i and λ have no impact on the other variables, we are in the presence of a standard classical-keynesian model without autonomous demand. In all the other cases, financial decisions have an impact on real economy. For instance, since workers' solvency condition is more stringent than firms' solvency condition, a relatively high interest rate increases the probability of workers' insolvency, even if firms' solvency condition holds with strict equality⁹⁷; the resulting outstanding debt (shortcoming income) of workers can reduce next-period consumption, with possible depressive effects. A particularly high interest rate can also increase the probability of firms' insolvency; if investment loans and/or investments decided by firms don't appropriately increase, on the one hand this can boost the capital centralization process, on the other hand the resulting outstanding debt (shortcoming income) can have depressive effects on the economy in the following periods. If monetary authorities want to ensure solvency condition for the whole system, they must set an interest rate not more higher than the lowest rate resulting from firms' and workers' solvency condition⁹⁸. Banks can also decide to set the interest rate at an intermediate level between the rate resulting from workers' solvency condition and the rate resulting from firms' solvency condition; in this case, the effects on economic dynamics in future periods are more ambiguous.

⁹⁶ This means that the central bank always sets the appropriate interest rate for any degree of debt refinancing.

⁹⁷ This can be the case when monetary authorities are only worried about firms' solvency.

⁹⁸ In this simplified model without workers' savings, the lowest rate of interest is always that resulting from workers' solvency condition.

The interest rate might also contribute to determine the normal rate of profit (even if here we have not considered this hypothesis). For instance, the normal rate of profit can be a positive function of the interest rate and the remuneration for “risk and trouble” (see for instance Pivetti 1985). Then, an increase in the interest rate, pushing up the normal rate of profit, can lead to a decrease (or a lower increase) of capacity utilization, with possible negative (or less strong) effects on rate of growth. However, this feature is not considered in this model, since both the rate of interest and the normal rate of profit are assumed to be exogenous variables⁹⁹.

2. Degree of debt refinancing. Controlling the share of repayments financed by loans, banks can affect economic dynamics in several ways. The higher is the degree of debt refinancing, the higher is the interest rate required to ensure the solvency conditions with strict equality. The combination between a high degree of debt refinancing and a low interest rate can lead to an ultraspeculative position fed by the injection in the circuit of increasing surplus incomes, which can lead, in the end, to an unsustainable credit boom followed by a financial crisis. On the contrary, a low degree of debt refinancing, together with a high interest rate, can depress the economy while stimulating the process of capital centralization. A not sufficiently high degree of debt refinancing can ensure firms’ solvency while making more probable workers’ insolvency, with ambiguous effect on economic dynamics and possible weakening of workers.

3. ratio between the interest rate on workers’ loans and the interest rate on firms’ loans. Changing α , banks can influence workers’ solvency condition. The higher is α , the more

⁹⁹ See for instance Pivetti (1985) for an analysis on this issue. Following a suggestion of Sraffa (1960), Pivetti (1985) deals with the idea that the normal rate of profit strictly depends on the rate of interest.

stringent is workers' solvency condition. An increase in α can increase insolvency of workers, if it is not followed by neither an increase in workers' loans, nor a decrease in interest rate. With a higher α , workers' solvency condition comes close to firms' solvency condition, easing the stabilization role of monetary authorities, but at the expense of workers' consumption.

4. Investment loans and wage loans. By deciding the amount of investment loans, banks can influence, among other things, income distribution. Indeed, given the amount of investments decided by firms, an increase in investment loans relative to wage loans leads to an increase in price deviations, and then to a reduction of real wages, assuming firms are able to maintain the real value of their investments.

Deciding the amount of wage loans, banks can influence, among other things, the level of production. Indeed, given the amount of investments decided by firms, an increase in wage loans relative to investment loans allows firms to hire a higher quantity of labour, leading to a possible increase in capacity utilization, and then employment and total wages.

This can be seen in the following way. Assume the maximum amount of investment loans is

$$I_b = (1 + g)\delta K$$

and the maximum amount of wage loans is:

$$W_b = wuL$$

Then, their ratio is given by:

$$\frac{W_b}{I_b} = \frac{wuL}{(1+g)\delta K} = \frac{u}{\delta} \frac{w}{(1+g)k}$$

It follows that, on the one hand, the more stringent is the constraint on wage loans (short-term loans) relative to investment loans (long-term loans), the higher is the impact on the production level; on the other hand, the more stringent is the constraint on investment loans (long-term loans) relative to wage loans (short-term loans), the higher is the impact on income distribution.

5. Consumption loans. Controlling consumption loans, banks can influence both the amount of goods consumed by workers and the solvency of workers.

