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Trade-off between research risk and major innovations: a theoretical discussion to understand optimal R&D

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Introduction

The activities of research and development have been transforming our society for the last centuries and nowadays their importance is even greater due to the high level of technology required by all players in the economy. It is important to firms, since R&D represents the key for profits in many growing sectors, such as the microelectronic, the automobile and the pharmaceutical ones; to customers, who have ‘modern’ needs that can be satisfied only with high standards of technology, such as the need to be constantly online, as it is proved by the spreading of social media, to receive real-time information, and to use the internet to perform everyday-tasks, such as online shopping with home delivery; and to public institutions, since technological development may be an engine for growth and welfare improvements.

It is no surprise that many economic papers, published mainly between the end of the 1980s and the beginning of the 2000s, have addressed the topic of R&D from various perspectives and enriched our knowledge in this regard; a survey of such literature is not repeated here because is provided throughout the following chapters. More specifically with the present thesis I plan to contribute to the literature concerning the field of R&D risk, that is the risk of conducting an R&D investment without producing any innovation, from both private producers and social planner perspectives. Focusing on the private aspect, the possibility for a research activity to fail represents a negative factor able to decrease profits of firms, especially when the amount of investment in R&D is considerably high. On the other hand many firms decide to embark on risky projects because they have greater potential in delivering ground-breaking innovations. In other words firms may be willing to take greater risk to obtain better innovations, at least to a partial extent; in fact a growing number of firms is forming research consortia with the objective to share the risk related to R&D and to aim at major innovations. As for the social perspective, the government has a limited availability of public funds and should carefully address resources in a welfare-maximising way. It is common in the real world to dispense subsidies to researching firms in order to foster technological development and to obtain welfare gains. In particular governments may decide to grant subsidies to either safer research projects that

have larger probability to produce an innovation or riskier projects which might deliver larger benefits to the population. It is the case for instance of the millions of dollars that governments invest into the pharmaceutical sector for the discovery of new medicines against dangerous diseases, which are worth investing even if a large share of the funded projects inevitably do not deliver the expected results. The present dissertation represents an opportunity to discuss these topics and other related to the activity of research and development.

The approach followed in the thesis is theoretical since it is based on a benchmark model of industrial organization in which two firms operate in the same sector and compete à la Cournot over the quantity of a homogeneous good. At the pre-competitive stage producers can decide to invest in research and development, where the research output is an innovation that raises firm efficiency. A peculiarity of the model is the presence of technological spillovers: the investment of one firm is likely to benefit the rival, at least to a partial extent, due to the possibility of observing and replicating innovations. The incentive of firms to invest is thus lowered if the spillover rate is large, since the rival can free-ride on technological development and obtain a large benefit at no extra cost. To overcome this issue firms have the possibility to sign a cooperative agreement: in this case they form a research cartel and choose the level of investment to maximise the joint level of profits of cartel members. From this kind of model, largely adopted in the literature, it is possible to show that cartels are willing to invest more than non cooperative firms and thus are welfare-improving, provided that the spillover rate is sufficiently large. On the contrary if spillovers are low research cartels invest lower amount of resources and are welfare-reducing.

In chapter 1 I add the element of research risk to the benchmark model. The R&D activity needs not deliver an innovation to the firms, which thus face a certain risk level represented by the negative event of research failure. Hence there are two possible outcomes: the research activity produces either an innovation or nothing. Firms may select the optimal risk rate by choosing among different research projects, which thus differ in their ‘degree of ambition’. The trade-off for greater risk is represented by higher marginal returns: given the level of investment riskier projects might deliver better innovations and higher gains in efficiency. The focus of the chapter is the solution of such trade-off between R&D risk and ambition for profit-oriented firms. I show that such solution depends on the presence of technological spillovers. In general they may serve as an insurance instrument to firms, which can implement a new technology even when own R&D fails provided that the rival’s is successful. If spillovers are sufficiently large, the insurance effect fosters the adoption of riskier research projects. However I show that this argument may be overturned when firms form a research cartel due to the impact of spillovers on investment: cooperative firms prefer to increase their level of investment when spillovers are large and thus are less keen to take greater

risk, contrary to the previous expectation.

Chapter 2 represents the natural evolution of chapter 1: I consider the optimal research project that a firm should undertake from the perspective of a social planner, thus the focus shifts from the private scope to the social one. This model develops upon the previous one by adding a government that sets subsidy policy to affect the decisions of the firms and obtain welfare improvements. In particular it may grant as many types of subsidy as the actions of the firms: one per unit of output supplied at the competitive stage and one per dollar of investment in R&D conducted at the pre-competitive stage. In the paper I show that in the first-best scenario with high spillovers the government desires firms to develop riskier projects due to the spillover-insurance argument, which preserves the average level of efficiency in the industry. Conversely in case of low spillovers safer activities are more likely to be welfare-improving. In the second-best case, where output subsidies are not available to the government and the output level cannot be influenced towards efficient values, simulations suggest that riskier activities are likely to be welfare-maximising also when spillovers are low because they may foster efficiency and hence production.

In chapter 3 I depart from the risk analysis to address strategic trade policy issues concerning research subsidies. The model is simplified relative to the ones adopted in chapter 1 and 2 by removing the element of R&D risk, but is also enriched by the presence of two different countries, say home and foreign, that freely trade in the international market; such model may thus be suited to describe trade policy between two European countries. The main difference between the two countries is the degree of intervention of the respective social planners, because only the home government is allowed to conduct policy and to dispense research subsidies to the domestic firm so as to increase the domestic level of welfare. I explore the relationship between level of subsidy, degree of research cooperation and amount of international spillovers. In general a large level of international spillover is able to transfer knowledge from one country to another, so that the foreign firm may take advantage from the domestic subsidy. Moreover if the two firms sign an international cooperative agreement the domestic subsidy is likely to benefit the foreign firm because the home firm pursues cartelwide profits maximisation. In principle both elements may lower the effectiveness of research subsidy policy by the government. In the paper I show that the optimal level of research subsidy is always non-negative when the two firms carry their research activities in a non-cooperative fashion. When spillovers are large the incentive to invest in R&D is so low that the government is willing to grant a positive subsidy even if it benefits foreign rivals. When firms form an international R&D cartel instead the subsidy is positive only when the spillover rate is sufficiently low. On the other hand, if it is large the government may optimally raise an R&D tax on the domestic firm which still maintains high level of efficiency due to international

cooperation. As a consequence the presence of an international cartel is always welfare-maximising and its creation should be encouraged through suited policy.

At the beginning and the end of each chapter I dedicate specific introduction and concluding remark sections. Moreover the reader may find a chapter-specific appendix to check assumptions and mathematical methods.

Contents

1 Research uncertainty and R&D cooperation: behaviour of the firms	9
1.1 Introduction	9
1.2 Model	12
1.2.1 Product-level competition	16
1.2.2 R&D investment decision	18
1.2.3 R&D mode-of-operation choice	25
1.2.4 Decision over the R&D project	26
1.3 Comparison between the two modes-of-operation and cooperative synergies	30
1.3.1 Research and development	30
1.3.2 Output and profits	32
1.3.3 Cooperative synergies	35
1.4 Concluding remarks	36
1.5 References	38
2 Research uncertainty and subsidy policy: behaviour of the government	40
2.1 Introduction	40
2.2 Welfare analysis	43
2.2.1 State-contingent welfare analysis	44
2.2.2 Welfare stochasticity in first-best	47
2.2.3 Welfare stochasticity in second-best	50
2.2.4 Welfare ranking of R&D-regimes	52
2.3 Policy package to the first-best	55
2.3.1 Output subsidy	55
2.3.2 R&D subsidy	58
2.4 Policy package to the second-best	61
2.4.1 Non-cooperative firms	62
2.4.2 Cooperative agreements	63
2.4.3 Comparison of the subsidies between the two R&D-regimes	64

2.5	Concluding remarks	64
2.6	References	66
3	Subsidy policy in an open economy framework	68
3.1	Introduction	68
3.2	Model	72
3.3	R&D competition	74
3.3.1	Subsidy under R&D competition	77
3.4	R&D cooperation	79
3.4.1	Subsidy to the R&D cartel	82
3.5	Difference between R&D competition and cooperation	84
3.5.1	Investment level	84
3.5.2	Profit level	85
3.5.3	The welfare level	86
3.6	Cost Heterogeneity	87
3.6.1	No-cooperation	88
3.6.2	Cooperation	90
3.7	Subsidy social cost	93
3.8	Concluding remarks	95
3.9	References	96
A	Appendix	98
A.1	Appendix	98
A.1.1	Proposition 1	98
A.1.2	Proposition 2	99
A.1.3	Proposition 4	100
A.1.4	Parameter discussion	100
A.2	Appendix	101
A.2.1	Proof of lemma 1	101
A.2.2	Proof of lemma 2	101
A.2.3	Proof of proposition 5	101
A.2.4	Proof of proposition 6	102
A.2.5	Proof of lemma 4	102
A.2.6	Proof of proposition 7	103
A.2.7	Proof of proposition 9	104
A.2.8	Proof of proposition 10	104
A.2.9	Proof of proposition 11	105
A.2.10	Equation 2.13	106
A.2.11	Equation 2.19	106
A.2.12	Second-order conditions	107
A.3	Appendix	107

A.3.1	Proof of lemma 7	107
A.3.2	Proof of lemma 9	107
A.3.3	Proof of lemma 8	107
A.3.4	Proof of proposition 12	108
A.3.5	Proof of lemma 10	109
A.3.6	Proof of proposition 13	109
A.3.7	Proof of lemma 11	110
A.3.8	Proof of conjecture 1	110
A.3.9	Proof of conjecture 2	111
A.3.10	Proof of conjecture 3	111
A.3.11	Proof of lemma 13	111
A.3.12	Proof of lemma 14	111

Chapter 1

Research uncertainty and R&D cooperation: behaviour of the firms

1.1 Introduction

R&D has represented a set of activities with a rising relevance since the 1990s. As shown by data¹, most countries have been raising the level of investment in research to produce new innovations. This holds true not only for high R&D-intensity industries, but also for mid-intensity ones, supporting the idea that such activities are of strategic importance to the survival of firms.

Due to the growing importance, many authors have developed studies to analyse the relevance of R&D from either an empirical and a theoretical point of view. Jaffe (1989) and Feldman (1994) highlight the fact that R&D does not limit its effectiveness within the firm carrying it, but produces technological spillovers that benefit other firms and institutions; Romer (1986) and Krugman (1991), among others, explore the way in which technological spillovers affects long-run growth; Audretsch and Feldman (1996) show how R&D spillovers affect geographic concentration of industrial activities. Brander and Spencer (1983) explore the strategic interaction among firms which carry research activities in an oligopoly; within the same market structure, Spence (1984) focuses on how technological spillovers influence the incentive to invest in R&D and shows that larger spillovers bring about lower investments due to the possibility of ‘free-riding’ on competitors’ R&D.

A specific area that receives attention by the literature is the organisational-level at which conducting R&D; in general the choice may entail either a within-firm research activity or the formation of an R&D consortium. The original line of thought followed by the American government was to limit any collaboration

¹Tables from the OECD dataset shows, for instance, that R&D expenses in Italy increased from 17'540.341 million USD in 1996 to 26'838.83 million USD in 2015 (at constant prices)

among firms for the development of new research, as it would constitute a (R&D) cartel able to reduce the competitiveness of the market and to increase market power concentration within industries. On the other hand, with the goal of increasing the international competitiveness of firms, the Japanese government in the 1960s encouraged cooperation among firms, copying the system first employed by the British in the early 1920s. As a result, Japanese industries experienced a period of substantial technological growth and became leaders at the worldwide-level, overturning the original skepticism over R&D cartels.

To analyse the Japanese experience, R&D cooperation among firms has received much attention from economic scholars. The first works in this respect consider R&D as a deterministic activity, and tend to address questions such as how firms determine the optimal amount of investment, whether they decide to cooperate and the impact of R&D on their output and profit levels. For instance, Katz (1986) analyses how R&D consortia may foster cooperation synergies; Suzumura (1992) and Ziss (1994) allow for many firms and general demands in Cournot competition; d'Aspremont and Jacquemin (1988) formalise an argument in favour of R&D consortia and conclude that they may be welfare-improving if technological spillovers are sufficiently large. The intuition is that R&D consortia (research joint ventures) are a way to ‘internalize’ technological spillovers and restore optimal levels of investments. These works were naturally extended to investigate public issues as competition policy and welfare analysis.

Afterwards an alternative approach was developed in the literature to take account of the stochastic nature of research (it is not possible ex-ante to know the economic outcome of the research activity). In this family of papers, firms take part in an R&D tournament (race) and select the optimal amount of investment to implement. On this basis, ‘nature’ selects randomly the winner of the R&D race, which is awarded a patent and the possibility to exploit market power for a limited period of time; while the losers become followers in the market. Notable works developing along this direction are Miyagiwa and Ohno (2002), Choi (1993), and Erkal and Piccinin (2010). While the advantage of the deterministic approach is that it allows to relate the quality of innovation to the amount of investment, the stochastic one regards uncertainty surrounding R&D outcomes and relates the investment in R&D with the probability to win the race.

An investment in research and development may be addressed towards different projects which focus on different types of innovation. Each project, by its nature, present a specific level of difficulties in the process towards actual innovation; and, for this reason, has a specific probability of being successful. This fact is relevant when firms decide whether to cooperate in research and development. Empirical evidence, in fact, shows that one of the main driver that pushes firms into joining a research joint venture is the need to manage the risk associated to R&D; and that the number of R&D cartels is growing, in spite of the fact that

many of them fail because they do not reach their goal; the motivation of failure may be represented by many factors, as, for instance, impossibility to obtain an innovation or impossibility to sell it on the market. In reality, R&D cartels are seen as instruments by which firms may share the risk and cost associated to R&D, and thus the decision over the R&D project may have an impact in this respect.

In this paper I combine the approach belonging to the classic set-up with the assumptions of stochastic R&D in order to explore how firms select the R&D project to undertake. More specifically, I develop a model of duopoly à la d'Aspremont and Jacquemin in which firms compete over the quantity to produce. In order to increase their efficiency, firms undertake investments in research and development and choose the optimal R&D project on the basis of the risk associated to it. The size (or quality) of the innovation implemented depends both on the amount of money invested and on the riskiness of the project (if it fails, no innovation is delivered; if it is successful, higher levels of risk relate to larger innovations). The industry is organised as an ex-ante symmetric duopoly, meaning that all firms have equal pre-investment marginal cost and that their behaviour responds to strategic considerations; in this respect Leahy and Neary (1997) disentangle the effect of strategic behaviour on the implications of the model. If the research activity is successful, firms are able to implement the innovation, the quality of which is measured as a reduction in the level of marginal cost; the paper refers thus to process innovations, although Spence (1984) argues that any innovation is ultimately cost-reducing; although present ex-ante, symmetry in the industry is not guaranteed ex-post (even if both firms invest the same amount of money) due to the assumption of R&D uncertainty. In line with the literature, R&D produces knowledge which benefit all firms in the industry. In other words, the research activity of a firm, if successful, is also beneficial to the competitor, the marginal cost of which is reduced by a term depending on the amount of investment in R&D of the former firm and the level of externality in the industry. Finally, firms may choose to carry their research either non-cooperatively or cooperatively within a research joint venture (the size of which is trivially industry-wide, being this a duopoly; other works as Poyago, 1995, and de Bondt, 1997, extend the argument to a n-firm oligopoly and study the equilibrium size of research joint ventures).

The contribution of this paper is three-fold. First, I regard the process by which firms select the research project to undertake. More specifically, I introduce the risk component into the benchmark model proposed by classic literature on the topic of research and development. I derive an optimally negative relationship between the risk associated to the project and the amount of innovation which the firm aims at implementing, which means that profit-maximising firms may produce better innovations by embarking on riskier activities without increasing

the amount of investment. Firms may have incentive to increase the risk level in replacement of higher investments. The model suggests that firms are more risk-averse for large values of technological spillovers and, consistently, embark on safer activities; and that R&D consortia are generally more keen towards risky projects.

Second, I analyse the difference between R&D cooperation and competition under the point of view of firms. A result obtained in the paper is that the expected level of profits is generally higher under cooperation, which thus represents the equilibrium of the model. However, profits are shown to depend on the outcome of research activity and R&D competition may turn out to be more profitable than cooperation in some state of the world. This consideration extends the result obtained by the deterministic R&D literature (d'Aspremont and Jacquemin, 1988 and Leahy and Neary, 1997, 1999, 2005, 2007), in which cooperation is always the more profitable option for firms; the interpretation for this fact is that research joint ventures serve as partial surrogate for product-market collusions.

Third, I run some simulation to understand how the incentive to form a cooperative agreement with other competitors is affected by the riskiness of R&D. The relevance of this analysis is due to empirical evidence, in which risk is perceived by firms as a critical element for promoting R&D cooperation. In my paper, though cooperation is always more profitable than no-cooperation, the level of profitability varies with respect to the level of R&D risk. In particular, simulations seem to suggest that a greater risk of failure in R&D enhances the incentive to form a research joint venture only when spillovers are sufficiently large, whereas it has a negligible impact for low values of spillovers.

The structure of the paper is as follows. Section 2 presents the duopoly model and analyse the characteristics of the possible market equilibria. In section 3, I compare the outcomes of R&D cooperation vis-à-vis no-cooperation, highlighting the differences both at the product-level and at the research-level; moreover, I explore the relationship between higher R&D riskiness and the incentive to form R&D consortia. Section 4 concludes.

1.2 Model

In this section I consider a model à la d'Aspremont and Jacquemin (1988) augmented with innovation uncertainty. It is a model of partial equilibrium in which two firms compete oligopolistically over the quantity to produce, and in which firms have the possibility to conduct a research and development activity to implement new innovations. The oligopolistic framework allows to take into account the strategic interaction that occurs among firms when deciding both the activity

in R&D and the production of goods, and to draw conclusions surrounding the motives underlying the adoption of a certain research activity.

Consider a Cournot-duopoly that produce a homogeneous good. The demand for the product is assumed to be linear

$$p = a - by \quad (a, b > 0) \quad (1.1)$$

where p is the price and y is the total quantity produced in the industry.

In the production of the good firms bear a marginal cost c , which is constant and does not depend on the output level. Such marginal cost relates to the use of an input (e.g. labour) assumed to be employed in homogeneous quantities by all firms; this symmetry reflects in an industry-wide homogeneous marginal cost equal to \bar{c} . In order to increase their efficiency and decrease the level of the marginal cost, firms can decide to invest money in R&D to develop a process innovation. However, the outcome of research and development is not predictable in advance; R&D is a stochastic activity and its realization into actual innovation is not guaranteed *a priori*. If the R&D activity delivers an innovation, the marginal cost decreases according to the ‘quality’ of the innovation itself. In this respect, I define x as such quality and α as the probability that an R&D project be successful; in the negative case in which the research activity does not deliver any innovation, I assume the returns are simply equal to zero. Within this framework, firms may choose not only the amount of money K to invest on research and development, as in the classic literature, but also the project to undertake and thus level of R&D risk to bear². To keep the analysis tractable I assume that only one project can be selected or, in other words, that firms cannot differentiate their investment.

The choice of the best project is not always driven towards the least risky. In fact, selecting riskier projects may pay off with better (of higher quality) innovations (even keeping constant the amount of money invested) and give the firm an edge against the competitor. In other words, embarking on riskier projects may increase the returns to the level of investment in R&D. In analytical terms, I assume the following relationship between returns on quality x and R&D investment K :

$$x = \frac{\sqrt{K}}{\Gamma(\alpha)} \quad (1.2)$$

where $\Gamma(\alpha)$ is a positive and increasing function with respect to the probability α of realization of the project. According to this relationship, returns on quality fall if firms select higher levels of α , i.e., with safer projects, and rise with the amount

²The choice of the ‘direction’ of the R&D appears in other contributions, as Kamien and Zang (2000), Molto et al. (2005) and Weithaus (2005), as affecting a firm’s absorptive capacity, a thorough discussion of which may be found in Leahy and Neary (2007)

of investment in R&D. Therefore, firms could in principle increase the level of quality of their innovation by selecting a lower level of α and keeping constant the amount of K . Differentiating equation 1.2 with respect to K gives the expression for marginal returns on quality to the level of investment:

$$\frac{\partial x}{\partial K} = \frac{1}{\Gamma(\alpha)} \frac{1}{2\sqrt{K}} \quad (1.3)$$

Such equation displays the relationship for marginal returns, which are clearly positive and decreasing with respect to the level of K ³. The level of α affects negatively marginal returns: they fall as firms select safer R&D projects. For simplicity, $\Gamma(\alpha)$ is assumed to be linear: $\Gamma(\alpha) = m\alpha + q$, where m is the slope of the function and q the vertical intercept⁴. Equation 1.3 describes a trade-off between safer project and better innovations: by embarking on safer project, marginal returns are lower and the innovation delivered present a lower quality, given the level of investment K ; α becomes therefore a control variable that firms select in an optimal way.

As observed by many authors⁵, an activity of R&D does not benefit only the researching firm. On the contrary, it produces knowledge that cannot be confined within the firm and in fact spills over within the industry (and more likely across different sectors as well) delivering competitors a costless advantage. In this respect, technology is similar to a public good, the ‘consumption’ of which cannot be totally privatised. For this reason, each firm benefits from the flow of knowledge coming from the rival’s R&D. The absorption of such knowledge concerns a fraction of the information produced through research and development. For the sake of simplicity, let us assume that this fraction be constant and equal to $\beta \in [0, 1]$ for both firms⁶. With this additional assumption, it is possible to define the expected value of the marginal cost for firm $i \in \{1, 2\}$

$$E(c_i) = \bar{c} - \alpha_i x_i - \beta \alpha_j x_j \quad (i \neq j) \quad (1.4)$$

³An economic justification for this case is in fact provided by the literature (see for example d’Aspremont and Jacquemin, 1988 and Dasgupta, 1986), as there is no evidence by which R&D activities exhibits economies of scale to the size of the firm undertaking R&D on innovative outputs.

⁴The role of the coefficient q is crucial in the analysis: as α get close to zero, returns to the investment grow rapidly to infinity provided $q = 0$ (see equations 1.2 and 1.3), rendering riskier projects always more profitable than safer projects (with α close to unity and finite returns), whereas a positive level of q is able to cushion this effect.

⁵See for example Kats (1986), d’Aspremont and Jacquemin (1988) and Leahy and Neary (1997).

⁶Leahy and Neary (2007) drop this assumption and explore how firms change the level of investment in R&D in the case in which it is able to raise their absorptive capacity, i.e., the amount of spillover they receive.

where α_i and x_i are respectively the R&D success rate and the innovation quality of firm i and the subscript j refers to firm i 's competitor. The pre-investment expected value of the marginal cost of firm i decreases with respect to the expected value of the quality of its own innovation and to the one of the competitor's (to the extent of the fraction represented by the spillover coefficient β). Given this notation, I can define the profit function of firm i as follows

$$\Pi_i = (p - c_i)y_i - K_i \quad (1.5)$$

where y_i represents its output level and K_i is its investment in research and development. Note that, by investing in R&D, a firm trades off higher fixed costs for a lower marginal cost which, in turn, allows for larger output levels. In this way, it pursues an 'aggressive' strategy with the aim of increasing its market share and of decreasing the one of the rival.

Finally, firms decide the organizational level at which it is convenient to conduct research and development. In addition to doing so at the natural firm-level, they can form an R&D consortium⁷ as so to coordinate efforts and share the results of basic research (but not the eventual innovation). In this case, firms are assumed to conduct their research in their own laboratories so to rule out any synergy-effect, in line with most contributions in the literature.

The model proposed throughout this paper has five stages, as shown in figure 1.1. In the first stage, firms select the R&D project to undertake. In particular, let us assume there exist two possible projects with different probability of realization, say $\bar{\alpha}$ and $\underline{\alpha}$ ($\bar{\alpha} > \underline{\alpha}$); thus, the project associated to $\bar{\alpha}$ represents the safer project, while the other one is the riskier. For semplicity, let us further assume the safer project relies on existing technologies and for this reason has a unit probability of realization $\bar{\alpha} = 1$; and that $\underline{\alpha} = 1 - \gamma$, where $\gamma > 0$ represents the risk parameter: the higher the value of γ , the riskier the project associated to $\underline{\alpha}$. After deciding the level of risk of their R&D activity, in stage two firms decide the level at which conducting R&D - firm-level or R&D consortium -; in the latter case firms select the amount of investment in R&D as to maximise the consortium-level revenues, rather than optimizing own profits. Let us remark that the cooperative solution concerns R&D activity only, and any collusion in the product market to keep low the level of quantity and high the level of price is ruled-out⁸. In stage three, firms choose the optimal amount of R&D investment K to maximise the level of profits (according to the decision taken at the second stage). In stage four, the model acquires a stochastic behaviour; this is the stage in which 'nature' decides whether the R&D activity turns into actual innovation and, therefore, into a reduction of

⁷Being this a duopoly, the R&D consortium must be at the industry-wide level, but smaller configurations are analysed in the literature (see for instance Poyago, 1995).

⁸In line with the literature, I assume that any collusive behaviour at the product-level is forbidden by law.

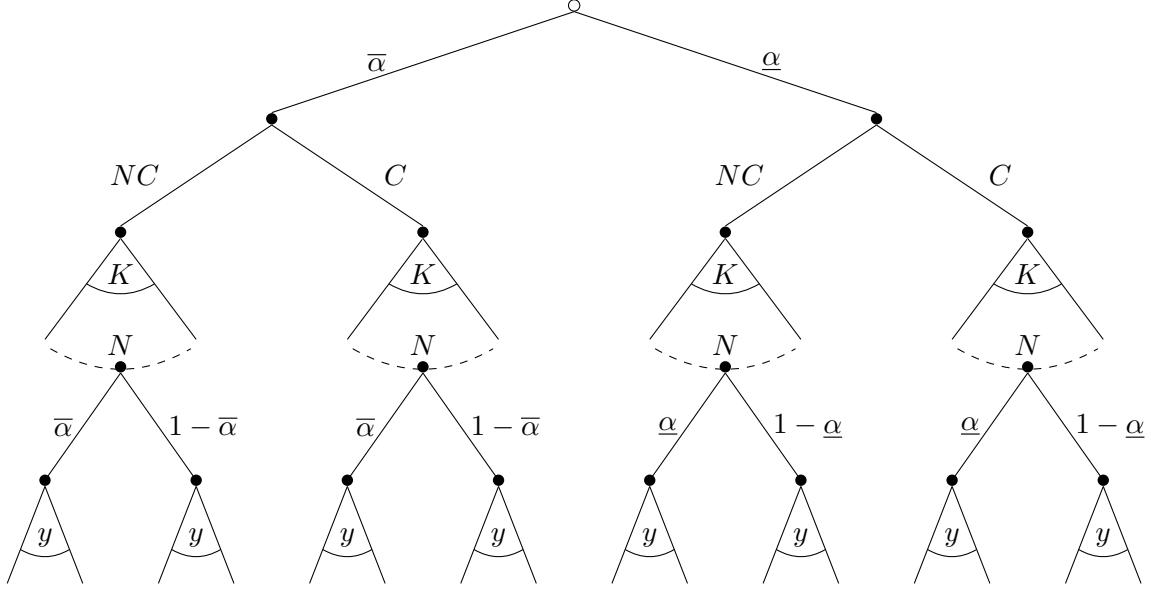


Figure 1.1: Extensive form game

the marginal cost (this happens with probability $\alpha \in \{\bar{\alpha}, \underline{\alpha}\}$). Finally, in stage five firms compete over the quantity of the homogeneous good to provide.

1.2.1 Product-level competition

I focus on the existence of subgame-perfect equilibria and, thus, I shall solve the model through backward induction. In the final stage, each firm maximises the level of own profit; the problem for firm i is as follows

$$\begin{aligned} \max_{y_i} \quad & \Pi_i = (p - c_i)y_i - K_i \\ \text{s.t.} \quad & p = a - by \end{aligned} \tag{1.6}$$

where the first order condition is

$$\frac{\partial \Pi_i}{\partial y_i} = p - c_i - by_i = 0 \tag{1.7}$$

Clearly, the level of marginal cost of firm i depends on the successfulness of the R&D activity conducted within the whole industry (through the working of spillovers); and thus the amount of output provided to the market depends on the realization of the stochastic process in stage four. In particular, there exist four possible states of the world: (I,I), (I,O), (O,I), (O,O), where the first component

refers to firm i and the second to firm j , and in which the letter I represents the positive event of innovation, while the letter O represents the negative event in which the R&D project fails to deliver any cost reduction. In case (I,I), the structure of marginal costs for both firms are as follows:

$$\begin{aligned} c_i^{(I,I)} &= \bar{c} - x_i - x_j \beta \\ c_j^{(I,I)} &= \bar{c} - x_j - x_i \beta \end{aligned} \quad (1.8)$$

Solving for both firms we obtain the following equilibrium output:

$$y_i^{(I,I)} = \frac{a + c_{-i} - 2c_i}{3b} = \frac{a - \bar{c} + x_i(2 - \beta) + x_j(2\beta - 1)}{3b} \quad (1.9)$$

As in standard Cournot models, one firm's output depends positively on competitor's marginal cost and negatively on own marginal cost. In the case (I,I), in particular, the level of output of firm i depends positively on both its own innovation x_i and the competitor's x_j - through the effect of spillovers. The derivative of 1.9 with respect to x_i establishes the optimal relationship between output level and investment in R&D by firm i :

$$\frac{\partial x_i}{\partial y_i} = \frac{3b}{2 - \beta} \quad (1.10)$$

which is always greater than zero.

Remark 1. *If the output level grows, firms have incentive to increase the investment in research and development activities for every value of the spillover parameter β .*

Analogously, the cost structures in the other cases for firm i are:

$$\begin{aligned} c_i^{(I,O)} &= \bar{c} - x_i \\ c_i^{(O,I)} &= \bar{c} - x_j \beta \\ c_i^{(O,O)} &= \bar{c} \end{aligned} \quad (1.11)$$

by which I can derive the equilibrium output levels for the other three states of the world:

$$\begin{aligned} y_i^{(I,O)} &= \frac{a - \bar{c} + x_i(2 - \beta)}{3b} \\ y_i^{(O,I)} &= \frac{a - \bar{c} + x_j(2\beta - 1)}{3b} \\ y_i^{(O,O)} &= \frac{a - \bar{c}}{3b} \end{aligned} \quad (1.12)$$

For the Cournot-equilibrium to be stable, a sufficient and necessary condition (see Seade, 1980) in a duopoly is

$$\frac{\partial^2 \Pi_i}{\partial y_i^2} + \frac{\partial^2 \Pi_i}{\partial y_i \partial y_{-i}} < 0$$

that is, the direct effect on marginal profits of own quantity plus the cross effect on marginal profits of competitor's quantity must be negative. In this work, this is equivalent to

$$\frac{\partial^2 \Pi_i}{\partial y_i^2} + \frac{\partial^2 \Pi_i}{\partial y_i \partial y_{-i}} = -3b$$

so the stability condition holds for any $b > 0$.

1.2.2 R&D investment decision

In stage three, each firm calculates the optimal amount of resources to address to R&D: greater levels of investment increase the amount of fixed cost with a negative impact on profits, but, at the same time, reduce in expected terms the level of marginal cost by producing better innovations, delivering a positive increase in revenues. Firms take this double-edge into account in their profit maximisation to select the optimal level of K . Given the level of α , there exists a one-to-one relationship between the level of returns on quality and the level of investment K , and for this reason I can solve the optimisation problem for the value of x . From equation 1.2 I obtain the following expression for K :

$$K = [x(m\alpha + q)]^2 \quad (1.13)$$

To understand the relationship between x and α , it is convenient to calculate the total derivative and impose $dK = 0$ as to obtain:

$$\frac{dx}{d\alpha} = -\frac{mx}{m\alpha + q} \quad (1.14)$$

which is always negative. This relation illustrates the trade-off, expressed in terms of the ‘marginal rate of transformation’, between innovation quality x and R&D success rate α ; as it can be seen from the equation, higher levels of x increase (in absolute value) the marginal rate of transformation, while larger values of α reduce such rate. Equation (1.14) means that the adoption of riskier projects (lower α) leads to better innovations (larger x) given the level of investment. Clearly, the optimal solution of this trade-off stems from the profit maximisation problem faced by the firm; the objective function depends on the R&D mode-of-operation chosen at the upper stage: in the non-cooperative regime firms maximise own profits, whereas in the cooperative solution the efforts are brought together to increase the value of the consortium-level profit.

‘No-cooperation’ equilibrium

Let me start with the analysis for firm i in the ‘no-cooperation’ equilibrium, in which firms maximise their own level of profits; since they do not know the realization of the state of the world, the objective function is expressed in expected terms:

$$\max_{x_i} E[\Pi_i] = \left[\sum_{s \in S} pr(s)(p^s - c_i^s)y_i^s \right] - K_i \quad (1.15)$$

subject to the holding of the output first-order condition (1.7) calculated at a lower stage and for this reason considered in the profit-maximisation process⁹. $E[\Pi_i]$ represents the expected value of profits, the superscript s indicates that variables are calculated in that specific state of the world and $pr(s)$ is its probability of realization. The constraint of the problem can be rearranged and plugged into the profit function, as to obtain:

$$\max_{x_i} E[\Pi_i] = \left[\sum_{s \in S} pr(s)b(y_i^s)^2 \right] - K_i \quad (1.16)$$

From this problem I obtain the following first-order condition with respect to x_i :

$$2\alpha_i \frac{2-\beta}{9b} [a - \bar{c} + x_i(2-\beta) + \alpha_j x_j(2\beta-1)] = 2x_i(m\alpha_i + q)^2 \quad (1.17)$$

At the left-hand sides is represented firm i ’s marginal benefit from R&D, while the respective marginal cost is isolated at the right-hand sides. By rearranging the equation we get an expression for x_i :

$$x_i = \frac{a - \bar{c} + \alpha_j x_j(2\beta-1)}{\frac{9b(m\alpha_i+q)^2}{\alpha_i(2-\beta)} - (2-\beta)} \quad (1.18)$$

where both the term $(a - \bar{c})$ and the denominator must be larger than zero in order for x_i to be a positive quantity. This condition represents the reaction function of x_i with respect to x_j . It is convenient to understand the kind of relation that exists between the decisions made by the two firms. Looking at the reaction function, it is easy to notice that the value of the spillover coefficient β determines the sign of the effect of x_j on x_i : there exists a critical value of β (equal to 1/2) below which any increase in the value of x_j shifts downwards the reaction function, and above which the reaction function shifts upwards. As I will show more clearly by a graphical analysis, if $\beta < 1/2$ the investments in R&D of the two firms

⁹Similar applications are present in the literature: for instance, Leahy and Neary (1997, 1999, 2005, 2007), and d’Aspremont and Jacquemin (1988) on R&D; and Leahy and Montagna (2017) on firm optimal vertical size.

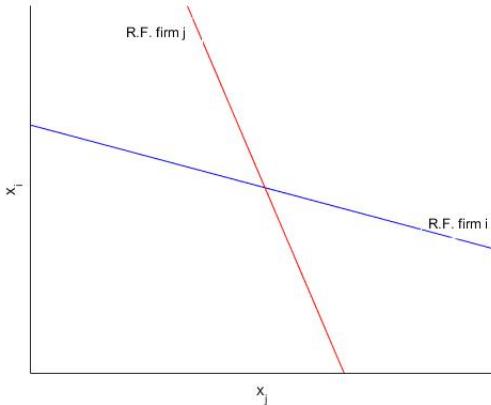


Figure 1.2: Reaction function in 'no-cooperation' for low values of the spillover (picture drawn at $\beta = 0.25$)

are strategic substitutes: any increase in the investment by one firm reduces the incentive to invest of the other firm; by contrast, if $\beta > 1/2$ the investments of the firms are strategic complementary.

Figures 1.2 and 1.3 offer a graphical outlook of the reaction functions; the level of innovation quality selected by firm j is indicated on the horizontal axis, while the one chosen by firm i is on the vertical axis. Figure 1.2 illustrates the relation for a low value of the spillover parameter ($\beta = 0$, lower than the critical value 0.5), to represent an industry in which the knowledge produced by R&D remains mainly within the firm in which such activity is carried. Looking at the figure, let us imagine that firm j increase the investment in R&D, so to increase the value of x_j . Firm i must decide how to respond to such strategy. If firm j is successful at innovating, its marginal cost will drop by a larger value, and consequently it will acquire a leading position in the market: firm j will be awarded a portion of firm i 's market share. In addition, this has a lowering effect on the price level and on the output level of firm i (see equation 1.9), which in turn drives down its optimal investment in R&D by remark 1. Taking account of this, firm i 's best response is to pursue a minor innovation as so to decrease the impact of investment on profits. For this reason, if the spillover value is sufficiently low (lower than 0.5) the R&D actions of the two firms are substitutes (as represented in figure 1.2, the curves are negatively inclined).

Provided that the spillover parameter is high, the R&D activities implemented by each firm do not bring advantages only to the firm itself, but spreads out into other firms within the industry (and possibly across different sectors, as analysed by Leahy and Neary, 1999); figure 1.3 depicts this kind of situation. A firm's

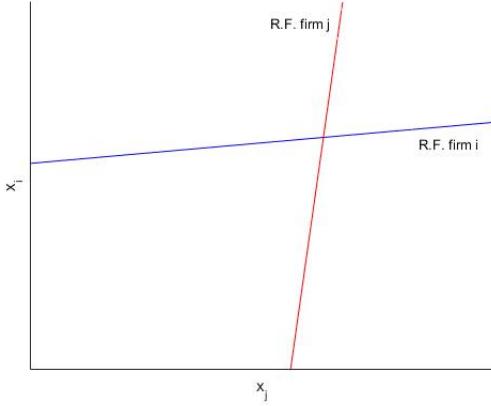


Figure 1.3: Reaction function in 'no-cooperation' for large values of the spillover (picture drawn at $\beta = 0.75$)

private incentive to invest in R&D is low¹⁰, since all competitors receive large spillovers at zero cost and may choose to 'free-ride' on technological development. Let us now suppose that firm j decide to reduce the level of investment and to free-ride on R&D; the investment-drop embeds a fall in x_j , which produces a decrease not only in y_j , but also in y_i due to the large spillover effect on firm i 's marginal cost. In this case, firm i 's best response is also to cut the level of investment (remark 1) so to limit free-riding from firm j . Thus, the R&D actions of the firms are now strategic complementary, as shown in figure 1.3 (the curves are positively sloped).

In order to obtain an expression for x_i as a function of α_i and α_j , I can analogously derive the expression for x_j and then plug it into equation 1.18. The expression I get is the following:

$$x_i^N = \frac{(a - \bar{c}) \left[\frac{9b(m\alpha_j + q)^2}{(2-\beta)a_j} - (2 - \beta) \right] + (2\beta - 1)\alpha_j(a - \bar{c})}{\left[\frac{9b(m\alpha_j + q)^2}{(2-\beta)a_j} - (2 - \beta) \right] \left[\frac{9b(m\alpha_i + q)^2}{(2-\beta)a_i} - (2 - \beta) \right] - \alpha_j\alpha_i(2\beta - 1)^2} \quad (1.19)$$

where the superscript N stands for the 'no-cooperation' regime. Equation 1.19 expresses a closed form for the amount of quality of innovation that firms have incentive to pursue, determined as a function of the parameter α_i and α_j . Clearly, if the two firms adopt the same riskiness of R&D ($\alpha_i = \alpha_j$), the value of x_i is equivalent to the x_j 's and the equilibrium is symmetric. Away from this situation, different levels of risk by the firms correspond to asymmetric values of innovation quality. Moreover, it can be proved the following (see the appendix):

¹⁰See Spence (1984) for a theoretical discussion

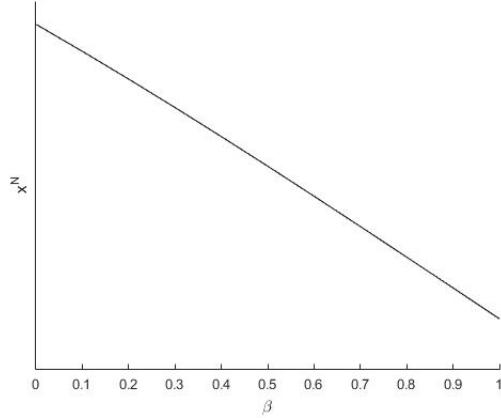


Figure 1.4: Behaviour of x with respect to β in the non cooperative regime

Proposition 1. *In ‘no-cooperation’ the quality of innovations and the R&D project success rate are in a negative relationship, i.e., they are strategic substitutes, provided that the success rate is not excessively close to zero.*

The adoption of a safe project reduces the marginal returns to the investment level and renders costly for a firm to pursue high-quality innovations. For this reason, a high success rate corresponds to the implementation, provided the research activity is successful, of minor innovations.

To relate this work with other in the deterministic R&D literature, it is opportune to understand the kind of relationship between the level of innovation x and the amount of spillovers β . To obtain a description of the relationship, I employ a graphical representation of equation 1.19 given the level of the success rates $\alpha_i = \alpha_j = 1$. As it can be seen from figure 1.4, the graph is decreasing, which means that an increase in the amount of spillovers produces a fall in the quality of the innovation to implement. The reason is the possibility for rivals to free-ride on technological development: as the value of β rises, the research activity conducted by a firm benefits greatly all firms in the industry and for this reason does not open a significant gap among the firm and the competitor. As a result, for large spillovers decreases the incentive to invest into high-quality innovations.

Remark 2. (*Regularity conditions*) *The equilibrium arising from the strategic interaction of the two firms has the properties of (i) uniqueness (and existence), that is, there is only a possible outcome stemming from the market; (ii) stability, by which firms have no incentive to deviate from the equilibrium strategy¹¹.*

¹¹For a thorough analysis on the stability of equilibria in this kind of models see Henrique (1990)

The holding of these property is required to allow a global comparison among different equilibria, without restricting the analysis to local comparisons.

‘Cooperation’ equilibrium

In the ‘cooperation equilibrium’ firms aim at maximising the revenues at the consortium-level; more specifically, the problem for firm i is as follows

$$\max_{x_i} E[\bar{\Pi}] = \left\{ \sum_{s \in S} pr(s)b \left[(y_i^s)^2 + (y_j^s)^2 \right] \right\} - K_i - K_j \quad (1.20)$$

where $E[\bar{\Pi}]$ indicates the expected level of joint profits of the two firms. The first-order condition for this problem is

$$\frac{\alpha_i}{9b}[(a - \bar{c})(1 + \beta) + x_i(5\beta^2 - 8\beta + 5) + 2\alpha_j x_j(2 - \beta)(2\beta - 1)] = x_i(m\alpha_i + q) \quad (1.21)$$

where at the left-hand side is expressed the marginal benefit arising from greater investment in R&D, whereas the right-hand side represents its marginal cost; the left-hand side of this equation takes account not only of the benefit of firm i , but also of its rival’s: unlike the non cooperative case, in the cooperative regime firms aim at maximising the *social* benefit of R&D. Rearranging the first-order condition I obtain the following reaction function:

$$x_i = \frac{(a - \bar{c})(1 + \beta) + 2\alpha_j x_j(2 - \beta)(2\beta - 1)}{\frac{9b(m\alpha_i + q)^2}{\alpha_i} - (5\beta^2 - 8\beta + 5)} \quad (1.22)$$

The investment in quality of firm i depends on the investment of the competitor according to the amount of spillovers: if $\beta < 1/2$, investments in R&D by the two firms are strategic substitutes (see figure 1.5), i.e., an increase of the one determines a decrease of the other, while they are strategic complementary if $\beta > 1/2$ (see figure 1.6); and if β is exactly equal to the threshold value of $1/2$, the investment of one firm is independent of the other’s; the interpretation of this result is analogous with the one presented in the ‘no-cooperation’ case and therefore not further discussed in details.

Plugging the condition for x_j (obtained from firm j first-order condition) into equation 1.22 delivers:

$$x_i^C = \frac{\left[\frac{9b(m\alpha_j + q)^2}{\alpha_j} - (5\beta^2 - 8\beta + 5) \right] (a - \bar{c})(1 + \beta) + 2\alpha_j(a - \bar{c})(2\beta - 1)(2 - \beta)(1 + \beta)}{\left[\frac{9b(m\alpha_i + q)^2}{\alpha_i} - (5\beta^2 - 8\beta + 5) \right] \left[\frac{9b(m\alpha_j + q)^2}{\alpha_j} - (5\beta^2 - 8\beta + 5) \right] - 4\alpha_j\alpha_i(2\beta - 1)^2(2 - \beta)^2} \quad (1.23)$$

where the superscript C stands for the cooperative equilibrium; clearly, the equilibrium arising from the intersection of the two reaction functions is symmetric

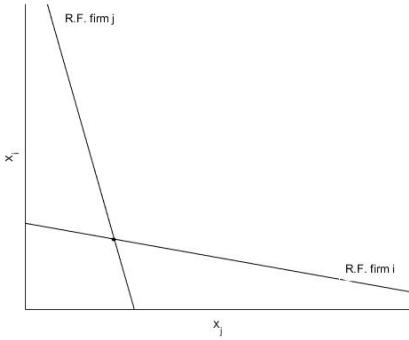


Figure 1.5: Reaction function in 'cooperation' for low values of the spillover (picture drawn at $\beta = 0.25$)

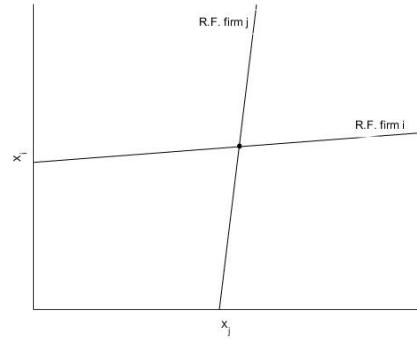


Figure 1.6: Reaction function in 'cooperation' for large values of the spillover (picture drawn at $\beta = 0.75$)

if the two firms select the same R&D project, otherwise the firms will undertake different levels of investment in research activities. As proved in the appendix, the following statement holds:

Proposition 2. *In 'cooperation' the quality of innovations and the R&D project success rate are in a negative relationship, i.e., they are strategic substitutes, provided that the success rate is not excessively close to zero.*

In the cooperative regime, as well as in the non cooperative, selecting riskier project entails the possibility to pursue better innovations; the trade-off between the two variables x and α holds not only given the level of investment, as analysed in equation 1.14, but generally along the optimal path determined by the first-order conditions, and firms in an *optimal* manner requires risky project to implement high-quality innovations. In other words, investments in R&D can grow very large only with risky activities, since safe projects deliver too low returns and make the investment in quality too costly.