5. Conclusions

In this chapter we have shown how the classical-keynesian model of growth and distribution and the keynesian multiplier process can be consistently introduced in the monetary circuit framework. We have outlined a monetary rule - called solvency rule and alternative to the Taylor rule - which central banks can follow in order to assure the average solvency conditions of firms. We have also shown how both monetary authorities and the banking sector can affect economic dynamics through decisions about the variables they can control, in particular the interest rate, the wage, investment and consumption loans, the degree of debt refinancing, the ratio between the interest rate on workers' loans and the interest rate on firms' loans.

This scheme has allowed to partially overcome some dichotomies typical of the critical literature, such as the separation between the real and the monetary part of the economic system, between the long and the short period and then between normal and actual prices, between the macroeconomic adjustment of demand to supply and of supply to demand.

Of course, the model outlined in this chapter presents several simplifications. Suffice it to say that it describes an economic system in which only one (perishable) commodity is produced, with an explicit reference to only one rate of interest and one rate of profit, and with the simplified assumption of negligible interests to be paid on wage loans (see the appendix on this issue). However, these simplifications have allowed to single out the crucial features of this model, which remains logically alternative to the neoclassical one.

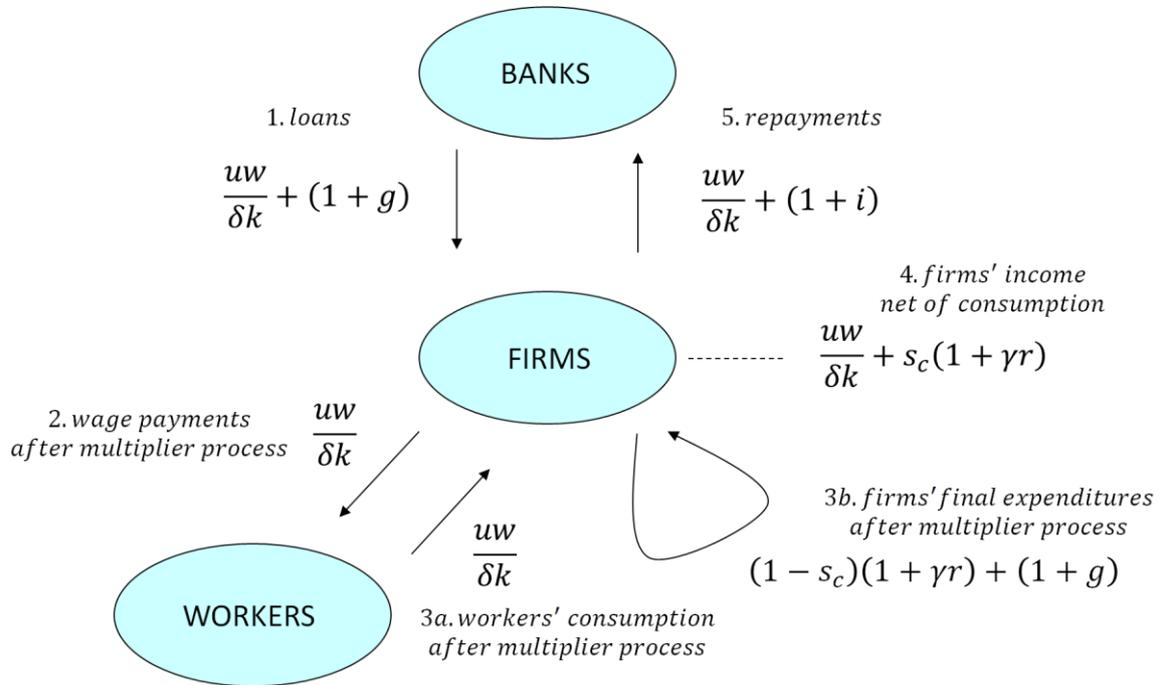
APPENDIX

THE KEYNESIAN MULTIPLIER PROCESS

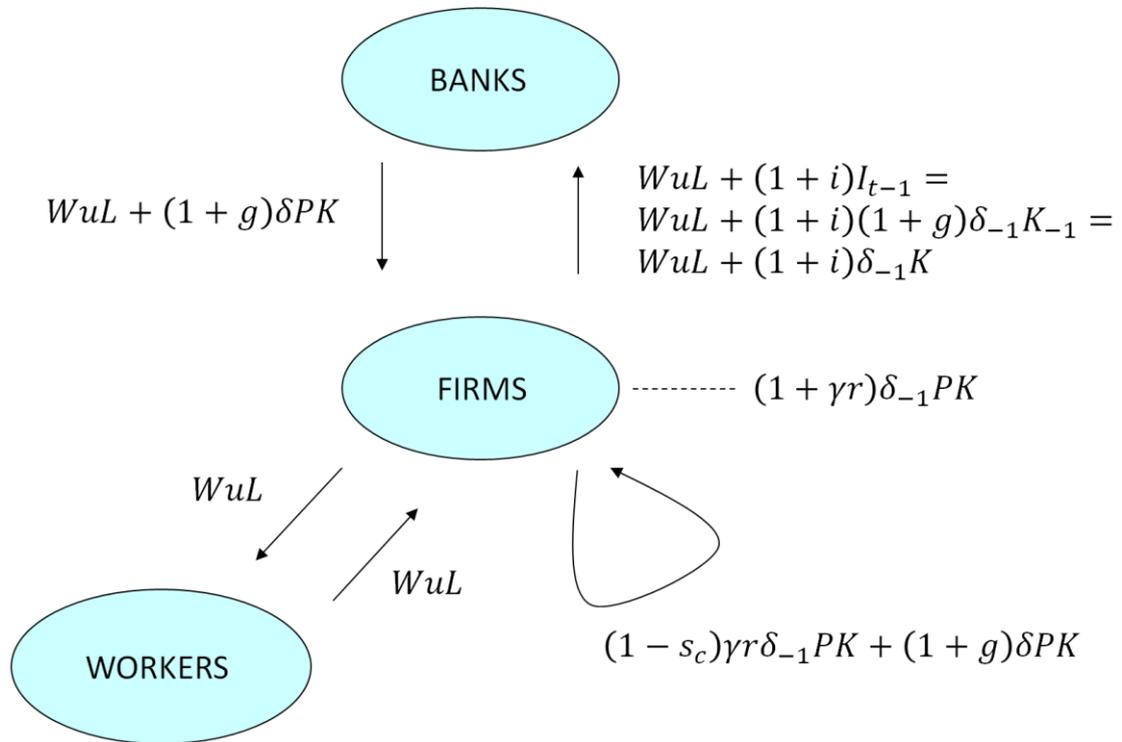
		$(1+g) k$	$(1+\gamma r) k$	c_c	$s f(k)$
Subperiod 1		x	x		
Subperiod 2			$(1-s_c) x$	$(1-s_c) x$	$s_c x$
Subperiod 3			$(1-s_c)^2 x$	$(1-s_c)^2 x$	$s_c^2 x$
Subperiod 4			$(1-s_c)^3 x$	$(1-s_c)^3 x$	$s_c^3 x$
Subperiod 5			$(1-s_c)^4 x$	$(1-s_c)^4 x$	$s_c^4 x$
			
∞			x / s_c	$(1-s_c) (x / s_c)$	x

Monetary circuit

(real per-capital actual variables)



Monetary circuit