From equation 1.23 I can analyse the kind of relationship existing between innovation quality and spillover coefficient; figure 1.7 is depicted by imposing $\alpha_i = \alpha_j = 1$. From a graphical inspection, the relationship between x and β is rising (unlike the 'no-cooperation' case): an increase in the value of spillovers determines an increase in the investment into higher innovation quality. The aim of cooperation, in fact, is to internalize the presence of technological spillovers by maximising the benefits at the industry-level, rather than at the firm-level. In this case, the greater β , the greater the benefit received by all firms, and therefore the greater the incentive to invest in R&D. As a result, the graph (β, x) assumes an increasing pattern.

The equilibria arising from the two regimes ('cooperation' and 'no-cooperation')

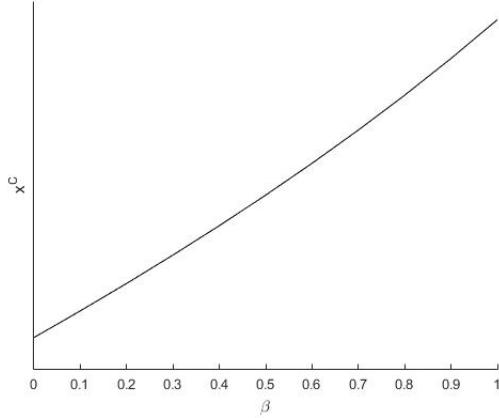


Figure 1.7: Behaviour of x with respect to β in the cooperative regime

are generally different, as will be shown later, and therefore the adoption of the regime affects the quality of the innovation to develop. The cooperative equilibrium presents the regularity conditions of uniqueness and stability as the non cooperative one, making thus possible a global comparison between the two regimes.

1.2.3 R&D mode-of-operation choice

In stage two, firms must decide whether forming an R&D consortium to carry the research and development activity; if so, they coordinate their efforts as so to maximise the joint level of profits. As proposed by d'Aspremont and Jacquemin (1988), such R&D regime is strictly correlated to the presence of technological spillovers in the industry. The larger the value they assume, the higher the incentive to free-ride on R&D, and the lower the level of investment carried by the firms. More technically, the issue lies in the fact that the private benefit from R&D is generally different from the social benefit, defined as the sum of the private benefits of each firm in the industry (more precisely, of any other firm which receives the technological spillover). In such framework, constituting an R&D consortium is a way for participating firms to ‘internalize’ the effect of spillovers and choose the socially optimal level of R&D. The choice between the two regimes depends on a comparison of the respective level of expected profit, which is equal to

$$E[\Pi_i^r] = b \sum_{s \in S} \{pr(s) [(y_i^{s,r})^2]\} - K_i^{s,r}$$

where the superscript $r \in \{N, C\}$ indicates the equilibrium values in the specific R&D mode-of-operation.

The amount of output and of investment in each regime depends on the R&D

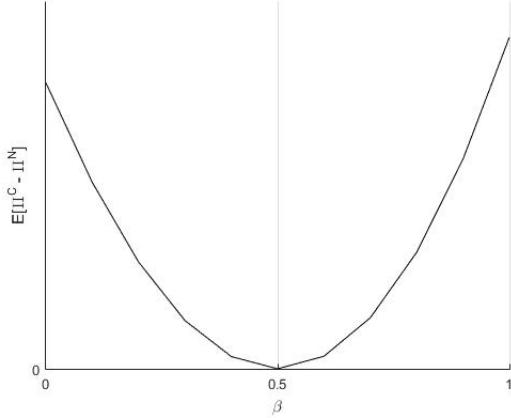


Figure 1.8: Difference of the expected profit levels in the two regimes $\Pi^N - \Pi^C$ with respect to technological spillovers

project selected by the two firms. In principle, they can both select the safe project, corresponding to the equilibrium $(\bar{\alpha}, \bar{\alpha})$, the risky project, associated to the equilibrium $(\underline{\alpha}, \underline{\alpha})$, or different projects, related to mixed equilibria. Figure 1.8 allows a graphical comparison between the expected profits in the two different regimes; the comparison over the profit of each R&D regime is based on the equilibrium $\alpha_i = \alpha_j = 1$ (deterministic R&D), but can be extended to every configuration of risk (stochastic R&D). On the horizontal axis is listed the amount of spillover β , whereas on the vertical axis is represented the difference between the expected profit under a consortium and the one in ‘no-cooperation’. The first observation concerns the fact that the function is positive for every value of the spillover β ; this allows to conclude that the cooperative solution is always more profitable than the non cooperative one, as in Leahy and Neary (1997). Second, the graph follows a falling trend for low values of the spillover coefficient, reaches the horizontal axis at $\beta = 1/2$ and tends to rise as spillovers grow large. This means that only at the critical value $\beta = 1/2$ the two regimes embed the same level of expected profits, and firms are indifferent between the two modes of operation; otherwise cooperation represents a profitable solution for firms. In other words, the decision of cooperation weakly dominates the non cooperative equilibrium for every value of the parameter β and is always chosen by the firms.

1.2.4 Decision over the R&D project

Finally, firms decide the R&D project to undertake to maximise their level of profit. They can pick between two possibilities: a safer one, associated to the probability of realization $\bar{\alpha}$, and a riskier one, related to the probability $\underline{\alpha}$. For

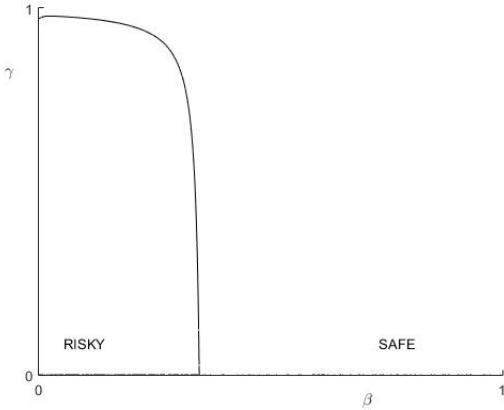


Figure 1.9: Isoprofit of firm i when firm j selects the safe project

simplicity, I shall assume that the ‘safe’ project relies on the exploitation of existing technologies and, for this reason, may be assumed to be risk-free ($\bar{\alpha} = 1$). In this way I can conveniently define the probability of the risky project $\underline{\alpha}$ as the difference between one and a parameter γ , that measures the degree of risk of the project ($\underline{\alpha} = 1 - \gamma$). The higher γ , the higher the probability of failure of the risky project. The existence of this parameter allows to conduct a comparative statics analysis to assess the impact of risk over the decisions of the firms, as will turn useful in the following.

Firms select the optimal project by evaluating which one is profit maximising. In the assessment each firm considers the behaviour of the other. In particular, four configurations may in principle arise: both firms choose the safe project, or the risky one, or they select different projects. To analyse all cases, let us consider firm i decision given the behaviour of firm j .

First, let us consider the case in which firm j chooses to undertake the safe project. The problem for firm i is to assess the profit-maximising behaviour. Being this a binary choice, firm i selects the safe project if and only if:

$$E[\Pi_i(\alpha_i = 1; \alpha_j = 1)] > E[\Pi_i(\alpha_i = 1 - \gamma; \alpha_j = 1)] \quad (1.24)$$

where the left-hand side of the inequality is the level of expected profits that firm i receives if it undertakes the safe project and the right-hand side represents the one in the risky-project case, provided that firm j selects the safe activity; the profit of the firm is associated to the cooperative regime for what concluded in the previous section. Figure 1.9 depicts the isoprofit line with respect to the parameters γ and β to analyse the behaviour of firm i . The area at the left-hand side represents the set of parameters (β, γ) by which firm i prefers to opt for the risky project. In general, the higher the level of spillovers, the more

profitable becomes to select the safe activity. A possible interpretation for this result may be the fact that high levels of risk are borne by the firm in order to obtain an advantage in terms of marginal cost relative to the rival's; however, if β is sufficiently large the result of the research activity is spread across all firms in the industry, limiting the possibility to create such positive competitive gap, and firms are less keen on embarking on risky activities. Moreover, let us observe that the function is rigid with respect to the risk level for a large range of the parameter γ . In fact, any increase in the risk parameter is offset by higher marginal returns to the investment in R&D; thus, it does not reduce the incentive towards the risky project. However, the line becomes more elastic as γ assumes greater values, as the returns may, at best, only partially compensate for the greater amount of risk (in fact they should grow infinitely large for a full compensation). Therefore, given the level of β , if γ grows close to unity the firm has more incentive to choose the safe R&D activity.

Remark 3. *The incentive towards the risky project is largely determined by the spillover parameter: if it is sufficiently large, the firm selects the safe activity. Moreover, the safe project might be optimal also at low values of β provided that the risk parameter γ is sufficiently large (close to one).*

The description provided applies also to the case in which firm j decides to embark on the risky activity. In this case the problem faced by firm i to assess optimal behaviour consists of comparing the levels of profit obtained in the two alternatives as follows:

$$E[\Pi_i(\alpha_i = 1; \alpha_j = 1 - \gamma)] > E[\Pi_i(\alpha_i = 1 - \gamma; \alpha_j = 1 - \gamma)] \quad (1.25)$$

Firm i can decide either to undertake project $\bar{\alpha}$ or project $\underline{\alpha}$ according to the indication of problem 1.25, the solution of which can be represented by an isoprofit function, as in the previous analysis, for every value of the parameters. Since problems 1.24 and 1.25 share the same structure, they are equivalent for a very large set of the parameters and thus produce analogous isoprofit functions; and therefore only symmetric equilibria take place. In particular, both firms select the risky activity for those values of the parameters that lie on the left-hand side of the isoprofit line in figure 1.9; or the safe activity if parameters assume values on the right-hand side of the isoprofit line in the same figure.

Relationship between R&D riskiness and incentive towards cooperation

The fact that R&D represents a risky activity is widely considered as one of the main issues that may push firms into joining a research joint venture. In fact, the argument is that firms sometimes desire to share the risk and the cost associated to a research project with a group of other firms. This is for instance the case in

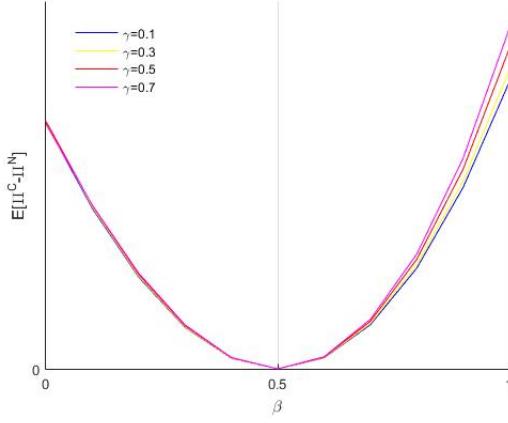


Figure 1.10: Variation of the incentive towards cooperation, $\Pi^C - \Pi^N$, with respect to the level of risk parameter γ

the microelectricals industry, in which evidence provides a series of successful R&D consortia working on high-risk projects; a notable example may be represented by the Cyc project on artificial intelligence started off at MCC (Microelectronics and Computer Technology Corporation, the first R&D consortium in the industry). The structure of this model allows to analyse this issue and draw some conclusion surrounding the relationship between R&D risk and cooperation; for this purpose I shall allow the risk parameter γ to vary in order to understand the impact on the incentive towards R&D cooperation.

On the vertical axis of figure 1.10 is shown the incentive for firms to form a research joint venture, expressed as the difference between the expected profit obtained under the cooperative regime and the one in no-cooperation, such difference is depicted for various values of the risk parameter γ ; whereas the amount of spillovers β is represented on the horizontal axis. As seen in figure 1.8, concerning the incentive to cooperate at a given level of risk, all lines share the same trend: they present a quadratic shape and assume value equal to zero in correspondence of $\beta = 0.5$. If spillovers are sufficiently low, all lines stay very close to each other, showing that a change in the risk parameter does not affect considerably the incentive for firms to cooperate. However, for large values of β some more relevant difference arises; in particular, the lines pivot upwards as the risk parameter grows large (in the figure it goes from $\gamma = 0.1$, represented by the blue line, to $\gamma = 0.7$ for the magenta line). This indicates that the incentive to form a research joint venture, while always greater than zero, is perceived by firms to a larger extent when the industry is characterised by high spillovers, whereas is ‘more rigid’ to the risk level for low values of β . This fact has an interpretation consistent with the results of the model. From remark 3 it is stated the fact that firms are more

keen to embark on the safe project when spillovers are large; consistently, they may be interpreted to be more risk-averse with respect to R&D. Provided that R&D consortia represent an instrument by which a firm can cut the level of risk of R&D by sharing it among more participants, then the cooperative regime may become more appealing as spillovers grow large.

Proposition 3. *Simulations suggest that an increase in the risk rate increases the incentive to form a research joint venture only when spillovers are sufficiently large.*

1.3 Comparison between the two modes-of-operation and cooperative synergies

In order to understand thoroughly the different effect produced by R&D cooperation over no-cooperation, it is convenient to compare the two different regimes; the comparison concerns either the research-level, i.e., the implications on innovation quality and R&D investment, and the product market, to outline any difference in the output level. I shall rely on a graphical inspection to highlight the outlook of the results; all the graphs presented are drawn at the following values of parameters: $q = 0.025, m = 1, a - \bar{c} = 100$. This specific configuration belongs to the broader set of parameter able to assure the holding of the second-order conditions, the regularity conditions¹² in remark 2 and the presence of returns to investment as in equation 1.2 able to justify the adoption of either safe or risky projects¹³.

1.3.1 Research and development

Cooperation entails the internalisation of technological spillover, as they remain confined within the consortium; this in turn has an effect on the amount of capital that a firm is willing to invest in R&D activities and, as a consequence, on the quality of innovations to implement. D'Aspremont and Jacquemin (1988) have proved that the greater the amount of spillover of the industry, the greater the positive effect of cooperation on R&D investment.

Figure 1.11 gives a graphical insight of the comparison between the non cooperative innovation quality and the cooperative one. On the horizontal axis is measured the spillover parameter β ; whereas the function depicted represents the difference between innovation quality in the cooperative case and in the non

¹²Works as Seade (1980), Henrique (1990) and Hahn (1962) discuss the topic of stability in greater details.

¹³See note 4.

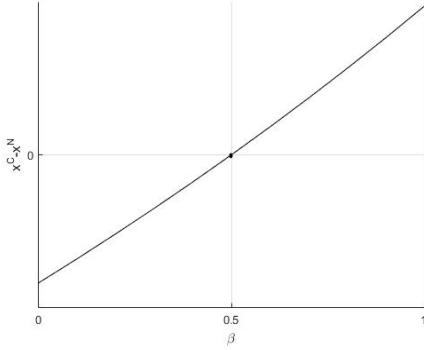


Figure 1.11: Difference of the innovation quality in the two regimes $x^N - x^C$ with respect to technological spillovers β

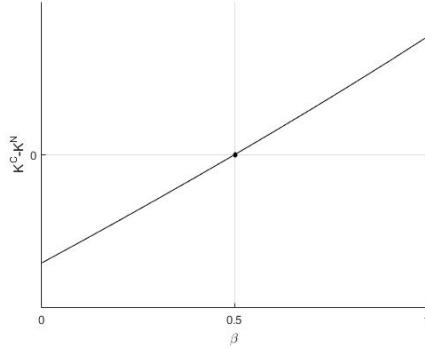


Figure 1.12: Difference of the R&D investment in the two regimes $K^N - K^C$ with respect to technological spillovers

cooperative one, therefore positive values of the function captures the fact that cooperation increases the quality of the innovations in the industry. The figure shows that the equilibrium level of x generally differs in the two regimes. In particular, ‘cooperation’ entails higher levels of x , and thus the implementation of better innovations, if and only if the spillover parameter is sufficiently high (larger than $1/2$); otherwise the non cooperative solution embeds a greater level of innovation quality. Moreover, when β is equal to $1/2$, the two regimes are equivalent: they deliver in expected terms the same level of innovation quality. This result is in line with the literature by d’Aspremont and Jacquemin (1988) and Leahy and Neary (1997) on deterministic R&D.

Proposition 4. *The size of innovations in the industry are greater under the cooperative regime if, and only if, the spillover parameter is sufficiently high.*

Moreover, figure 1.12 illustrates the relationship between the investment in R&D borne in ‘no-cooperation’ relative to ‘cooperation’ and the degree of technological spillovers. As it can be seen, such relationship is negatively inclined and the level of investment is higher within R&D consortia only for large spillovers, confirming the previous conclusions on the level of quality of innovation in the industry.

Remark 4. *At $\beta = 1/2$ ‘cooperation’ and ‘no-cooperation’ bring about an equivalent amount of investments in R&D.*

Through the comparison between the two regimes it is possible to draw some conclusion over the effect of cooperation on the project to undertake. In other words, one may investigate whether joining in a research joint venture may influence the R&D project on which firms desire to embark. In order to develop

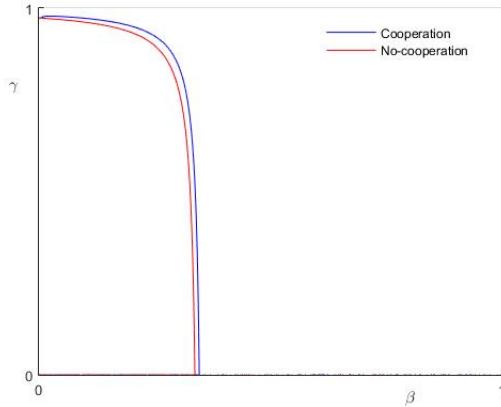


Figure 1.13: Comparison of the isoprofits in the two regimes

this kind of analysis, I shall compare the isoprofit lines in the two regimes as a function of the risk parameter γ and the spillover parameter β , as in figure 1.9 (which concerns the cooperative regime). A comparison of such isoprofits is depicted in figure 1.13, where the blue line represents ‘cooperation’ and the red line indicates ‘no-cooperation’. As in the previous section, the area lying at the left-hand side of the isoprofit is the one associated to those parameter by which firms are keen on selecting the risky research activity. As it can be seen, the ‘riskier’ area determined by the cooperative regime is larger than the one in the non cooperation solution. This is not sufficient to provide a formal result, in fact a more precise analysis - possibly within a model in which firms select continuously the value of α , instead of facing only a binary problem as in the current work - is required to assess to which extent cooperation in R&D triggers the adoption of riskier projects, however, the graphical inspection suggests that research joint ventures are more likely to embark on the riskier project than ordinary firms.

1.3.2 Output and profits

Equipped with the considerations concerning the research-level decisions, let us turn to analysing the impact of the mode-of-operation on the product market. The solving of stage five of the model, the cournot competition on the product market, produces a condition of optimal output, which is given by equation 1.9 when both firms are able to implement the innovation out of the research activity. The derivative with respect to the variable x_i delivers the following:

$$\frac{\partial y_i}{\partial x_i} = \frac{2 - \beta}{3b} > 0 \quad (1.26)$$

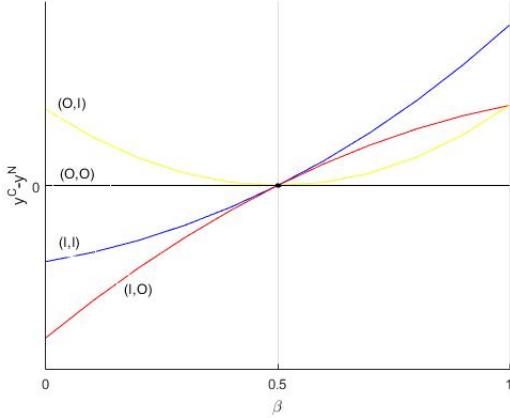


Figure 1.14: Difference of the output levels in the two regimes $y^N - y^C$ with respect to technological spillovers

from which one may conclude that an increase in the size of innovation by firm i has a positive impact on its output level. In other words, the level of investment in R&D might affect positively the amount of goods supplied to the market, provided that the research activity of the firm delivers an actual innovation.

Figure 1.14 represents the working of this argument showing how the term $(y^N - y^C)$ varies with respect to the value of the spillover parameter β ; the function is depicted in all possible states of the world. To understand the economic intuition, let me stress that a rise in β results in a rise in K^C , a fall in K^N and a consequent rise in their difference, as shown in figure 1.11. If the investment is successful and produces an innovation for both firms, the level of output varies according to condition 1.26: as spillovers grow high, the non-cooperative equilibrium embeds a lower level of production (and high level of price¹⁴), opposite result in the cooperative regime; and the term $(y^C - y^N)$ displays an increasing tendency with respect to β . This situation is represented in figure 1.14 by the function (I,I); as expected, at the value of spillover 1/2 the two regimes are equivalent. If no firm is able to innovate, the state of the world is (O,O). In such a case, the level of output is independent of the amount of investment, since the marginal cost does not undergo any reduction. This is the reason why the respective function in the figure is horizontal for every value of the spillover parameter. In the state of nature (I,O) only firm i can implement an innovation. Any increase in β determines in the cooperative regime an increase in its investment, the size of its innovation and its level of output; the respective function in figure 1.14 has therefore an increasing pattern. Interestingly, in the case (O,I) firm i is the one that cannot

¹⁴This consideration raises the issue whether R&D consortia are welfare-improving. I shall analyse such topic in a following chapter of this thesis.

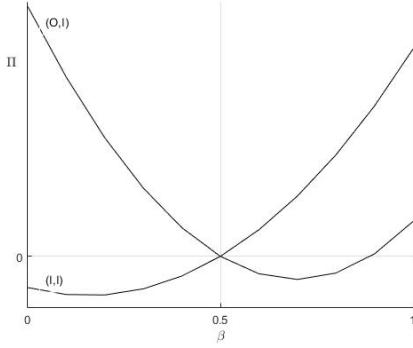


Figure 1.15: Incentive towards cooperation with respect to β for firm i when firm j succeeds at innovating

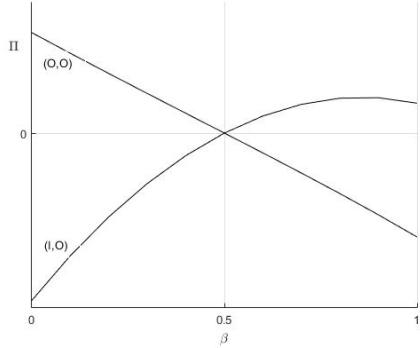


Figure 1.16: Incentive towards cooperation with respect to β for firm i when firm j fails at innovating

implement the innovation, nonetheless the cooperative solution increases the level of its output for every value of β . At $\beta = 1/2$ the two regimes are equivalent as expected. Moving towards left ($\beta < 1/2$), the switch towards cooperation determines a fall in K_j and x_j , but y_i is allowed to increase as shown by condition 1.12 on Cournot competition. Moving to the right, instead, K_j and x_j rise, and y_i rises as well, since by the same condition the variables are now complementary. These considerations give the function that quasi-quadratic shape, by which the non-innovator undergoes an increase in the level of output when joining a research joint venture.

As for profits, it requires a specific mention the fact that in the traditional approach since d'Aspremont and Jacquemin profits are always greater under the 'cooperation' equilibrium. The argument is that R&D consortia serve as partial surrogate for product-market collusion (Leahy and Neary, 1997), and therefore firms may never have falling profits. In expected terms, the same result as Leahy and Neary applies in this work, as confirmed by figure 1.8; therefore firms opt for the cooperative regime. If the projects chosen by the firms are the safe ones, the expected profit corresponds to the actual profit and no economic insight can be provided relative to the previous literature. However, if the equilibrium is the one associated to the risky activity, in stage four 'Nature' selects the state of the world; in each of which firms earn a different level of profit. In this respect let us observe figures 1.15 and 1.16, which show the level of profit obtained by firm i when firm j , respectively, succeeds or fails at innovating. On the vertical axis is indicated the term $(\Pi^C - \Pi^N)$, to show the difference between the R&D regimes, whereas the level of spillover is expressed on the horizontal axis. Not surprisingly, at the critical value $\beta = 0.5$ all functions cross the horizontal axis: the two regimes bring about the same level of profits; generally, the cooperative regime

entails higher profits than ‘no-cooperation’ for those values of the parameter β for which the lines lie above the horizontal axis. If firm j is able to implement a new innovation, figure 1.15 proves that firm i might in fact be better off under the non cooperative regime, oppositely to what one could expect relying on the conclusions provided by the classic literature on deterministic R&D, depending on the state of the world and on the amount of spillovers. The same kind of argument is valid also in the state of the world in which firm j does not innovate, as shown by figure 1.16. Therefore, risk-neutral firms always decide to join a research joint venture in order to maximise expected profits, however cooperation does *not* represent necessarily the more profitable solution.

1.3.3 Cooperative synergies

In the paper, as in large part of the literature (see Leahy and Neary, 1997, d’Aspremont and Jacquemin, 1988 among others), the decision of firms to form a research joint venture does not affect the flow of knowledge moving across firms. However, this is unrealistic, as some author has noticed; the level of spillovers should in fact reasonably rise as firms decide to cooperate, as they would share not only the results of basic research, but also the development activity and the innovation, in case they produce any; Kamien et al. (1992) distinguish between ‘R&D cartel’ and ‘R&D cooperation’, when the former denotes the notion assumed throughout the present work and the latter refers to the case of complete information-sharing (in other words, to the case $\beta = 1$). In such situation, it is interesting to understand the impact of forming a research joint venture on the results of the model.

If switching to cooperation embeds the fact that the spillover coefficient becomes equal to unity, the research joint venture should in principle be more inclined to select the safe research project ($\alpha = \bar{\alpha}$), since the eventual extra-gain, in terms of marginal cost reduction, associated to the risky project is fully absorbed by the competitor, as observed in remark 3; figure 1.9 gives a graphical representation to this argument. As for the quality of the innovation pursued, the effect is two-fold: on one hand, it should decrease as firms select safer and lower-return projects, on the other hand, a larger level of spillovers raises the social benefit of R&D and for this reason increase the incentive to invest in research and development. The net result of these two effects depends on their order of magnitude; unfortunately, this model is not able to supply definite results as they may be parameter-sensitive. Possibly, a set-up in which the decision over the level of risk is continuous, rather than binary as in the present work, is more likely to deliver interesting insights. Finally, the high amount of spillovers is likely to reduce the marginal cost of both firms. This is certainly true in the scenario in which the level of quality of innovation rises; in this case a firm undergoes a marginal cost expected-reduction

from both better innovations and larger spillovers. To see this, let us recall that equation 1.9, provided symmetric equilibria and the fact that both firms innovate, produces the following expression

$$y = \frac{a - \bar{c} + x(1 + \beta)}{3b}$$

where y is the level of output of the firm and x is the quality of its innovation. Clearly, if both β and x rise, the level of output must increase. Moreover, such increase may occur even when the technological process concerns the implementation of lower-quality innovations - case in which the term x diminishes - provided that a firm receives a sufficiently large benefit from the competitor's R&D. In such cases, the output level in the industry is likely to increase, with a consequent reduction of the price level.

1.4 Concluding remarks

My paper contributes to the strand of the literature on R&D by developing a model in quantity competition in which firms have the possibility to decide the research project to undertake (as well as the amount of research and development to carry out and the organizational level at which performing it - firm level or research joint ventures); in line with the literature, R&D activities determine the implementation of new process innovation which cuts the marginal cost level. The decision over the project is made on the basis of R&D risk associated to it: on the one hand, greater risk represents a larger probability of failure of the innovation; on the other hand, a risky project, when it is successful, delivers better innovations.

The main contribution of this work is the introduction of a range of different R&D projects, which differ by the level of risk. Unlike most papers in the stochastic R&D literature (e.g. Erkal and Piccinin, 2010) in which there exists one race (one project), in this paper firms are allowed to select the optimal innovation project. Besides rendering the risk level endogenous, such approach has the upside to describe how the decision of an R&D project impacts on the goods-market and the amount of investment made by firms.

The paper is able to replicate some of the well-established results in the literature. For instance, the level of investment reacts with the level of technological spillovers, and rises under R&D cooperation only if the amount of spillovers is sufficiently large; moreover, profits are higher under cooperation agreements; and output is (in expected terms) an increasing function of the amount of R&D investment.

I formalise the existence of a trade-off between the level of risk associated to R&D and the quality of innovation to implement, and derive its functional

form, which is proved to be independent of the presence of an R&D cooperation agreement among firms. Such trade-off holds given the level of investment; in this situation an increase of the risk level increases the quality of the innovation implemented (if research turns out to be successful). Moreover, the trade-off is confirmed along the optimal level of investment: when firms select the investment in R&D in an optimal way, whatever the organizational level (firm or consortium), selecting a riskier project pushes firms to increase the quality of their innovations.

The relationship between the investment activity of the firms is proved to depend on the spillover parameter: if they are low, the R&D investments of the two firms are strategic substitutes, in other words when a firm lowers its investment, the rival has the incentive to raise it; whereas they are strategic complementary when spillovers are high. This is due to the relationship between investment in R&D and output level of the firms.

The analysis suggests that the choice of the R&D regime produces an impact on the optimal research project to undertake. In general, R&D cartels seem to be more keen on embarking on riskier activities than non cooperative firms. However, this result may be inverted when R&D cartels share not only the results of basic research, but also the development of the innovation, as so to produce cooperative synergies. In this case, research joint ventures embed the fact that the spillover parameter becomes equal to one, discouraging firms from embarking on risky activities.

An other contribution of the paper is to show the impact of riskiness of the projects on the incentive to form an R&D consortium. A graphical analysis based on comparative statics suggests that a low level of spillovers corresponds to the case in which a greater level of risk in R&D does not significantly change the incentive for firms to cooperate and to form an R&D cartel. However, when spillovers grow large an increase in the risk parameter affects positively the incentive to join a research joint venture; this may confirm the empirical evidence by which risk is a primary driver for firms to cooperate.

The lines for future research are suggested by the literature. In fact, the model set-up is opportune to address public policy analysis and quantify the impact of cooperation on the level of social welfare, as well as to design an optimal package of fiscal stimulus and competition policy to achieve the social optimum. In particular, a natural extension of this model is able to study the socially optimal level of R&D risk and how the policy maker can affect the firm decisions to implement it. This topic will be the object of a chapter of my thesis.

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Chapter 2

Research uncertainty and subsidy policy: behaviour of the government

2.1 Introduction

A large part of research activities carried within a country is funded by public subsidies, confirming the fact that R&D plays a key role for the development of an economy. For example, as pointed out by Stiglitz and Jayadev (2010), in the United States two-thirds of the R&D investment in the pharmaceutical sector derives from public funding. The relevance of R&D is higher in some strategic sectors, such as the automobile and the microelectronic ones, where technological development represents the main engine for the growth - and the survival - of firms in the market.

Investments in R&D are affected by a high level of uncertainty because it is not possible to know *a priori* the outcome of the research activity. Such uncertainty may arise in various forms: the research project needs not bring about an innovation; the innovation, in any, may be difficult to place on the market; or it may entail an economic advantage which is lower than the one expected. To take account of the risk related to R&D, Choi (1993) departs from the deterministic R&D literature¹ and models the development of a new technology as a tournament, in which many firms participate but only one is awarded the innovation; the level of investment in R&D is directly related to the probability of success in the tournament: the higher the investment, the higher the likelihood of winning the race. The winner obtains monopoly power over the new technology for a given period of time, after which the losers are able to replicate the innovation and increase the degree of competition in the market. Such period of time measures

¹Notable examples: Katz (1986), d'Aspremont and Jacquemin (1988), and Leahy and Neary (1997).

the level of technological spillovers in the industry. Furthermore, firms have the possibility to cooperate at the research-level by forming a research joint venture². In this case they sign a cooperative agreement by which they commit to sharing the innovation, provided that one of the participants wins the race. Miyagiwa and Ohno (2002) consider an infinite horizon, instead of a two-period game, and prove that the incentive to form a research joint venture is independent of the amount of spillovers in the industry or the degree of uncertainty. Erkal and Piccinin (2010) assume free-entry into the R&D tournament and show that research cooperation may be less profitable than research competition if it is formed by a sufficiently large number of firms; and that cooperative agreements in which R&D efforts are coordinated but the innovation is not shared (R&D cartel) are always welfare-reducing, since they are never profitable and does not increase consumer surplus, whereas agreements in which the innovation is shared among the participants (research joint venture) may be welfare-improving under specific conditions.

The literature on stochastic R&D adopts the research race approach used by Choi (1993). Although differing by the focus of their analysis, all contributions consider the risk level within a given research project. However, none of them explores the implications of differences in the research risk *across different projects*. In general, a research project aiming at the development of a minor innovation may rely upon existing technologies and embed a higher probability of success. Thus, the ‘degree of ambition’ of the research project may affect the risk related to R&D investment. Under a social point of view, maximisation of welfare does not necessarily require firms to choose the lowest level of risk. On the one hand, a low-risk research project increases the probability that firms develop an actual innovation. On the other hand, a project with a higher level of risk may pay

²The possibility for firms to sign a cooperative agreement inevitably cast doubts about welfare implications. The primary fear is based on the fact that cooperation may reduce the degree of competition of the market and in turn the level of welfare. However, as seen in the first chapter, there exists much evidence supporting the idea that research joint ventures foster technological development. Romer (1986) explores the effect of R&D on long-run economic growth, modifying the approach proposed by Ramsey (1928); Spence (1986) analyses the impact of technological spillovers on the incentive for firms to invest in research and development activities. D’Aspremont and Jacquemin (1988) propose a duopolistic model able to explain the condition for which R&D cooperation is welfare-improving; more specifically, this requires technological spillovers to be sufficiently large, since research joint ventures become instruments to internalize such spillovers; Suzumura (1992) extends their analysis considering n-firm oligopolistic competition. On the other hand, when knowledge creation is adequately protected and does not spill over to other firms, R&D cooperation entails lower level of welfare but higher industry-profits, as research joint venture serves as a partial surrogate for output-level cooperation, with a rise in firms’ monopoly power (Leahy and Neary, 1997). Kats (1986) and Kamien et al. (1992) investigate the effect of different R&D agreements, selected on the basis of cost-sharing rules, on profits and welfare, and shed light on the role played by the degree of competition in the product market.

off with a major innovation, able to raise sharply the level of welfare - let us think, for instance, of a new drug able to treat dangerous diseases. Thus, existing contributions are not able to address the following issue: which research project is best for welfare?

I depart from the research race approach and follow the d'Aspremont and Jacquemin (1988) set-up, enriched with research stochasticity. I consider two firms competing à la Cournot for the supply of a homogeneous good. At first they have the same level of technology and, thus, the same marginal cost. However, they can also decide to invest in R&D to increase their efficiency. The decision over R&D regards three dimensions: the research project to undertake, where each project has a specific level risk, exogeneously given; the level of investment, which affects the quality of the innovation pursued; and the degree of cooperation, in particular firms can both compete or collude at the research level³. Each research project presents either a different level of risk and a different level of marginal return to investment: given the amount of resources invested in R&D, a riskier project delivers a better innovation to the firm. However, producers do not know *a priori* the outcome of the research activity; it may both succeed or fail, and only in the former case the innovation is obtained. In this regard, the riskier the research project, the higher the probability of ending up without an innovation. The decision over the research project is thus the result of a trade-off, which firms address aiming at the maximisation of expected profits. Finally, I consider the possibility for the social planner to grant subsidies, both at the research and at the output levels, and induce the social optimum. This allows to explore the conditions for first-best and second-best scenarios.

A contribution of this paper is to provide a new benchmark to analyse the effect of R&D decision by firms on the level of welfare, taking account of both the level of investment and the project-specific risk rate. In particular, I derive the conditions for maximum welfare in first-best, in which both output and R&D-investment can be controlled by the social planner, and second-best, situation in which the government is able to grant a subsidy only for R&D activities; I analyse them in every possible state of the world: both firms implement an innovation, only one firm does or no firm innovates; and I describe the optimal level of risk for the government with respect to the amount of technological spillovers in the industry. As for the latter point, in particular, graphical representation suggests that the social planner in first-best is more inclined towards riskier projects as spillovers grow large.

Moreover, I set out a ranking of market outcomes with respect to their level of welfare. This is an analysis existing in deterministic R&D models which I reproduce in this stochastic framework. The main result is maintained: cooperation

³Collusion at the goods-level is forbidden by law.

between firms is desirable under a social point of view - cooperation is welfare-maximising - if, and only if, spillovers are sufficiently large; the reason for this is that only under high technological spillovers the cooperative regime embeds a higher level of investment in R&D - relative to no-cooperation. This framework has the advantage to describe the role of uncertainty: an increase in the risk rate is shown to enlarge the gap between socially optimal technological development and the one obtained in equilibrium. In other words, the effect of risk is to reduce the desirability of market outcomes in terms of quality of innovations.

Finally, I analyse the optimal scheme of subsidy, either in first best and in second best, that the government should grant to firms to attain the social optimum. A tax on R&D is justified in first-best only when spillovers are sufficiently low, otherwise a positive subsidy should always take place. Under second-best, somewhat counter-intuitively, the government should grant higher subsidy to riskier projects, because an increase in the risk level enlarges the gap between equilibrium innovation-quality and socially optimal one. Moreover, the government should concede subsidies conditional on the undertaking of a specific project with the goal to encourage the development of the welfare-maximising one; whereas the other projects should not receive any subsidy or, even, could be taxed.

The outline of the paper is as follows. Section 2 describes the working of the model and draw some conclusions surrounding welfare. Section 3 and section 4 provide a positive analysis for the public authority to induce, respectively, first-best and second-best equilibria. Section 5 concludes with some remark and future line of research.

2.2 Welfare analysis

The model of this work is based upon the one developed in the first chapter. Two symmetric firms compete in a given industry over the quantity of a homogeneous product to supply to the market. They may reduce their marginal costs of production, assumed to be constant with respect to the output level, by investing in technological development, namely, in research and development activities. However, such development is not deterministic: every investment may either be successful or fail, and only in the former case an innovation is delivered. Consequently, provided that every firm decides to invest in R&D, there are four possible states of the world surrounding R&D: both firms get an innovation, only one firm does and the other one does not, or no innovation is obtained. In the case of success, the level of investment is positively correlated with the quality of the innovation; in other words, the larger the investment, the better the innovation. It is necessary, also, to recall the fact that firms may decide either to compete or to cooperate on the ground of R&D; in the latter case, they constitute a research

joint venture and maximise the joint level of profits, rather than the firm-specific one. Finally, unlike other contributions in the literature, firms have the possibility to decide the research project to undertake. Projects are different in their level of risk, capturing their different ‘degree of ambition’; however, selecting riskier projects may pay off, provided that the R&D activity is successfully carried, with higher marginal returns on quality of innovation to the level of investment. In other words, each dollar spent in riskier R&D activities produces an innovation with higher quality compared to a dollar spent in safer, less ambitious projects.

This paper regards the problem of a government which has the objective to affect the actions of the firms so as to maximise the level of welfare, which may be possible through subsidy payments. I consider the classic welfare function defined as the sum of consumer surplus and profits. In a general state of the world $s \in S$ this is equal to

$$W^s = \int_0^{\bar{y}} p(u) du - p\bar{y} + \bar{\Pi}^s = \int_0^{\bar{y}} p(u) du - c_i^s y_i - c_j^s y_j - 2K \quad (2.1)$$

where \bar{y} is the total level of output in the industry, W is the level of welfare; p is the price level, K is the amount of investment in R&D selected by each firm and $\bar{\Pi}$ are industry-wide profits; the state of the world determines whether firms are able to implement an innovation as a result of their research activity. Let us recall from chapter 1 that the level of investment is given by the function $K = x^2\Gamma(\alpha)^2 = x^2(m\alpha + q)^2$, where m and q are positive scalars; and α and x are, respectively, the probability with which the R&D activity delivers an innovation and the quality of such innovation. In equilibrium, two symmetric firms are shown to invest the same amount of money on the same project (see chapter 1). As a result, if both firms are either successful or unsuccessful, their marginal cost stays equal and, thus, the symmetry property within the industry is maintained. In this case, the welfare analysis is carried assuming $c_i = c_j = c$. However, in the remaining states of the world, in which only one firm - either firm i or j - is successful at implementing a new innovation, the industry loses its cost-symmetry. The government does not know *a priori* the realization of the state of the world and, for this reason, can maximise welfare in expected terms only. However, it is convenient first to describe the maximisation of welfare *for each state of the world* so as to provide some benchmark results, this will prove useful in the following sections.

2.2.1 State-contingent welfare analysis

In this section I shall explore the optimal levels of output \bar{y} , innovation quality x and success rate of research projects α from the perspective of the government,

where the latter variable represents a binary choice ($\bar{\alpha}$ or $\underline{\alpha}$, with values exogenously given). In continuity with the previous analysis and in order to provide a clear distinction, the two projects are divided into ‘risky’ and ‘safe’. The latter is assumed to be deterministic and thus to have probability $\bar{\alpha} = 1$. The former, instead, is stochastic and has a probability of realization $\underline{\alpha} = 1 - \gamma$, where γ measures the risk rate. In this section I shall assume the validity of the following:

Assumption 1. *From a social perspective, the optimal research activity is represented by the risky project $\underline{\alpha}$.*

This assumption allows to focus on the stochastic case, being the deterministic case well-explored in the literature. First, let us consider the case in which both firms innovate, which means that in equilibrium⁴ we have: $c_i = c_j \equiv c = \bar{c} - x(1 + \beta)$. The optimal levels of output and investment are described by the total derivative of equation (2.1) with respect to \bar{y} and x :

$$dW = [p - c]d\bar{y} + [(1 + \beta)\bar{y} - 2K_x]dx \quad (2.2)$$

where K_x is the derivative of K with respect to the variable x ; and by imposing the coefficients of $d\bar{y}$ and dx equal to zero, as follows:

$$\begin{cases} p - c = 0 \\ \bar{y}(1 + \beta) - 4x\Gamma(\underline{\alpha})^2 = 0 \end{cases}$$

The first expression concerns output and delivers the well-known condition for maximal welfare $p = c$, by which the price level must be equal to the marginal cost; while the second regard the decision on R&D. From these equations we get:

$$\begin{cases} \bar{y}^{I,I} = \frac{a - \bar{c} + x(1 + \beta)}{b} \\ x^{I,I} = \frac{\bar{y}(1 + \beta)}{4\Gamma(\underline{\alpha})^2} \end{cases} \quad (2.3)$$

where the superscript (I, I) denotes the state of the world in which an innovation is obtained by both firms.

Lemma 1. *When both firms innovate and given the total level of output in the industry, the socially optimal level of investment in R&D is an increasing function of spillovers.*

This is due to the fact that the greater the spillover rate, the higher the social benefit arising from R&D and, thus, the socially optimal level of investment.

⁴As shown in chapter 1, in equilibrium $x_i = x_j = x$ and $\alpha_i = \alpha_j = \alpha$.

Let us now turn to the case in which no firm becomes an actual innovator. In this case the optimal levels of output and investments are given by the following:

$$\begin{cases} p - c = 0 \\ -4x\Gamma(\alpha)^2 = 0 \end{cases}$$

From the second equation we get $x^{O,O} = 0$, where the superscript (O,O) denotes the no-innovation case. This is a trivial result: if the R&D activity fails, any rise in x represents an increase in the cost term related to investment, but does not produce any fall in the level of marginal cost, with a consequent reduction in the level of profit and of welfare. The level of output is given by:

$$\bar{y}^{O,O} = \frac{a - \bar{c}}{b} \quad (2.4)$$

which is clearly independent of the level of investment in R&D.