INTRODUCING THE INTERESTS ON WAGES

The time gap between repayments on wage loans and repayments on investment loans is preserved even removing the simplifying hypothesis of negligible interests and profits on wage loans. The time gap between the employment of labour and the use of the means of production is maintained, and then also the existence of two different time lags between loans and repayments is preserved. Indeed, workers are paid at the beginning of each period and are employed in the same period, whereas the means of production are paid at the beginning of each period but they can only be used in the following period, since we assume one period is needed to produce them. Thus, on the one hand, when considering wages, the loans provided at the beginning of a period must be repaid at the end of the same period; on the other hand, when considering the means of production, the loans provided at the beginning of a period must be repaid at the end of the following period. Then, the rate of profit computed on the advanced wages is only referred to one period, whereas the rate of profit computed on the loans needed to purchase the means of production refers to two periods. The same is true for the interest rate.

The system is modified as follows:

$$1) f(k) = (1 + r)w + (1 + r)^2k \rightarrow w$$

$$1^*) uf(k) = u \frac{w}{\delta} + u\gamma r \frac{w}{\delta} + (1 + \gamma r)^2k \rightarrow \gamma, u \text{ (or } \delta)$$

$$2) k = \bar{k} \rightarrow k$$

$$3) sf(k) = (1 + g)k$$

$$3'[3 + 4]) s_c \left[u\gamma r \frac{w}{\delta} + (1 + \gamma r)^2 \right] = (1 + g) \rightarrow \gamma, u \text{ (or } \delta)$$

$$4) s = s_c \left[u\gamma r \frac{w}{\delta} + (1 + \gamma r)^2 \right] \frac{k}{f(k)} \rightarrow s$$

$$5) s_c \left[u\gamma r \frac{w}{\delta} + (1 + \gamma r)^2 \right] = (1 + i)$$

$$5'[5 + 3']) g = i \rightarrow i$$

This is a system of six equations in six unknowns: k , w , γ , u (or δ), s , i . The solution is as follows. Given k from 2 and r , 1 determines w . Given g and δ (or u), 3' and 1* determine γ and u (or δ) and 4 determines s . Given g , 5' determines i .

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