The property of symmetry in the industry does not hold on, even in symmetric equilibria, when only one firm implements an innovation - provided that spillovers are strictly lower than one. Technically, the marginal cost of the innovator falls to a larger extent than the rival's, which, however, receives a benefit determined by the spillover parameter β . Assuming that firm i innovates, we have that $c_i = \bar{c} - x$ and $c_j = \bar{c} - \beta x$. The welfare function in equation (2.1) becomes:

$$W^{I,O} = \int_0^{\bar{y}} p(u) du - c_i y_i - c_j y_j - 2K$$

where the case (I,O) denotes that only firm i is innovating. The derivative of this equation with respect to \bar{y} cannot be calculated in a straightforward manner; for this reason further assumptions are to be made. In particular, I shall allow for a generic parameter δ to capture the market share the government wants to assure to the innovator (this could be equal to $1/2$, but it might be different). By posing thus $y_i = \delta \bar{y}$, where firm i is the innovator, we can rewrite welfare as follows:

$$W^{I,O} = \int_0^{\bar{y}} p(u) du - c_i \delta \bar{y} - c_j (1 - \delta) \bar{y} - 2K$$

I can compute the derivatives with respect to output and investment to get:

$$p = \delta c_i + (1 - \delta) c_j \quad (2.5)$$

$$\bar{y}[\beta + (1 - \beta)\delta] = 2K_x \quad (2.6)$$

Lemma 2. *If there is only one innovator, the level of marginal cost becomes heterogeneous across firms. Under the assumption of pre-fixed market share, the socially optimal level of output requires that price be equal the sector-average marginal cost.*

If the government desires to maintain a homogeneous level of market share ($\delta = 1/2$), equations (2.5) and (2.6) develop into:

$$\bar{y}^{I,O} = \frac{2(a - \bar{c}) + x(1 + \beta)}{2b} \quad (2.7)$$

$$x^{I,O} = \frac{\bar{y}(1 + \beta)}{8\Gamma(\underline{\alpha})^2} \quad (2.8)$$

2.2.2 Welfare stochasticity in first-best

The fact that R&D is a stochastic activity, the outcome of which cannot be known *ex-ante*, produces some relevant consequence on welfare analysis. The social planner does not know in advance which state of nature will occur. For this reason, any social commitment to increasing the value of welfare and attain the social optimum has to be weighted by the uncertainty factor. Notwithstanding, there exists a critical difference between output and R&D-investment decisions: the realization of the state of the world occurs *after* firms conduct their R&D activity, thus the result of the investment is uncertain; however, they know the outcome *before* competing on the product market. The decision concerning the level of quantity to supply to the market is therefore taken by firms with perfect knowledge around the state of the world.

Remark 5. *Provided that the social planner is able to grant output subsidies to affect the level of production of the firms and differentiate the subsidy payments according to the state of the world, it can induce the optimal level of output independently of the research outcome.*

Given the amount of investment and its outcome, the government is aware of the fact that the determination of output is not affected by any degree of uncertainty. In fact, the state of the world affects the efficiency of the firms and their capability to produce, but the government is able to correct this by increasing or decreasing the level of output subsidy according to the realization of the state of the world. In other words, the optimality conditions with respect to output in first best are equivalent to the state-contingent ones expressed by equations (2.3), (2.4) and (2.7). Instead, the optimality condition with respect to investment must be obtained on the basis of expected welfare, because at this stage neither firms nor government have knowledge around the research outcome. The expected level of welfare is as follows:

$$E[W] = \sum_{s \in S} pr(s)W^s \quad (2.9)$$

where the term $pr(s)$ indicates the probability that the state of nature s occurs (which depends in turn on the research project selected).

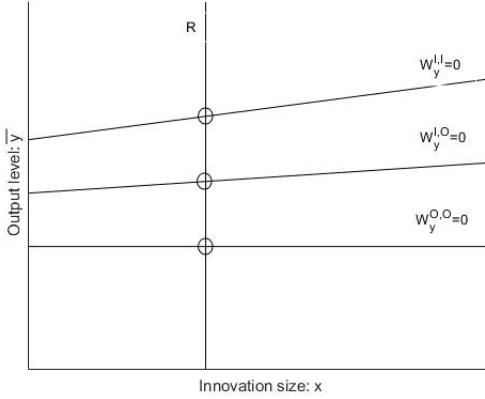


Figure 2.1: Welfare first-order conditions

The derivative of equation (2.9) with respect to x can be arranged to obtain the following:

$$x = \frac{\alpha(1 + \beta)(\alpha\bar{y}^{I,I} + (1 - \alpha)\bar{y}^{I,O})}{4\Gamma(\alpha)^2} \quad (2.10)$$

If firms select the ‘safe’ project, the probability of success is equal to one; and we get:

$$x = \frac{(1 + \beta)\bar{y}^{I,I}}{4\Gamma(1)^2} \quad (2.11)$$

This condition will be used in the following to compare the innovation level of the present model - stochastic - with the one obtained by the deterministic R&D literature. Equation (2.10) describes the socially optimal level of investment in R&D carried by each firm; as one could expect, it falls to zero in the limit case in which $\alpha = 0$, i.e., if the project is too ambitious to produce an innovation. Substituting the output levels $\bar{y}^{I,I}$ and $\bar{y}^{I,O}$, given by equations (2.3) and (2.7), into equation (2.10), we get:

$$x = \frac{a - \bar{c}}{\frac{4b\Gamma(\alpha)^2}{\alpha(1+\beta)} - \frac{(1+\beta)(1+\alpha)}{2}} \quad (2.12)$$

From this equation is possible to state the following:

Lemma 3. *Given the research project α , the amount of spillovers β is positively correlated with the amount of investment in R&D required in first-best.*

Given the value of $\alpha \in \{1; 1-\gamma\}$, an increase in technological spillovers unambiguously reduces the denominator in equation (2.12) and raises the optimal amount of investment in R&D. This is due to the fact that investments may produce greater

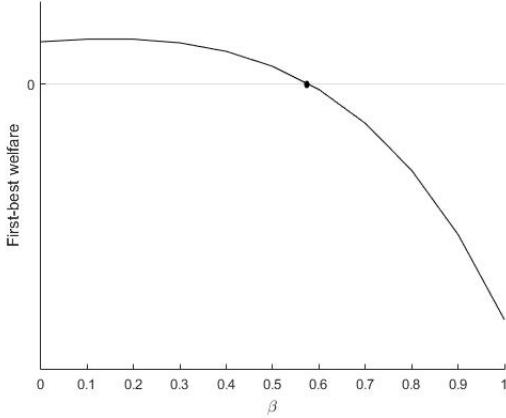


Figure 2.2: Project comparison in terms of first-best welfare

gains in efficiency at the industry-level when the spillover rate is large, as one firm's R&D benefits the rival to a relevant extent. Figure (2.1) shows the first-order conditions of welfare in the stochastic case ($\alpha = 1 - \gamma$). The innovation size x is measured on the horizontal axis and the industry-wide output level \bar{y} is on the vertical axis. The line R represents the optimal amount of innovation size pursued by each firm (equation 2.12); while $W_y^{I,I} = 0$, $W_y^{I,O} = 0$ and $W_y^{O,O} = 0$, are, respectively, the locus of optimal output in the three cases: both firms innovate, one firm does or any firm does not (equations 2.3, 2.4 and 2.7); the intersection points express the social optima in each state of the world. As it can be seen from a graphical inspection, condition $W_y^{I,I} = 0$ lies above $W_y^{I,O} = 0$ and, in turn, above $W_y^{O,O} = 0$, which means that, given the level of investment in R&D of each firm, the larger the number of firms that are successful in the development of an innovation, the larger the level of output which can be supplied in first-best.

We can now turn to the determination of the social optimum in terms of research project. Since α is a binary variable which cannot be used for computing derivatives, we shall rely on a graphical argument to rank the two possible options. Let us observe figure (2.2); on the axes we can measure the spillover rate β and the welfare differential, expressed as the difference in expected welfare between the safe project and the risky one ($E[W(\beta; \alpha_i = \alpha_j = \bar{\alpha})] - E[W(\beta; \alpha_i = \alpha_j = \underline{\alpha})]$). The welfare differential considers the symmetric choice $\alpha_i = \alpha_j$ because the asymmetric case is related to an intermediate level of welfare and thus is never maximising or minimising. In other words, for any level of the spillover rate the government desires both firms to undertake the same project. The function lies above the horizontal axis when spillovers are sufficiently low, which means that, in this case, the safe project should be preferred to the risky project. On the contrary, if the spillover rate grows large the adoption of a risky activity may produce larger gains

in terms of expected welfare.

Proposition 5. *In first-best the risky activity is likely to be preferred by the social planner when spillovers are sufficiently large.*

First, let me point out that this proposition is based on graphical arguments and should be interpreted as a simulation. In fact, if the risk parameter is close to unity we may have that the risky project becomes not worth undertaking and that the safe project delivers the higher level of welfare even when spillovers are large. However, it is interesting to interpret this result and understand the role of technological spillovers. When they are negligible, the efficiency of a firm largely depends on its own research activity. In this situation the government may prefer the adoption of a safe activity, because it is more likely to foster efficiency at the firm-level and, in turn, at the industry-level, with positive effects on welfare. On the other hand, when spillovers are large, efficiency at the firm-level does not depend only on own R&D: even in case of research project failure, firms can imitate the technology of the rival. In other words spillovers serve as an insurance to firms and to the government, because the average level of efficiency is more likely to rise. In the presence of such insurance the risky activity may have the upside to pay off with a major innovation and deliver higher welfare gains.

2.2.3 Welfare stochasticity in second-best

We can now turn to a ranking of the two possible R&D activities in second-best, situation in which the government is only able to control R&D subsidy, whereas output is freely determined by the market (this definition of second-best follows the one adopted by Leahy and Neary, 1997). More specifically, the level of output is determined along the locus $\Pi_y^s = 0$, which represents firm-level profit maximisation with respect to output, in every state of the world $s \in S$. The social planner solves welfare maximisation subject to the holding of these constraints, as follows:

$$\max_x E[W] \text{ s.t. } \Pi_y^s = 0 \quad (2.13)$$

This problem may be solved by plugging the constraint⁵ into the objective function and calculating the derivative with respect to x , so as to get:

$$\begin{aligned} \frac{\partial}{\partial x} E[W] &= \frac{4}{3}\alpha^2(1+\beta)\bar{y}^{I,I} + 2\alpha(1-\alpha) \cdot \\ &\cdot \frac{(1+\beta)[4a - 4\bar{c} - x(1+\beta)] + 12x(\beta^2 - \beta + 1)}{9b} - 2K_x \end{aligned} \quad (2.14)$$

⁵The constraint may be computed straightforwardly as shown in chapter 1 of this thesis. In this chapter the formula is shown in the appendix.

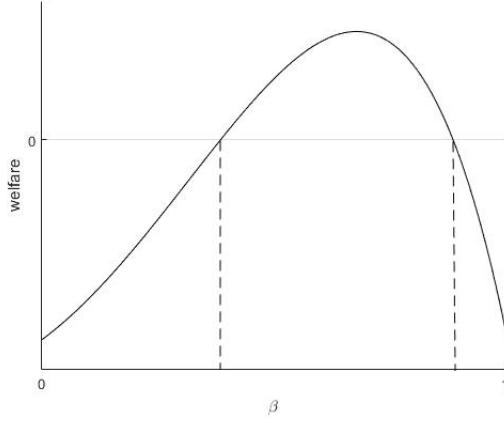


Figure 2.3: Project comparison in terms of second-best welfare

In the particular case $\alpha = 1$ we obtain:

$$x = \frac{(1+\beta)\bar{y}^{I,I}}{3\Gamma(1)^2} \quad (2.15)$$

The comparison between conditions (2.15) and (2.11), that is, between innovation-quality in, respectively, second best and first best, shows that firms in the former case overinvest in innovation quality relative to the latter case *given the level of output*; this result confirms the one established in the deterministic literature by Leahy and Neary (1997). I can rearrange equation (2.14) to obtain the second-best value of x :

$$x = \frac{a - \bar{c}}{\frac{9b\Gamma(\alpha)^2}{2\alpha(1+\beta)} - \frac{(1+\beta)(5\alpha-1)}{4} - \frac{3(1-\alpha)(\beta^2-\beta+1)}{1+\beta}} \quad (2.16)$$

Given the research project, this equation describes the behaviour of optimal investment in R&D in terms of maximal second-best welfare. I shall make use of equation (2.16) in the following sections to make a comparison with the first best case and explore positive analysis issues, especially analyzing the level of research subsidy to attain the social optimum.

Let us now turn to research projects. From the social perspective, the second-best optimal research project depends on the spillover rate, as in the first-best case. Figure (2.3) depicts the welfare differential between the safe and the risky projects. In this simulation the function presents a quadratic shape and crosses the β -axis twice, the first time upwards and the second time downwards. This means that the risky activity may be preferred for either small and large amount of spillovers; for an intermediate amount of spillovers, on the other hand, the safe project might represent the more desirable one.

Proposition 6. *In second-best the risky activity is likely to be preferred by the social planner when the spillover rate is both sufficiently large or sufficiently small.*

Unlike the first-best case, in second best the level of output is determined by oligopolistic conditions and may assume very low values, especially when spillovers are low because they do not sustain the average efficiency in the industry. In this case the price level may increase with a drop in consumer surplus. The simulation in figure (2.3) suggests that such decline may be offset for by undertaking riskier projects, since they may deliver major innovations and restore a higher level of output. On the other hand, a large amount of spillover serves as an insurance instrument, encouraging the adoption of a riskier project as seen in the first-best case. Noteworthily, the result of proposition (6) derives from simulation and thus should not be considered as a formal statement; in fact, the safe project may generally be the welfare-maximiser even for $\beta = 0$ or $\beta = 1$ if the risk parameter of the risky project is close to unity. Notwithstanding, it allows to understand at which values of the spillover parameter the risky project *is more likely* to be preferred by the government.

2.2.4 Welfare ranking of R&D-regimes

The two firms may decide to carry out their activity of research in a cooperative manner by forming a research joint venture, which in fact represents the equilibrium outcome in the model displayed in the first chapter of the thesis, as it delivers a higher level of expected profits. The R&D-regime affects the level of investment in R&D and the research project chosen by firms, and consequently the level of welfare. The government should encourage the optimal regime through an opportune scheme of subsidies. The research joint venture may embark on either the safe or the risky project. The former case is the one analysed in the literature⁶ and, for this reason, does not require further discussion in this chapter. The main conclusion is that cooperation produces the higher level of welfare if, and only if, spillovers are sufficiently high ($\beta > 0.5$), since in this case cooperation entails a larger level of investment in R&D. The second case, instead, allows to discuss novel elements into the literature and is the one analysed in this section. The main difference with the previous case is that a higher level of investment in R&D does not necessarily deliver an increase in welfare, because the research activity is allowed to fail. Remarkably, the social planner may have an influence over the research project undertaken by firms. Such influence, for instance, may be exerted by relating the payment of a subsidy to the adoption of a specific R&D activity, whereas the other one does not receive the same incentive, or it may even get taxed. On the basis of this argument, I shall assume that the social planner is

⁶See for instance Leahy and Neary (1997).

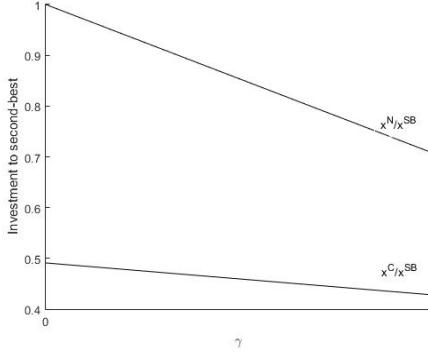


Figure 2.4: Investment level in equilibrium relative to the value required in second-best (picture drawn at $\beta = 0$)

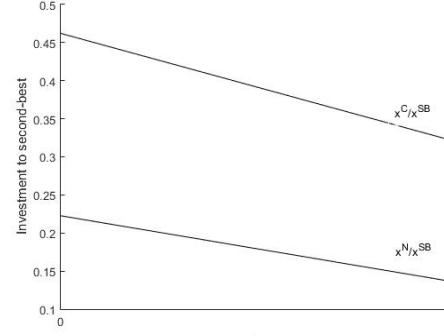


Figure 2.5: Investment level in equilibrium relative to the value required in second-best (picture drawn at $\beta = 1$)

always able to induce the socially-optimal project, whatever the mode-of-research chosen by firms.

In equilibrium, the amount of investment in R&D carried by firms is determined by the following conditions, as stated in the first chapter of the thesis⁷:

$$\begin{cases} x^N = \frac{(a - \bar{c})(2 - \beta)}{\frac{9b\Gamma(\alpha)^2}{\alpha} - (2 - \beta)^2 - \alpha(2 - \beta)(2\beta - 1)} \\ x^C = \frac{(a - \bar{c})(1 + \beta)}{\frac{9b\Gamma(\alpha)^2}{\alpha} - (2 - \beta)^2 - (2\beta - 1)^2 - 2\alpha(2 - \beta)(2\beta - 1)} \end{cases} \quad (2.17)$$

where the superscripts N and C refer, respectively, to the non cooperative regime and the cooperative one. Since we are analysing the case in which both firms select the risky project, the value of α is given by $(1-\gamma)$, where γ is the risk parameter.

Lemma 4. *Cooperation entails higher investments in research and development if and only if the spillover rate is larger than $1/2$. Moreover, when either β and γ are equal to zero, the investment level embedded in no-cooperation is equal to the one required in second-best.*

The proof derives from equations (2.16) and (2.17). This lemma confirms the result obtained in the deterministic R&D literature, in which no-cooperation delivers the second-best investment when spillovers are equal to zero. It may be insightful to draw the functions in condition (2.17) on a graph γ/x and compare them with

⁷In the present paper, the equations are written considering that in equilibrium firms are R&D-symmetric: $x_i = x_j$ and $\alpha_i = \alpha_j$.

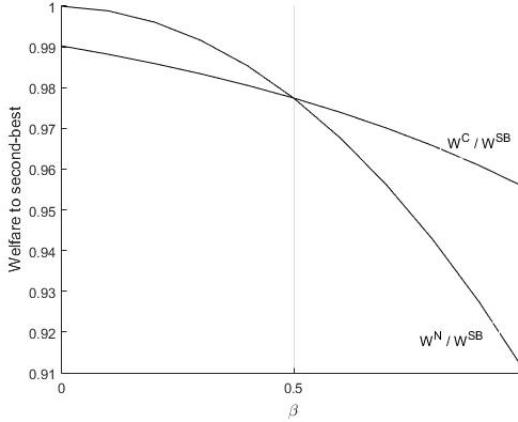


Figure 2.6: R&D-regime ranking with respect to welfare

the respective value required in second-best, expressed by condition (2.16); see in this regard figures (2.4) and (2.5), where the former is drawn for $\beta = 0$ and the latter for $\beta = 1$. From a graphical inspection of the former figure it is easy to check the validity of lemma (4): the ratio between non-cooperative investment and second-best investment is equal to unity for $\gamma = 0$. For larger values of the risk parameter, instead, the investment ratio tends to fall, meaning the fact that the market is progressively unable to deliver the socially optimal level of investment in research and development; this occurs regardless the mode-of-research chosen by firms. In this regard, R&D-risk may be interpreted to produce a distortion in the social effectiveness of the market⁸. A consequence of these considerations is that the market-equilibrium investment is *always* lower than the second-best value (the only exception is represented by lemma 4). Finally, in figure (2.5) the level of investment is greater under cooperation, unlike the situation depicted in figure (2.4) in which the higher investment is brought about by no-cooperation.

Lemma 5. *Provided that firms embark on the risky project, an increase in the R&D-risk parameter γ reduces the social desirability of the market-induced level of investment.*

Equipped with this analysis, let me turn to describing the ranking of the two modes-of-research with respect to welfare. As goods demand is linear, the welfare

⁸The implicit assumption is represented by the fact that such change in the value of γ does not shift the preference of the social planner onto the safe activity. If this assumption fails, in fact, the social planner encourages firms to embark on the safe project, the risk parameter of which is fixed and equal to zero; dropping thus the validity of the result. Even if the assumption might fail generally, it may hold locally, since *small* changes in the risk level are reasonably less able to modify the preference structure of the social planner.

function in expected terms can be rearranged in the following form:

$$E[W] = E \left[a(y_i + y_j) - \frac{b}{2}(y_i + y_j)^2 - c_i y_i - c_j y_j - 2K \right]$$

This function can be evaluated by substituting the equilibrium values of output and investment for each R&D-regime, so as to determine the regime ranking⁹. Figure (2.6) shows the level of welfare of each regime in relative terms to the level of welfare in second-best; the amount of spillovers β is given on the horizontal axis, whereas on the vertical axis is measured the welfare ratio equilibrium-to-second best. As it can be seen, the no-cooperation line intersects the vertical axis at the point $(0;1)$ and lies above the cooperation line if and only if spillovers are lower than $1/2$. The conclusion that can be drawn is that the regime ranking holding in deterministic R&D is maintained also when firms opt for the risky project: cooperation entails higher levels of welfare if and only if spillovers are sufficiently large.

Proposition 7. *The cooperative mode-of-research delivers the higher expected level of welfare if, and only if, spillovers are sufficiently large.*

This result is to read through the lens of lemma (4): the regime that entails the higher level of investment in R&D is the one delivering the higher expected level of welfare, independently of the research project developed by the firms.

2.3 Policy package to the first-best

A necessary condition for attaining the first-best is that price equal marginal cost, as in perfect competition, or industry-average marginal cost, in case of asymmetric firms, as seen in section 2.2.1. In an oligopolistic setting the output level supplied by the market is lower than the one required for the first-best, for this reason the public authority could design a scheme of output subsidies to raise the level of production. Moreover, the government can grant research subsidies to induce the socially optimal level of R&D.

2.3.1 Output subsidy

Considering the possibility for the government to grant subsidies, firm i 's profit function becomes equal to:

$$\Pi_i = (p + \tau_i - c_i)y_i - K_i + \sigma_i x_i \quad (2.18)$$

⁹The equilibrium conditions concerning output are reported in the appendix section A.2.10.

where τ_i and σ_i represent, respectively, subsidies per unit of output and per unit of expected innovation quality. The former is equivalent to an increase in price or to a reduction in the level of marginal cost, whereas the latter is given for each dollar spent on R&D investment¹⁰. The first-order condition of equation (2.18) for firm i with respect to output is equal to:

$$-by_i + p - c_i + \tau_i = 0$$

Since the cost c_i varies according to the state of the world, it is convenient to examine the four cases separately. Let us begin with the case (I,I) in which the R&D activity of both firms delivers an innovation. In equilibrium we have that $c_i = c_j = \bar{c} - x(1 + \beta)$; from which we obtain:

$$y_i^* = \frac{a - \bar{c} + x(1 + \beta) + 2\tau_i - \tau_j}{3b}$$

Market-determined output, indicated by an asterisk, depends positively on the innovation size x and the output subsidy τ_i received by the government; on the other hand, firm i 's optimal output y'_i , where prime denotes first-best, stems from condition $p = c_i$ and, as stated in equation (2.3), is equal to:

$$y'_i = \frac{a - \bar{c} + x(1 + \beta)}{2b}$$

Finally, by posing $y_i^* = y'_i$ and $y_j^* = y'_j$ we can derive the optimal expressions for τ_i and τ_j , so as to get:

$$\tau_i = \tau_j = \frac{a - \bar{c} + x(1 + \beta)}{2} = by'_i = by'_j$$

The optimal level of τ_i depends on first-best quantity: a rise in optimal output entails a rise in the unit output subsidy level. Further, the quantities supplied by the two firms are equal, and so are the firm-specific subsidies: $\tau_i = \tau_j$. Somewhat similar is the case (O;O), when the two firms do not innovate, because their marginal cost remains symmetric and equal to \bar{c} . In this situation we obtain analogous expressions:

$$y_i^* = \frac{a - \bar{c} + 2\tau_i - \tau_j}{3b}$$

$$y'_i = \frac{a - \bar{c}}{2b}$$

¹⁰Relating the R&D subsidy to the quality of the innovation instead of the level of investment is common practice in the literature, due to the one-to-one relationship between the two variables (given the research project α).

from which:

$$\tau_i = \tau_j = \frac{a - \bar{c}}{2} = by'_i$$

Symmetry grants that the quantities supplied to the market be equal across the two firms and that subsidies be equivalent; so no additional insights are provided. Let us now turn to the asymmetric case, in which there is only one innovator (say firm i). In equilibrium marginal costs are heterogeneous across firms: $c_i = \bar{c} - x$ and $c_j = \bar{c} - x\beta$. Since in first-best the market shares of the two firms are not determined by competitive dynamics, I shall assume that they are exogeneously set by the government. In particular, I shall analyse the case in which the government desires firms to produce the same amount of output, independently of the outcome of their research activity¹¹.

Assumption 2. *If there is only one innovator, government in first-best requires firms to provide the market with the same amount of output.*

From profit maximisation of each firm with respect to output we get:

$$\begin{cases} y_i^* = \frac{a - \bar{c} + x(2 - \beta) + 2\tau_i - \tau_j}{3b} \\ y_j^* = \frac{a - \bar{c} + x(2\beta - 1) + 2\tau_j - \tau_i}{3b} \end{cases}$$

These equations describe the equilibrium levels of output of the two firms. The optimal amount of subsidies is computed by setting equilibrium output equal to the first-best output given in equation (2.7). We obtain:

$$\begin{cases} \tau_i = \frac{2a - 2\bar{c} + x(3\beta - 1)}{4} \\ \tau_j = \frac{2a - 2\bar{c} + x(3 - \beta)}{4} \end{cases}$$

Lemma 6. *Given assumption 2, the innovator should receive a lower amount of output subsidy.*

From the expressions above only the special case $\beta = 1$ assures that the two subsidies be equivalent - in this case the marginal cost of the firms amounts to the same value - otherwise τ_j is always larger than τ_i . Moreover, given the level of investment in R&D, the difference between such subsidies is larger, the lower

¹¹One could be interested to make a broader analysis and set the market share of firm i equal to a generic parameter δ .

the amount of spillovers in the sector - corresponding to the situation of highest cost heterogeneity across firms¹².

2.3.2 R&D subsidy

Section 2.2.2 contains a discussion around the socially desirable research project in the case of first-best. In general the government may encourage a specific project, for instance by raising a tax on the other one so as to lower its profitability and the incentive of firms to invest on it¹³. Once selected the optimal project, the government needs to quantify the optimal amount of subsidy to address to it. On the one hand, if firms select the risky research project, the government assesses the concession of a subsidy on the basis of expected welfare, since it does not know in advance the research outcome. On the other hand, the amount of investment selected by the producers may aim at the maximisation of profits either at the firm- or at the cartel-level, according to the research regime; in turn, this affects the incentive of firms to invest and the level of subsidy that the government should grant. For exposive reasons, I shall analyse R&D cooperation and R&D no-cooperation in a separate fashion.

Non-cooperative firms

Firms maximise the expected level of profits with respect to the amount of investment K in research and development; given the level of α , this is equivalent to maximising with respect to the quality of innovation x . At this stage firms take the output first-order conditions, one for each of the possible states of the world, as a given. Thus, the problem faced by firm i is as follows:

$$\max_{x_i} E[\Pi_i] \text{ s.t. } \Pi_y^s = 0 \quad \forall s \in S \quad (2.19)$$

As shown in the appendix, in equilibrium the first-order condition of problem (2.19) is equal to:

$$\frac{\alpha(2-\beta)}{3b} \left[a - \bar{c} + \frac{x(1+\beta)(1+\alpha)}{2} \right] = K_x - \sigma$$

¹²A different result is produced dropping the assumption of symmetric production; the government could pursue different goals, for example, it could grant a higher market share to the innovator, and this would change the subsidy structure. However, symmetric market shares may represent an insurance to firms from the negative event of R&D failure and a support to their decision to invest in R&D.

¹³This argument requires the implicit assumption that firms always desire to invest in R&D, and cannot choose not to; this is likely to be more plausible in industry with high R&D-intensity, such as the automobile sector, the microelectronics sector and the like.

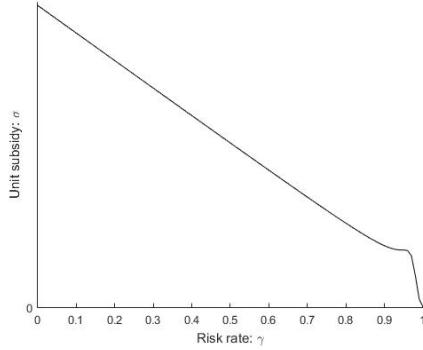


Figure 2.7: Relationship between R&D risk and unit R&D subsidy in first-best under no-cooperation

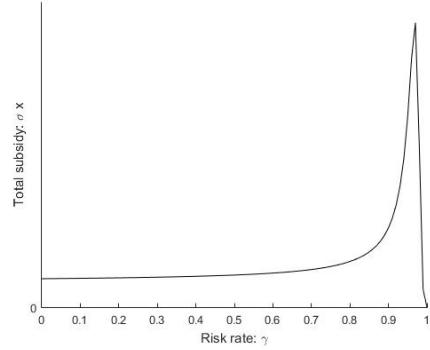


Figure 2.8: Relationship between R&D risk and total R&D subsidy in first-best under no-cooperation

where the left-hand side represents the marginal benefit from extra R&D investment, while at the right-hand side is expressed its marginal cost, net of the subsidy received by the government. By elaborating this expression we get the equilibrium level of innovation quality:

$$x^* = \frac{a - \bar{c} + \frac{3b\sigma}{\alpha(2-\beta)}}{\frac{6b\Gamma(\alpha)^2}{\alpha(2-\beta)} - \frac{(1+\beta)(1+\alpha)}{2}} \quad (2.20)$$

Since firms are symmetric, they invest the same amount of money on the same R&D project, as seen in the previous chapter of the thesis. As a result, the government is willing to grant them the same level of subsidy, that is, $\sigma_i = \sigma_j$. Equation (2.20) is to compare with equation (2.12), describing the optimal amount of innovation quality x' required in first-best. By posing $x' = x^*$ we get the optimal R&D subsidy in ‘no-cooperation’:

$$\sigma = \frac{2\Gamma(\alpha)^2(5\beta - 1)}{3(1 + \beta)} x' \quad (2.21)$$

When firms do not form an R&D cartel, the level of σ that assures the attainment of first-best depends positively on the optimal innovation size x' ; moreover, it may assume negative values when spillovers are sufficiently low (in particular, if $\beta < 1/5$).

When $\beta > 1/5$ and producers select the risky project, it is interesting to investigate the relationship between the risk rate and the amount of unit subsidy σ received by a firm. Figure (2.7) depicts such relationship for $\beta = 1$; it shows the amount of subsidy on the vertical axis and the amount of risk associated to the risky project (following the definition given in the first chapter of the present

thesis, we have that $\alpha = 1 - \gamma$, where γ captures the level of risk of the project) on the horizontal axis. The graph is falling for all values of the risk parameter, which means that the government should pay lower unit subsidies for riskier projects. Figure (2.8) shows the relationship at the aggregate level, that is, by considering the total amount of subsidy given by the term $\sigma x'$. Surprisingly, the function is nearly steady or slightly rising for most values of the risk rate, meaning that riskier projects should in general receive a higher amount of subsidy¹⁴. However, as the risk rate approaches unity, the function assumes a negative trend and quickly falls to zero, because at the limit point $\gamma = 1$ the optimal level of investment becomes null independently of the value of the unit subsidy.

Proposition 8. *In first-best and under no-cooperation, the concession of a positive subsidy requires the spillover rate to be larger than 1/5; otherwise the optimal subsidy becomes negative. If the spillover rate is larger than 1/5 and the risk level is not excessively close to unity, the government should grant a lower unit subsidy but larger total subsidies to riskier projects.*

Cooperative agreements

In the subgame-perfect equilibrium, firms carry research activities in a cooperative manner forming a research joint venture - see chapter 1. In this case the problem of firm i consists of the maximisation of cartel-wide profits, as follows:

$$\max_{x_i} E[\bar{\Pi}] = \sum_{s \in S} pr(s)[\Pi_i^s + \Pi_j^s] \quad (2.22)$$

subject to the holding of output first-order conditions for each state of the world. We can calculate the derivative of the objective function and set it equal to zero as to get:

$$\frac{\alpha(1+\beta)}{3b} \left[a - \bar{c} + \frac{x(1+\beta)(1+\alpha)}{2} \right] = K_x - \sigma$$

in which the marginal benefit and the marginal cost of R&D are expressed, respectively, at the left-hand side and the right-hand side. To describe the equilibrium amount of innovation of each firm, we can arrange this condition into the following:

$$x^* = \frac{a - \bar{c} + \frac{3b\sigma}{\alpha(1+\beta)}}{\frac{6b\Gamma(\alpha)^2}{\alpha(1+\beta)} - \frac{(1+\beta)(1+\alpha)}{2}}$$

¹⁴This result, perhaps counter-intuitive, finds in fact evidence in many real-world examples; it is sufficient, for instance, to consider the pharmaceutical industry, where the average risk of R&D failure is very high but, at the same time, the larger part of the investment in R&D derives from public funding. See Stiglitz and Jayadev (2010) relative to the US case.

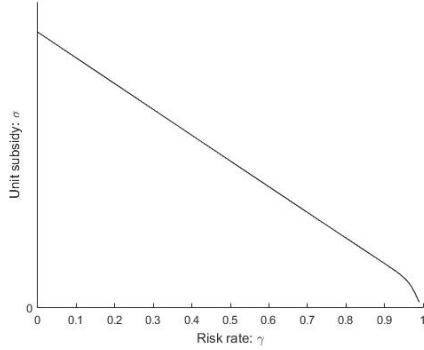


Figure 2.9: Relationship between R&D risk and unit R&D subsidy in second-best under no-cooperation

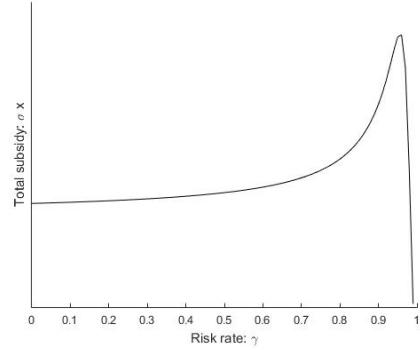


Figure 2.10: Relationship between R&D risk and total R&D subsidy in second-best under no-cooperation

Let us observe that this expression is increasing in the amount of σ , since the greater the unit subsidy, the greater the incentive to invest. Finally we can set $x^* = x'$, the value of which is given in condition (2.12), to get the optimal amount of unit subsidy:

$$\sigma = \frac{2}{3} \Gamma(\alpha)^2 x' \quad (2.23)$$

Unlike no-cooperation, a low amount of spillovers does not justify alone the imposition of a tax on cooperative agreements, since σ in equation (2.23) is non-negative. Moreover, it is possible to show that the unit subsidy falls when the risk rate grows larger; and that total subsidy rises, like the case of non cooperative firms, provided that the risk rate is not excessively close to unity. For this reason the government should give larger subsidy to riskier projects (see the appendix).

Proposition 9. *In first-best and under cooperative agreements, it does not exist a level of technological spillovers β such to justify the imposition of a research tax. Moreover, the total amount of subsidy $\sigma x'$ should be larger, the larger the risk level of the research project, provided it is not excessively close to unity.*

2.4 Policy package to the second-best

In this section it is assumed that the social planner is not able to impose a subsidy to output, which is therefore determined by market dynamics. To induce firms to behaving in an optimal manner, the only instrument available to the government is thus an R&D subsidy. Such behaviour is described in condition (2.16), which indicates the optimal amount of innovation in second-best.

2.4.1 Non-cooperative firms

If firm i selects the ‘no-cooperation’ regime, it maximises profits subject to the first-order conditions for output:

$$\begin{aligned} \max_{x_i} E[\Pi] &= \sum_{s \in S} pr(s)\Pi_i^s \\ \text{s.t. } \forall s \in S, \Pi_y^s &= 0 \end{aligned}$$

In equilibrium the first-order condition is equal to:

$$\frac{2\alpha(2-\beta)}{9b}[a - \bar{c} + x(2-\beta) + \alpha x(2\beta-1)] = K_x - \sigma$$

This can be rearranged so as to obtain the equilibrium amount of innovation:

$$x^* = \frac{2\alpha(2-\beta)(a - \bar{c}) + 9b\sigma}{18b\Gamma(\alpha)^2 - 2\alpha(2-\beta)^2 - 2\alpha^2(2\beta-1)(2-\beta)}$$

To compute the optimal level of unit subsidy, this value has to be set equal to the second-best amount of innovation x'' (double prime denotes second-best) given by condition (2.16). We get:

$$\sigma = \frac{2\alpha[\frac{9b\Gamma(\alpha)^2}{\alpha} - (2-\beta)^2 - \alpha(2\beta-1)(2-\beta)]x'' - 2\alpha(a - \bar{c})(2-\beta)}{9b}$$

The proof of the following proposition is in the appendix:

Proposition 10. *In the second-best non cooperative case, the optimal amount of unit research subsidy is a linear function of the amount of innovation x'' . Simulations suggest that it is always non-negative and that for most values of the parameters the government should grant lower unit subsidies but larger total subsidies to riskier research activities.*

This statement clarifies the relationship between second-best amount of innovation and the level of unit subsidy, which, unlike the first-best case, exceeds zero for any value of the risk and the spillover rates. More specifically, figure (2.9) represents how a variation in the risk rate, measured on the horizontal axis, impacts the level of σ , on the vertical axis. As it can be seen, the function is decreasing, which means that the subsidy per unit of innovation should fall as the risk rate grows large. On the other hand, figure (2.10) depicts the relationship between risk rate and total subsidy $\sigma x''$. Even if σ falls, the total amount of subsidy is steady, and eventually rising, for most value of γ , because x'' outgrows the reduction in σ . Consequently, this simulation suggests that, in case of R&D no-cooperation and provided that the risk level is not too close to unity, the government should grant higher subsidies to riskier projects not only in first-best (see proposition 8), but also in second-best.

2.4.2 Cooperative agreements

The same analysis must be carried in second best under the ‘cooperative’ equilibrium, in which firms form a research joint venture and maximise joint-profits, as follows:

$$\begin{aligned} \max_{x_i} \quad & E[\bar{\Pi}] = E[by_i^2 - K_i + \sigma_i x_i + by_j^2 - K_j + \sigma_j x_j] \\ \text{s.t.} \quad & \forall s \in S, \Pi_y^s = 0 \end{aligned}$$

In equilibrium, the first-order condition of this maximisation problem is represented by the following equation:

$$2\alpha \frac{(a - \bar{c})(1 + \beta) + x((2 - \beta)^2 + (2\beta - 1)^2) + 2\alpha x(2 - \beta)(2\beta - 1)}{9b} = K_x - \sigma$$

from which we get:

$$x^* = \frac{(a - \bar{c})(1 + \beta) + \frac{9b\sigma}{2\alpha}}{\frac{9b\Gamma(\alpha)^2}{\alpha} - (2 - \beta)^2 - (2\beta - 1)^2 - 2\alpha(2 - \beta)(2\beta - 1)}$$

This represents the equilibrium amount of innovation pursued by a research joint venture. The level of subsidy is determined by the government so as to obtain the equality $x^* = x''$, from which we get:

$$\sigma = \frac{2\alpha \left[\frac{9b\Gamma(\alpha)^2}{\alpha} - (2 - \beta)^2 - (2\beta - 1)^2 - 2\alpha(2 - \beta)(2\beta - 1) \right] x'' - 2\alpha(a - \bar{c})(1 + \beta)}{9b}$$

In the appendix I prove the following:

Proposition 11. *In the second-best cooperative scenario, the optimal amount of unit research subsidy is a linear function of the amount of innovation x'' . Simulations suggest that it is always non-negative and that the government should grant lower unit subsidies but larger total subsidies to riskier research activities.*

The proposition is in line with the results obtained in no-cooperation; in particular, in second-best the level of unit subsidy is positive independently of the spillover rate, thus it does not exist a level of β able to justify the imposition of a tax. Moreover, whereas the unit subsidy σ is decreasing with respect to the risk rate, the total subsidy $\sigma x''$ is increasing, since x'' outgrows the reduction in σ as the risk rate becomes larger. Therefore, provided that the risk rate is not too close to unity, the government funds riskier projects to a larger extent.

2.4.3 Comparison of the subsidies between the two R&D-regimes

The analysis carried out throughout section 2.4 allows to make a comparison between second-best subsidies in the two different research regimes. When granting a subsidy, the goal of the social planner is to affect the behaviour of the firms so as to vary the equilibrium induced by the market and to increase the level of welfare. In general, it may grant a positive subsidy if the equilibrium investment needs to be *increased*, whereas it may raise a tax if such value should be *decreased*; and the higher the differential between market outcome and social optimum, the larger the level of the subsidy (in absolute value). Let us recall proposition (7), by which the welfare expected level produced under the cooperative regime is closer to the social optimum than the one related to the non-cooperative regime if, and only if, the spillover rate is sufficiently large; and lemma (4), which confirms that, when spillovers are large, cooperative investments are higher than non-cooperative investments in absence of subsidies. Thus, an implication of these results is as follows:

Corollary 1. *In second-best, research joint ventures require a lower level of subsidy than non-cooperative firms if, and only if, spillovers are sufficiently large.*

The intuition is very clear: with high spillovers, research joint ventures has larger an incentive to invest than non-cooperative firms; the differential between social optimum and market outcome is thus smaller when firms cooperate and, thus, the government should reduce the amount of subsidy.

2.5 Concluding remarks

My paper contributes to the strand of the literature on R&D by developing a model in quantity competition in which firms have the possibility to decide the research project to undertake (as well as the amount of research and development to carry out and the organizational level at which performing it - firm level or research joint ventures). While the first chapter of the present thesis was centred around private incentives towards R&D, the focus in this work is on the social planner, in particular, its preferences concerning R&D and the instruments which should be used in order to attain the social optimum.

The main novelty of this work is the introduction of a range of different R&D projects which differ by the level of risk. Unlike most papers in the stochastic R&D literature in which there exists one project, in this paper firms are allowed to select the optimal innovation project. Such approach has the upside to describe how the decision of an R&D project may produce an impact on the level of welfare

in a country, and how a government should encourage the development of a project rather than the others.

The model is able to produce results in line with the relevant literature. In second-best, compared to the social optimum represented by the first-best, firms overinvest in R&D, as pointed out also by Leahy and Neary (1997); the spillover rate affects positively the optimal amount of investment in R&D, given the research project firms want to undertake; cooperation entails a higher level of welfare than no-cooperation if, and only if, spillovers are sufficiently large; and a tax on R&D should be raised only in first-best under no-cooperation, but never under cooperative agreements.

The scope of the paper goes beyond these aspects. The stochasticity of welfare is at the centre of the analysis, and I showed the optimality conditions in each possible state of the world, distinguishing according to the ability of firms to innovate. In particular, in the case in which only one firm is able to obtain a new technology, the price level should be equal to the average of the marginal cost of the two firms, due to the presence of cost-asymmetry. Moreover, simulations suggest that in first-best government prefers to embark on safe activities when spillover are sufficiently low, whereas high a spillover rate is associated with riskier projects; on the other hand, in second-best analysis riskier activities are likely to be welfare-maximising also when spillovers are sufficiently low.

Furthermore, I analysed the effect of the risk rate related to R&D on the equilibria arising from the market. I show that the R&D-risk raises the gap between social optimum and market outcome, meaning that greater risk is able to lower the social desirability of markets. As a consequence, by assuming that a given project is the welfare maximiser, an increase in the risk rate of such project is shown to require greater subsidies from the government. The result finds confirmation in some relevant evidence: Stiglitz and Jayadev (2010) for example point out that only one-third of R&D in the pharmaceutical industry, typically characterised by high degree of failure, derives from the private sector.

Finally, I prove that different projects deliver different levels of welfare, according to which the social planner forms its preferences concerning R&D. As a result, there exists an economic justification for the government to grant subsidies to a specific activity, whereas the other non welfare-maximising ones should either not receive a subsidy or, even, be taxed, in order to dissuade firms from undertaking them. The only case in which a tax on the socially desirable project should be raised by the government is represented by firms researching in a non-cooperative manner within an industry characterised by low spillovers.

In this model there exist only two possible projects representing the ‘safe’ and the ‘risky’ R&D activity. This set-up allows to describe the ‘preference for R&D-risk’ of a government; the precision of the results, however, could be increased by considering a continuum of possible projects among which firms can choose; this

would in turn allow to measure the exact change in the government's risk-attitude with the variation of other variables, such as the spillover rate and the degree of cooperation in R&D. Additionally, as a further channel of future research, this model may constitute a benchmark to analyse how subsidy policy is influenced by strategic interactions with foreign countries. This aspect in fact represents the centre of the third chapter of the thesis.

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Chapter 3

Subsidy policy in an open economy framework

3.1 Introduction

The concession of subsidies from a government to the domestic firms in an international competition framework has been common practice in the real world with the purpose of increasing competitiveness. However, pioneering works on the area, based upon models of perfect competition, were not able to justify the adoption of such subsidies. The Japanese success alleged to this kind of policy stimulated the debate and led to further methods of analyses and contributions. Brander and Spencer (1985) are the first authors to apply a model of imperfect competition, particularly a Cournot duopoly. Their main finding is that export subsidies, conceded by the government to the firm per unit of output exported, triggers a ‘profit-shifting’ from the foreign competitors to domestic producers, with an increase in domestic welfare. Eaton and Grossman (1986) show that this result is strictly connected to model qualifications. They adopt a model of Bertrand competition and prove that the optimal export subsidy becomes *negative* (a tax). As Neary (1994) points out, however, Cournot plays such a central role in international trade literature that its implications remain highly influential in the debate. The author explores the effect of cost heterogeneity across countries on government intervention and finds that subsidies should be addressed towards those industries where the country has a cost-advantage over foreign ones. Also Collie (1993) investigates cost heterogeneity; under asymmetric oligopoly export subsidies produce a rationalisation effect able to either enhance or weaken the profit shifting argument according to the convexity of goods demand.

These contributions shed light on the economic rationale underlying export subsidy and may be extended to analyse further kinds of strategic trade policy (Collie, 1993). However, the policy of granting subsidies to R&D activities de-

serves to be addressed separately; such kind of subsidy is generally payed by the government to the firm per dollar of investment spent in research and development. First, research activities are of great importance to the survival and growth of firms, particularly in mid- and high-technology sectors; in fact the expense in R&D, defined as the sum of private and public investments, has been steadily growing across countries since the 1990s. Second, export and research subsidies may pursue common objectives only to a partial extent and, thus, are not perfect substitutes. More specifically, the former aims at increasing the amount of export of the domestic firms so as to increase profits; for the producer it is equivalent to a reduction in marginal costs. The latter, on the other hand, contributes to a reduction in marginal costs through process innovations; however, it may also encourage the development of new products. Third and most importantly, research activities produce an impact not only on the researching firm, but also on its competitors through the working of technological spillovers, as pointed out by Jaffe (1989) and Feldman (1994); moreover firms may decide to collude at the R&D level by forming an R&D cartel. Technological spillover and research collusion are two elements that are not taken account of by general export subsidy models, since the former is strictly related to the development of a new technology and the latter is forbidden by law at the output-level. For this reason a different approach needs to be considered.

The literature discusses some implications for research subsidy policy. Leahy and Neary (1997) develop a closed economy model where research subsidies only impact internal markets, and discriminate between R&D competition and R&D cooperation. They find that a government should generally grant positive subsidies; the only exception is represented by R&D competition in case of low spillovers, where the imposition of an R&D tax is justified. Moreover Leahy and Neary (1999) address the issue in an open economy framework. The authors focus on the role of strategic behaviour on the implications for policy and analyse two kinds of models, one in which the R&D of a firm affects the output behaviour of the rival and one where it does not, and assess how optimal policy changes. In particular, they show that in the latter model international spillovers make a case for R&D tax, whereas in the former the results are less clear-cut because strategic behaviour may require positive subsidies. Zhou *et al* (2002) focus on strategic trade policy with endogeneous choice of quality of goods. Firms belong to two different countries (developed economy vs developing economy) and may decide to invest to raise the quality of their products and maximise profits from export. The authors prove that both a tax or a subsidy may be motivated depending on the competition model (Bertrand vs Cournot) and on the possibility of intervention (one-country intervention vs both-countries intervention).

I provide a discussion on R&D subsidies making use of the model of R&D developed in this thesis. The effects of technological spillover and the possibility

for firms to cooperate at the research level in open economies are aspects which can be further explored¹. On the one hand, if the spillover rate is sufficiently large, a research subsidy stimulating domestic investment in R&D is likely to produce a significant benefit for the foreign firm, weakening the argument for profit-shifting of subsidy policy. On the other hand, also R&D cooperation among firms, widely encouraged at the international-level within both the US and the EU², may in principle help the transmission of subsidy benefits to foreign firms.

I develop a partial equilibrium model of international competition with two firms. They produce the same homogeneous good but belong to different countries (Home and Foreign), so they are monopolist at the national level. Moreover, there exists a third-country which satisfies entirely the demand of the good by importing from the two producers. At the international level the two firms compete in a Cournot duopoly. In addition to goods production, the two firms may invest in research and development to increase their efficiency. The investment of each firm is able to produce a new technology which can be used by the researching firm to reduce marginal costs. However, also the rival receives a benefit due to technological spillovers. The extent of such benefit is measured by the spillover rate, assumed to be given. The presence of spillovers is strictly connected with the possibility of free-riding on technological development: one firm may decide to lower the investment level because it can increase its efficiency by receiving the spillover of the rival. To overcome this issue, the two firms have the possibility to coordinate their research activity by forming a research cartel. It consists of an agreement by which firms commit to the amount of investment that maximises profits at the cartel-level, rather than at the firm-level. In general, they are willing to sign such an agreement if it is profitable for both. Finally, the home government may unilaterally decide to grant a research subsidy to attain the social optimum, that is, to maximise the level of welfare. Given the structure of the model, domestic welfare is given by domestic profits minus the subsidy bill. Consumer surplus does not enter the welfare function due to the assumption of export towards a third-country. The solution of subgame perfect equilibria allows to establish a relationship between the level of home subsidy and the level of domestic welfare, and to describe the variables that affect the optimal degree of government intervention.

The main contribution of the paper is to provide a description of the factors that affect the adoption of a research subsidy. On the one hand a research subsidy

¹Leahy and Neary (1999) consider a first-best scenario in which the government is able to grant output subsidies to domestic firms. This type of policy however is limited within economic unions and other free-trade agreements. In a second-best analysis, where only research subsidies may be dispensed, I show that some results may be overturned.

²Horizon2020 is a programme funded by the European Commission with the goal of spreading technology across countries by enhancing the instrument of R&D cooperation.

raises the incentive to invest in R&D, and is particularly needed when private incentives are low due to specific market conditions. This situation corresponds to the case of large spillovers, since they are linked to free-ride issues. On the other hand, the presence of large spillovers may also have the opposite effect of reducing the need for subsidies, providing thus a counter-argument itself, because the spillover rate transfers the improvement in efficiency fostered by the subsidisation activity from the recipient firm to the foreign rival. For this reason, in presence of large spillovers a positive subsidy is likely to benefit both countries and reduce the capability of profit-shifting. The degree of cooperation among the two firms determines which force prevails over the other. In case of R&D competition, the former argument is proved to be stronger and requires the application of positive subsidies. In the case of R&D cooperation, however, the free-ride issue is dampened by the cooperative agreement: the latter argument prevails and the optimal amount of subsidy becomes *negative* (a tax).

Secondly, I carry out an analysis to explore the private incentives of firms to form an international R&D cartel, as well as the preference of governments in this regard. In an international competition framework the firms may have the incentive to compete at the R&D level, especially when spillovers are large, unlike previous contributions in closed economies showing that R&D cooperation is always more profitable³. Furthermore, in the open economy framework simulations suggest that R&D-cooperation is always preferable from a social perspective, and that government intervention is required to encourage its development. This result is in contrast with closed economy analyses, in which cooperation is welfare-maximising only when spillovers are sufficiently large.

Finally, I test the relevance of some traditional assumptions, that is, cost homogeneity and absence of subsidy social cost, on the results of the paper⁴. Introducing cost heterogeneity allows to show that the size of a firm has an impact on the amount of subsidy that a government should grant. The sign of such impact, however, depends on the research regime: only in R&D cooperation an increase in the domestic firm size reduces the need for government intervention; otherwise, more efficient firms enhance the profit-shifting argument of trade policy and require greater subsidies. On the other hand, the presence of subsidy social cost always reduce the level of subsidy, as shown by previous contributions on subsidy policy.

The paper is organised as follows. Section 2 describes the components of the model. Section 3 focuses on the case of R&D competition, whereas section 4 highlights the implications of R&D cooperation. Section 5 is dedicated to the differences between the two research regimes, particularly with respect to profits

³See Leahy and Neary (1997) and Erkal and Piccinin (2010) for a discussion.

⁴Neary (1994) and Leahy and Montagna (2001), for instance, relax these two assumptions in the regard of export subsidies.

and welfare. Section 6 and section 7 address, respectively, the relevance of cost heterogeneity and subsidy social cost. Section 8 concludes.

3.2 Model

This is a partial equilibrium model of international trade consisting of two different firms located in two different countries, say country ‘Home’ and country ‘Foreign’ (I shall refer to them with the letters H and F). They provide a third-country with the supply of a homogeneous good; therefore, in such country the two firms compete in a duopoly, assumed to be à la Cournot. The demand of the good is assumed to be linear and equal to:

$$p = a - b\bar{y} \quad (a, b > 0)$$

where p is price and \bar{y} is the total amount of output in the market.

At the beginning, the firms share the same level of technology, thus they have the same marginal cost \bar{c} , constant with respect to the output amount; however, they have the possibility to invest in research and development in order to implement better technologies into the production process, with a consequent reduction in the marginal cost level. Such reduction x is related to the amount of R&D investment K according to the function:

$$x = \sqrt{K} \tag{3.1}$$

equivalent to $K = x^2$. This function, largely adopted in the literature (see for example Leahy and Neary, 1997) describes falling marginal returns⁵, which means that the amount of investment required to obtain a given level of cost reduction is progressively larger. Moreover, a firm is able to observe the technology developed by the competitor: R&D activities produce knowledge which cannot be entirely held within the firm and, in fact, partially spills over to the rival; the spillover rate is captured by the parameter β lying between zero and one and affects directly the level of technology. The marginal cost of a firm, say the domestic one, is thus:

$$c = \bar{c} - x - \beta x^*$$

where the superscript ‘star’ denotes the foreign firm. The marginal cost c is equal to the pre-investment level \bar{c} minus the reduction associated to own R&D, minus the spillover deriving from the rival’s R&D.

As observed by Spence (1984) and formalised by d’Aspremont and Jacquemin (1988), the presence of technological spillovers affects the private incentive of firms

⁵An economic justification is that a large fraction of innovations derive from small firms even if the capital they can invest is relatively low.

towards R&D investment: the larger the capability of the rivals to benefit from one firm's R&D, the lower its incentive to invest. To overcome this issue, firms can decide to cooperate and coordinate their research activity, forming an R&D cartel (research joint venture). The goal of the participants in the research joint venture is to maximise the joint-level of profits, rather than own revenues (I assume that the creation of an R&D cartel does not impose extra-costs to the firms). Let us remark the fact that cooperation is allowed only for R&D activities, whereas any collusive behaviour at the product-level is forbidden by law.

The government of country H has the possibility to subsidize (or tax) the R&D activity carried by the domestic firm. The subsidy is paid for every dollar spent in research and development and is endogenously determined by the social planner so as to maximise domestic welfare, defined as the amount of profit of the domestic firm minus the subsidy bill. Noteworthily, consumer surplus does not enter the objective function of country H 's government because production is directed towards a third-country⁶. Finally, I assume that the social planner of country F cannot provide any subsidy to the foreign firm. By following this approach, country H 's behaviour is not affected by country F 's best response, so that I may disentangle economic reasons from strategic ones.

The model is set out in four stages. In the first one, the government of country H chooses the level of the subsidy to R&D, able to modify the incentive to invest of the domestic firm. In stage two, the firms decide the R&D regime. They can either carry out their research activity at the firm-level or cooperate by forming a research joint venture. In stage three, the level of investment in R&D activities is selected so as to maximise profits, which might be defined at the firm-level or at the cartel-level according to the R&D regime selected at the previous stage. In stage four, firms compete à la Cournot over the quantity of the good to produce.

I shall focus on subgame perfect equilibria, thus the method of solution of this game is backward induction. In stage four the two firms play Cournot and choose the output amount to maximise the level of profits. The analysis carried hereafter concerns the domestic firms but is analogously applicable to the foreign one. The profit function is equal to:

$$\Pi = (p - c)y - K + \sigma x \quad (3.2)$$

where y is the output level of the firm and σ is the unit R&D subsidy, which is related to the size of innovation x instead of the level of investment K - such simplification is conceptually harmless because K and x are in the one-to-one

⁶This assumption is largely adopted in international trade literature, because it has the upside to have greater tractability and allows to focus on the impact of trade policy on international competition. However, it ignores the effect of trade policy on internal markets, the discussion of which requires different approaches.

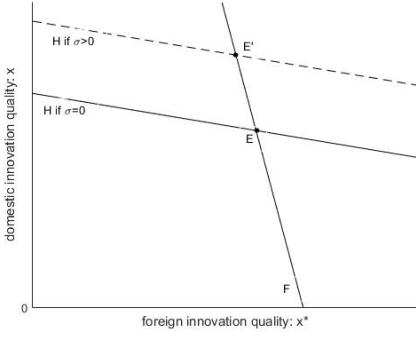


Figure 3.1: Reaction function in 'no-cooperation' for $\beta = 0$

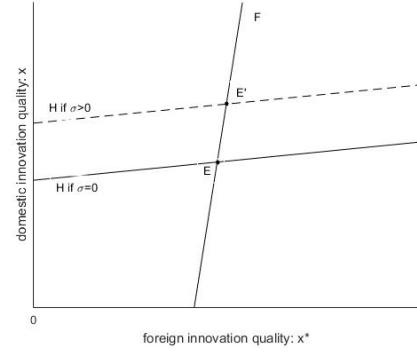


Figure 3.2: Reaction function in 'no-cooperation' for $\beta = 1$

relationship described by equation (3.1). The maximisation problem with respect to y yields the following first-order condition:

$$by = p - c \quad (3.3)$$

from which we get:

$$y = \frac{a - \bar{c} + x(2 - \beta) + x^*(2\beta - 1)}{3b}$$

as in standard Cournot duopoly models.

Remark 6. *The amount of investment in R&D of the domestic firm increases its output level; whereas the investment in R&D of the foreign firm raises domestic output if, and only if, spillovers are larger than 1/2.*

3.3 R&D competition

The degree of cooperation at the R&D level affects the decisions taken by firms. To help the reader, it is thus convenient to address the different cases separately: in this section I shall investigate the case of R&D competition, in which firms do not cooperate in the development of a new technology. They maximise the profit level subject to the holding of the output first-order condition - represented by condition (3.3). The control variable of the firm at this stage is represented by K , however this is equivalent to maximise with respect to x given the one-to-one relationship existing between this two variables (stated in equation 3.1).

Plugging first-order condition (3.3) into the profit function of the firm we obtain the following maximisation problem of non-cooperative firms:

$$\max_x \Pi = by^2 - K + \sigma x$$

The derivative of this objective function with respect to x gives:

$$2by \frac{dy}{dx} = \frac{dK}{dx} - \sigma$$

where the marginal benefit of R&D investment is expressed at the left-hand side and consists of the increase in output due to the better technology, whereas the marginal cost is at the right-hand side and is formed by the rise in the investment term minus the subsidy received by the government. After some algebraic manipulation we get:

$$x = \frac{9b\sigma + 2(2 - \beta)[a - \bar{c} + x^*(2\beta - 1)]}{18b - 2(2 - \beta)^2} \quad (3.4)$$

This equation represents the reaction function of the domestic firm vis-à-vis the rival, as it states a relationship between the investment levels of the two firms, x and x^* . As it can be seen, the sign of the relationship depends on the spillover rate: if β exceeds 1/2, any increase in x^* determines a rise in x , whereas the opposite holds true when β is smaller than 1/2. In other words, the spillover rate affects the degree of complementarity existing between the investments of the two firms; in particular, I refer to strategic complementarity in the former case and to strategic substitutability in the latter.

Lemma 7. *Under no-cooperation, the investment decisions of the two firms are complementary if, and only if, the spillover rate is larger than 1/2. Moreover $\beta = 1/2$ cancels off any strategic interaction between the two firms.*

When spillovers are large, firms may decide to lower the level of investment and free-ride on the rival's technological development, as pointed out by Spence (1984). This argument provides the lemma with a clear interpretation: if the foreign firm reduces the effort in R&D, the domestic firm should respond by lowering its investment, not to pay for the technological development of its rival; a similar guidance is provided by Kats (1986). When the spillover rate is low, instead, free-riding concerns becomes negligible and the investment decisions of the firms cease to be complementary; in fact, if the foreign firm lowers its investment, the domestic firm should increase it due to strategic substitutability.

The equilibrium value of x under no-cooperation is obtained substituting the reaction function of the foreign firm, analogous to the one of the domestic firm

once set $\sigma = 0$, into equation (3.4). We get:

$$\begin{aligned}
x_n &= \frac{9b[18b - 2(2 - \beta)^2]}{[18b - 2(2 - \beta)^2]^2 - [2(2\beta - 1)(2 - \beta)]^2} \sigma + \\
&+ \frac{2(2 - \beta)(a - \bar{c})}{18b - 2(2 - \beta)^2 - 2(2\beta - 1)(2 - \beta)} \equiv \\
&\equiv m_n \sigma + q_n
\end{aligned} \tag{3.5}$$

where the subscript n refers to the ‘no-cooperation’ regime. Let us analyse the relationship existing between the subsidy σ and the investment of the domestic firm. Equation (3.5) constitutes a linear function, since σ appears only once with a unit exponent, where m_n measures the ‘slope’ and q_n is the ‘intercept’ on the vertical axis on a (σ, x) -graph. For such relationship to have economic meaning, we require the coefficient m_n to be positive, so that an increase in σ could produce an increase in domestic investment⁷. In addition, the subsidy of country H may affect the behaviour of the foreign firm for the strategic argument expressed in lemma (7). It is possible to state the following:

Lemma 8. *Under no-cooperation, if the spillover rate is larger than 1/2, a positive subsidy to R&D from the government of country H induces higher investment in R&D for both the domestic firm and the foreign firm.*

In fact, it is easy to show that the equation of optimal amount of foreign investment is as follows:

$$\begin{aligned}
x_n^* &= \frac{18b(2 - \beta)(2\beta - 1)}{[18b - 2(2 - \beta)^2]^2 - [2(2\beta - 1)(2 - \beta)]^2} \sigma + \\
&+ \frac{2(2 - \beta)(a - \bar{c})}{18b - 2(2 - \beta)^2 - 2(2\beta - 1)(2 - \beta)} \equiv \\
&\equiv m_n^* \sigma + q_n
\end{aligned} \tag{3.6}$$

where the sign of m_n^* depends on β : this is positive if and only if the spillover rate is larger than 1/2. Further, the intercept on the vertical axis is equivalent for both x and x^* and equal to q_n . This is due to the assumption of symmetry: since firms have the same level of pre-investment technology, in absence of subsidies ($\sigma = 0$) they invest the same amount of resources on R&D activities.

Figures (3.1) and (3.2) depicts graphically the intuition behind lemma (8) in the case of $\beta = 0$ and of $\beta = 1$; x^* is measured on the horizontal axis while

⁷Analitically, we need for any β : $\frac{9b[18b - 2(2 - \beta)^2]}{[18b - 2(2 - \beta)^2]^2 - [2(2\beta - 1)(2 - \beta)]^2} > 0$, which is verified if $b > 1/4$

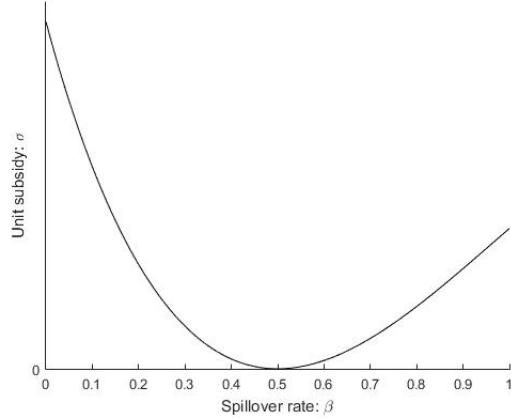


Figure 3.3: Relationship between the unit subsidy σ and the spillover rate β under R&D competition

x lies on the vertical one. The former figure represents the case of investment substitutability because the spillover rate is equal to zero. Line F is the reaction function of the foreign firm, whereas the continuous line H is the best response of the domestic firm in case of zero subsidy; they are both downward sloped due to investment substitutability, because an increase in x^* determines a fall in x and viceversa. The equilibrium is identified by the intersection point E , which dictates symmetric investments due to the assumption of symmetric firms. With government intervention and the implementation of a positive subsidy, the domestic firm's reaction function shifts to the locus represented by the dashed line, and at the new equilibrium, E' , the level of investment raises for the domestic firm and falls for the foreign one. Instead, the latter figure considers the case of investment complementarity because the spillover rate amounts to unity. The reaction functions have an upward slope and, as result, the presence of a positive subsidy, shifting the equilibrium from E to E' , embeds higher levels of investement in R&D for both firms - as expected from lemma (8).

3.3.1 Subsidy under R&D competition

The government of country H , unlike country F 's, is able to grant a subsidy to research and development activities. The subsidy is dispensed in the form of investment reimbursement; however, given the one-to-one relationship between K and x , this is equivalent to relate the amount of subsidy to the size of the innovation implemented. The goal of the social planner is to maximise the level of national welfare W , given by the amount of domestic profits minus the subsidy bill. The surplus of consumers, usually taken into account in welfare evaluation,

can be ignored because the export activity concerns a third-country, where the dynamics of the price level does not affect the consumers of country H . This approach allows to focus on the effect that R&D subsidies produce on international competition solely. The problem solved by the government H is as follows:

$$\max_{\sigma} W = \Pi - \delta\sigma x = by^2 - K + (1 - \delta)\sigma x$$

The subsidy bill is affected by the parameter δ which captures the social cost associated to subsidies. There are several reasons to believe that δ is larger than one, which means that the government is not willing to trade off one dollar of extra profits with one dollar of subsidy payments. For instance, the resources for the subsidy may be obtained by raising a distortionary tax on the population, with negative effects on efficiency; or the firm may be partially owned by foreign investors, with a shift in profits from the national account towards foreign countries⁸. In fact, one may consider $\delta = 1$ as a rather special case. Computing the derivative with respect to σ , we get the following first-order condition:

$$2by \left[\frac{dx}{d\sigma} \frac{2 - \beta}{3b} + \frac{dx^*}{d\sigma} \frac{2\beta - 1}{3b} \right] = 2x \frac{dx}{d\sigma} - (1 - \delta) \left(x + \sigma \frac{dx}{d\sigma} \right) \quad (3.7)$$

At the left-hand side there is the marginal benefit deriving from greater subsidy, represented by the rise in domestic profits; the term at the right-hand side, instead, expresses the marginal cost of subsidy, which consists of two terms: the extra investment firms are induced to carry out and the social cost associated to subsidy payments. The optimal amount of subsidy corresponds to the level that equalizes marginal benefit and marginal cost. By making use of equations (3.5) and (3.6) to calculate the derivatives, the subsidy first-order condition can be developed to obtain the optimal value of σ :

$$\sigma_n = \frac{2[(2 - \beta)m_n + (2\beta - 1)m_n^*][a - \bar{c} + (1 + \beta)q_n] - 9bq_n(2m_n - 1 + \delta)}{18bm_n(m_n - 1 + \delta) - 2[(2 - \beta)m_n + (2\beta - 1)m_n^*]^2} \quad (3.8)$$

First, let us analyse the impact of δ on the value of subsidy. An increase in δ , in particular, which correspond to a higher social cost of subsidy payments, unambiguously lowers the numerator and raises the denominator, decreasing the optimal amount of unit subsidy⁹. Second, it is convenient to study the sign of the subsidy, to understand whether an R&D subsidy is preferred to an R&D tax; in this section I shall assume $\delta = 1$ to state simpler results¹⁰. Figure (3.3) serves this purpose, showing on the axes the spillover rate and the unit subsidy. As it can

⁸For a thorough discussion and a microfoundation of δ , see Lahiri et al. (2000), and Leahy and Montagna (2001).

⁹A graphical analysis on the role of δ is provided in section 3.7.

¹⁰The general case is addressed in section 3.7

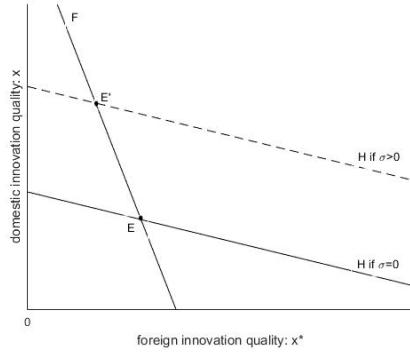


Figure 3.4: Reaction function in 'co-operation' for $\beta = 0$

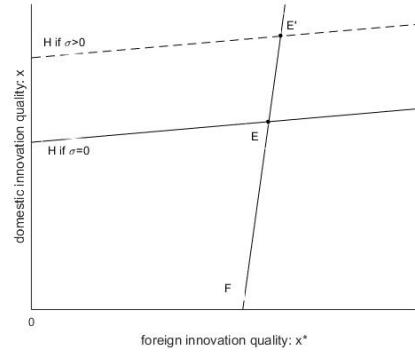


Figure 3.5: Reaction function in 'co-operation' for $\beta = 1$

be seen, for $\beta = 0$ the function lies above the horizontal axis: in the case of no technological spillovers the government should grant a positive subsidy to R&D. However, it falls as β grows larger and becomes null for $\beta = 1/2$. The reason is that this value of the spillover rate cancels off any strategic interaction between the two firms at the R&D level, which means that the investment of one firm does not affect the one of the rival (see lemma 7). In this case, each firm can select the level of investment as a quasi-monopolist, not being constrained by the rival best-response, and achieve both profit and welfare maximisation (provided welfare is not affected by consumer surplus, as in the current analysis). When β is larger than $1/2$, on the other hand, technological development of one firm spills over to its rival to a great extent. This enlarges the scope for free-riding, since a firm may become more efficient by observing and copying the rival's technology, without the need of engaging in R&D activities. In this case industry-wide investment is likely to tend towards inefficiently low values. As confirmed by the figure, when the spillover rate is large the government must grant a positive subsidy in order to restore a higher level of investment.

Proposition 12. *Under R&D competition and in the absence of subsidy social costs ($\delta = 1$), the optimal amount of unit subsidy σ is non-negative. Moreover, $\sigma = 0$ if and only if $\beta = 1/2$.*

3.4 R&D cooperation

I have recalled the fact that the presence of large technological spillovers may lower the incentive of firms to engage in the development of a new technology, since any benefit is largely shared with the rivals; and that investments at the industry-wide

level may drop to inefficient values. The creation of an R&D cartel however is able to internalize the flows of technological spillovers and correct this distortion (D'Aspremont and Jacquemin, 1988), because the objective of each member becomes the maximisation of *cartel* profits. More specifically, each member commits to the cartel-optimal level of R&D investment by signing a cooperative agreement. Various types of cartel may arise according to the sharing-rules of innovation. The members may agree, for example, to fully share the innovation produced by the cooperation. In this case the spillover rate within the cartel becomes equal to unity, since the knowledge produced by one member is shared with the other. Alternatively, the participants may only decide to coordinate their research activity, for instance to agree on the direction of the R&D effort or to conduct basic research, without any innovation sharing. This type of agreement does not have an effect on the spillover rate β , which thus remains equal to the non cooperative one¹¹. In the present paper I explore the latter kind of cooperation because it treats the spillover rate β as a generic value and allows to discuss the former kind of cooperation as a special case by setting $\beta = 1$; and to stress the fact that innovation-sharing is not an essential characteristic of cartels in the regard of spillover internalization: the critical feature is represented by the agreement over the amount of R&D investment.

Plugging the output first-order condition (3.3) into the function of cartelwide profits, the maximisation problem of the domestic firm becomes as follows:

$$\max_x \bar{\Pi} = \Pi + \Pi^* = by^2 - K + \sigma x + b(y^*)^2 - K^*$$

As it can be seen, the profit level of the foreign firm differs from the one of the domestic firm by the amount of subsidy granted by the home government. I can compute the derivative with respect to x and obtain the following first-order condition for the domestic firm:

$$2by\frac{2-\beta}{3b} + 2by^*\frac{2\beta-1}{3b} = 2x - \sigma$$

The private marginal benefit and the private marginal cost of increasing the level of investment are given, respectively, at the left-hand side and at the right-hand side. The former is measured by the rise in revenues for *both* firms, whereas the latter captures the increase in fixed costs minus the contribution from the government. This first-order condition can be developed to get:

$$x = \frac{9b\sigma + 2(1+\beta)(a-\bar{c}) + 4(2\beta-1)(2-\beta)x^*}{18b - 6(1+\beta)(1-\beta)} \quad (3.9)$$

¹¹A discussion around the different sorts of cooperative agreement can be found in Katz (1986).

This equation represents the reaction function of the domestic firm vis-à-vis the foreign firm, because the level of x is dependent on the variable x^* . As in the case of R&D competition, the sign of the response depends on the amount of spillovers: it is positive if $\beta > 1/2$ and negative if $\beta < 1/2$, as suggested by the coefficient of the term x^* .

Lemma 9. *Under cooperation, the investment decisions of the two firms are complementary if, and only if, the spillover rate is larger than 1/2.*

The reaction function of the foreign firm is analogous to equation (3.9) - the only difference being represented by $\sigma = 0$. By substituting the latter into reaction function (3.9) we obtain the equilibrium investment of the domestic firm:

$$\begin{aligned} x_c &= \frac{9b[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]}{[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]^2 - [4(2 - \beta)(2\beta - 1)]^2} \sigma + \\ &+ \frac{2(a - \bar{c})(1 + \beta)}{18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2 - 4(2 - \beta)(2\beta - 1)} \equiv \\ &\equiv m_c \sigma + q_c \end{aligned} \quad (3.10)$$

where the subscript c denotes R&D cooperation. The amount of subsidy is shown to affect in a linear manner the investment of the domestic firm; m_c is the angular coefficient of the line and q_c is the constant term; m_c should be positive for the subsidy to impact positively the amount of investment¹². Further, σ is expected to affect the foreign investment through the degree of complementarity with x stated in lemma (9). To confirm this, let us analyse the equilibrium value of foreign investment, given by the following expression:

$$\begin{aligned} x_c^* &= \frac{36b(2 - \beta)(2\beta - 1)}{[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]^2 - [4(2 - \beta)(2\beta - 1)]^2} \sigma + \\ &+ \frac{2(a - \bar{c})(1 + \beta)}{18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2 - 4(2 - \beta)(2\beta - 1)} \equiv \\ &\equiv m_c^* \sigma + q_c \end{aligned} \quad (3.11)$$

As it can be seen, the sign of m_c^* depends on the value of the spillover rate. In particular, it is positive if and only if $\beta > 1/2$; that is, if the two firms are strategic complementary at the R&D level. Moreover, in case of zero subsidy ($\sigma = 0$) the two firms select the same amount of investment, because the constant term q_c is the same for both firms; this is due to the assumption of symmetry. The implications of dropping this assumption will be discussed in section 3.6.

¹²For all β we have that m_c is positive if $b > 1$

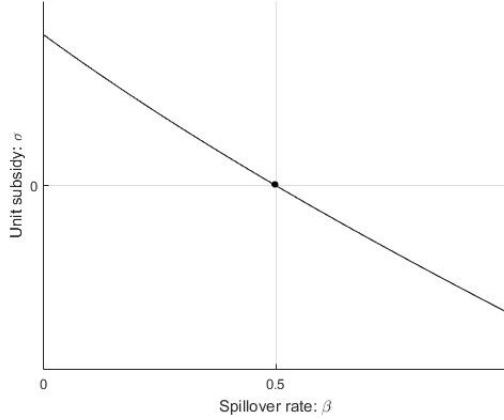


Figure 3.6: Relationship between the unit subsidy σ and the spillover rate β under R&D cartels

Lemma 10. *Under cooperation, if the spillover rate is larger than 1/2, a positive subsidy to R&D from the government of country H induces higher investment in R&D both for the domestic firm and the foreign firm.*

Figure (3.4) depicts the reaction functions of the two firms when $\beta = 0$ (case of investment substitutability); the continuous lines are associated to the case of zero subsidy, whereas the dashed line represents the behaviour of the domestic firm when a positive subsidy is applied. The investment decisions of the firms are strategic substitutes and, as it can be seen, the subsidy has the two-fold effect of lowering the foreign investment and raising the domestic one, moving the equilibrium from E to E' . Instead, figure (3.5) is depicted for $\beta = 1$ (investment complementarity). In this case, equilibrium E' , determined by a positive subsidy, entails higher effort in R&D than the zero-subsidy equilibrium E for either the domestic firm and the foreign one, confirming the validity of lemma (10) when spillovers are large.

3.4.1 Subsidy to the R&D cartel

The government of country H has observed the behaviour of the two firms and is willing to grant a subsidy to R&D in order to maximise national welfare.

$$\max_{\sigma} W = \Pi - \delta\sigma x = by^2 - K + (1 - \delta)\sigma x$$

subject to the constraints represented by equations (3.10) and (3.11). The first-order condition of this problem is the following:

$$2by \left[\frac{dx}{d\sigma} \frac{2 - \beta}{3b} + \frac{dx^*}{d\sigma} \frac{2\beta - 1}{3b} \right] = 2x \frac{dx}{d\sigma} - (1 - \delta) \left(x + \sigma \frac{dx}{d\sigma} \right)$$

which is analogous to the one associated to the case of R&D competition. For this reason, the expression for the optimal value of subsidy is the familiar:

$$\sigma_c = \frac{2[(2-\beta)m_c + (2\beta-1)m_c^*][a - \bar{c} + (1+\beta)q_c] - 9bq_c(2m_c - 1 + \delta)}{18bm_c(m_c - 1 + \delta) - 2[(2-\beta)m_c + (2\beta-1)m_c^*]^2} \quad (3.12)$$

The impact of δ on the value of σ does not require the computation of derivatives, since it is straightforward to observe that it lowers the denominator and increases the denominator, with a reduction in the unit subsidy level. Instead, the analysis of the sign of this expression is not trivial. To clarify this issue, I shall rely on a graphical inspection; to this purpose let us observe figure (3.6), which considers the case $\delta = 1$ (the implication of this assumption is discussed in section 3.7). The amount of spillovers is measured on the horizontal axis and the value of subsidy is on the vertical one. For $\beta = 0$ the function lies above the horizontal axis: the government is thus willing to grant a positive subsidy to R&D, so as to foster technological development and increase domestic profits. It progressively falls and, as in no-cooperation, crosses the horizontal axis for $\beta = 1/2$: the unit subsidy becomes equal to zero when the decision of the firm becomes independent of the rival's. Unlike figure (3.3) depicted for the R&D-competition regime, in figure (3.6) the function drops below the horizontal axis when spillovers are large, which means that the optimal level of subsidy becomes negative. This difference is due to the fact that there exist two opposite forces that affect the sign of the subsidy when spillovers are large. On the one hand, the larger the spillover rate, the larger the ability of the foreign firm to benefit from domestic subsidies, because domestic technological development may be largely absorbed. In fact, this argument may justify the adoption of an R&D tax, since the home government desires the domestic firm to reduce its investment and 'free-ride' on the technology of the foreign rival, with a net increase in domestic profits and welfare. On the other hand, the level of spillovers may reduce the incentive of firms to invest due to free-riding issues, requiring the application of a positive subsidy to restore higher levels of research activities; this is the case for instance of R&D competition analysed in section 3.3.1. Under R&D cooperation, however, the latter argument is sharply weakened, because high spillovers do not have the effect of lowering investment incentives - they in fact rise due to the larger benefit of R&D at the cartel-level. As a result, the former argument prevails and allows the government to justify the adoption of an R&D tax.

Proposition 13. *Under R&D cartels and in the absence of subsidy social costs ($\delta = 1$), the optimal amount of unit subsidy σ is positive if and only if $\beta < 1/2$. Equivalently, large spillovers may justify the application of an R&D tax.*

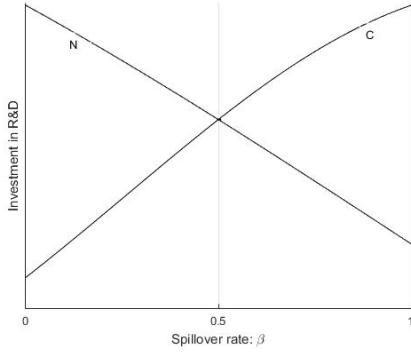


Figure 3.7: Comparison between domestic investments in the two modes-of-research

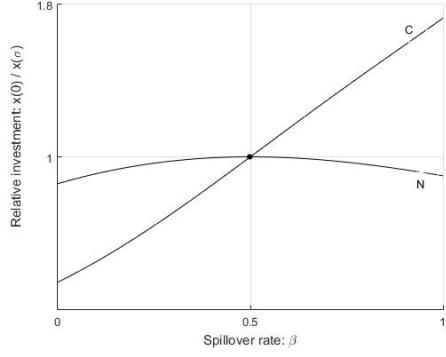


Figure 3.8: Relative investment: $\sigma = 0$ vis-à-vis optimal σ for both R&D-regimes

3.5 Difference between R&D competition and co-operation

As seen in the previous sections the degree of R&D cooperation affects the behaviour of firms, especially for what concerns the level of investment in R&D, since R&D cartels may internalize the amount of technological spillovers present in the industry. The level of investment, in turn, has an impact on output and profits. The assessment of this impact is of crucial importance to firms, because they are willing to adopt the mode-of-research that maximises their revenues. In this section I shall explore the differences between R&D competition and cartels to understand the implications on firms profitability. Due to analytical difficulties the analysis is based on graphical arguments, therefore its results should be considered as mere suggestions. In the appendix, however, I show that these graphical results hold for most values of the parameters.

3.5.1 Investment level

The investment decisions of the domestic firm in R&D competition and R&D cartel are determined, respectively, by equations (3.5) and (3.6). First, it is convenient to disentangle the firm incentive to invest from the government stimulus. For this purpose figure (3.7) depicts the levels of investment in the two regimes in the case of $\sigma = 0$; where N stands for no-cooperation investment and C denotes cartel investment¹³. The former line is falling with respect to spillovers because of the increasing incentive to free-ride on technological development. On the other

¹³Analytically, the figure is based on the inequality $x_n \geq x_c$ once set $\sigma = 0$

hand, the latter is rising, since an increase in the spillover rate raises the R&D benefit at the cartel-wide level. The two regimes bring about the same level of investment for $\beta = 1/2$, that is, when the firm is not constrained by the rival's best response function.

Lemma 11. *From a national perspective and without government intervention, cooperation entails greater investment in R&D than no-cooperation if and only if the spillover rate exceeds 1/2.*

However, to attain the maximal welfare-improvement, the government is willing to pay a subsidy, the optimal value of which depends on the mode-of-research and is represented by σ_n (equation 3.8) and σ_c (equation 3.12). Therefore, the amount of investment which corresponds to such welfare-improvement is represented by $x_n(\sigma_n)$ in R&D competition and $x_c(\sigma_c)$ in R&D cooperation. These 'optimal' investments can be compared to the investment levels obtained without government intervention to capture the effect produced by the subsidy. Let us observe figure (3.8). The spillover rate is captured on the horizontal axis; whereas the relative level of investment, defined as the ratio between the market-determined investment $x_i(\sigma = 0)$ (shown in figure 3.7) and the 'optimal' investment $x_i(\sigma = \sigma_i)$ for each $i \in \{n, c\}$, is on the vertical one. If the ratio is equal to one, market dynamics are fully desirable under a social welfare point of view; if it is lower (larger) than unity, the private incentive to invest is socially insufficient (excessive). As it can be seen, the only case of excessive investment is given by cooperation when spillovers are larger than 1/2. Otherwise, both regimes induce a level of investment which is not sufficient to welfare-maximisation. The two lines cross at the point (1/2; 1): for $\beta = 1/2$ both regimes tend towards welfare maximisation.

Conjecture 1. *At $\beta = 1/2$ the two research-regimes attain welfare maximisation without the need for government intervention. Excessive private incentive to invest may be observed in R&D cartels when spillovers are larger than 1/2. Only in this case an R&D tax is justifiable. Otherwise, a positive subsidy may help increase private incentives to invest and pursue the social optimum.*

3.5.2 Profit level

Figure (3.9) shows the change in the domestic firm's profit level with respect to the spillover rate in the two regimes. Cooperation at the R&D-level entails larger profits if and only if the spillover rate is lower than 1/2; but, if it exceeds this value, no-cooperation may be profitable to the domestic firm. This result forms a novelty to the R&D deterministic literature, where cooperation represents always the more profitable regime¹⁴. The key difference in this analysis is government

¹⁴See for instance Leahy and Neary (1997).

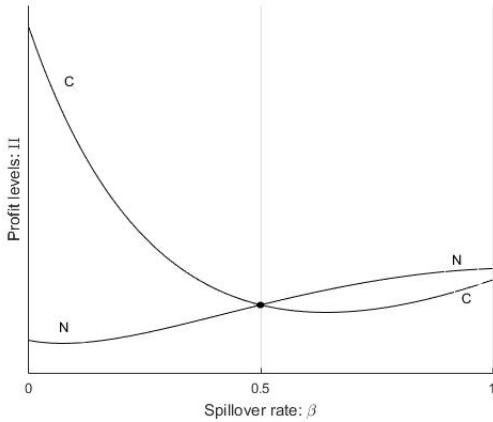


Figure 3.9: Comparison between profit levels in both R&D regimes with respect to the spillover rate

intervention, in particular the application of a tax to R&D cartels when spillovers are large, as stated in proposition (13). Such policy lowers the incentive to form a research joint venture and triggers R&D competition, which makes firms better off.

Conjecture 2. *Unlike previous contributions, in this analysis the no-cooperation regime may be more profitable than cooperation, especially when the spillover rate is sufficiently large and in presence of government intervention.*

3.5.3 The welfare level

The final scope of the comparative exercise concerns welfare. This is relevant to the social planner, which needs to encourage one regime or the other to bring about the maximal welfare. Figure (3.10) depicts graphically how the welfare level changes with respect to the spillover rate in the two regimes. First, in R&D competition the line is rising with respect to β , which means that the larger the share of knowledge that spills over, the larger the level of welfare of the country. Instead, under R&D cartels the effect of the spillover rate on welfare depends on the level of β and gives place to U-shaped a line. Second, R&D cooperation is always desirable under a social standpoint because it delivers the higher level of welfare. For such reason, the domestic government should always welcome the formation of R&D cartels. No social action is required when spillovers are low, since cartels are more profitable and autonomously arise from market dynamics (see figure 3.9). However, a social measure is needed to encourage the formation of cartels when spillovers are large, because they are less profitable than R&D

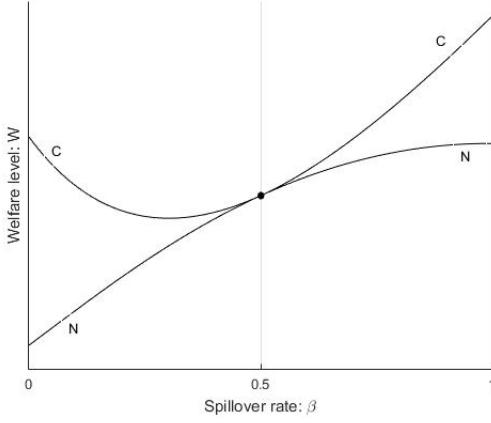


Figure 3.10: Comparison between welfare levels in both R&D regimes with respect to the spillover rate

competition. One way of achieving this, for instance, is through the application of an R&D tax to the no-cooperation regime, so as to prevent firms from competing at the R&D level and to foster cooperation.

Conjecture 3. *Provided that the government may apply an R&D subsidy to attain the social optimum, research joint ventures are always welfare-improving and thus should be encouraged. In this regard, a tax to ‘no-cooperation’ may serve this purpose by lowering the profitability of R&D competition and triggering the formation of R&D cartels. This is especially useful when spillovers are large, i.e., when cartels do not arise autonomously from market dynamics.*

3.6 Cost Heterogeneity

The assumption of symmetric cost across countries produces a relevant consequence on the results of the model: in absence of government intervention firms select the same amount of investment in R&D. Evidence in the real world however suggests that firms are quite cost-heterogeneous. Let us introduce this element into the analysis to shed further light on the topic. In an asymmetric duopoly the domestic firm may be either the more efficient or the less efficient. Let us suppose that the asymmetry coefficient is captured by ρ as follows:

$$c = \bar{c} - x - \beta x^*$$

$$c^* = \bar{c} + \rho - x^* - \beta x$$

The pre-investment difference between foreign and domestic costs is equal to ρ , which measures the cost-advantage of the domestic firm over the rival. Thus, a

positive value of this parameter represents the case where the domestic firm is *a priori* the more efficient.

The profit function of the domestic firm presents a two-fold difference from the profit function of the competitor, because, in addition to the cost asymmetry, the latter takes account of the public subsidy to R&D:

$$\begin{aligned}\Pi &= (p - \bar{c} + x + \beta x^*)y - K + \sigma x \\ \Pi^* &= (p - \bar{c} - \rho + x^* + \beta x)y^* - K^*\end{aligned}$$

Solving the model by backward induction, at the last stage firms select the amount of output. The equilibrium levels are:

$$\begin{aligned}y &= \frac{a - \bar{c} + \rho + (2 - \beta)x + (2\beta - 1)x^*}{3b} \\ y^* &= \frac{a - \bar{c} - 2\rho + (2 - \beta)x^* + (2\beta - 1)x}{3b}\end{aligned}\tag{3.13}$$

Let us focus on the role of ρ . If the domestic firm is *a priori* more efficient than the foreign one ($\rho > 0$), it is able to supply a larger amount of goods even in absence of R&D investments ($x = x^* = 0$). The larger the asymmetry parameter, the higher domestic output and the lower the foreign one. Moreover, by summing the two quantities y and y^* we get:

$$\bar{y} = \frac{2(a - \bar{c}) - \rho + (1 + \beta)(x + x^*)}{3b}$$

from which we can state the following:

Lemma 12. *Given the amount of investment in R&D of the two firms, the larger the asymmetry in the industry, the lower the total level of output \bar{y} . In particular, this reduction results from a rise in domestic output and a fall in foreign output.*

3.6.1 No-cooperation

The two firms select the optimal level of investment to maximise profits. The equilibrium outcome can be shown to be:

$$\begin{aligned}x_n &= \frac{9b[18b - 2(2 - \beta)^2]}{[18b - 2(2 - \beta)^2]^2 - [2(2 - \beta)(2\beta - 1)]^2}\sigma + \\ &\quad + \frac{2(2 - \beta)(a - \bar{c})}{18b - 2(2 - \beta)^2 - 2(2 - \beta)(2\beta - 1)} + \\ &\quad + \frac{2(2 - \beta)[18b - 2(2 - \beta)^2 - 4(2 - \beta)(2\beta - 1)]}{[18b - 2(2 - \beta)^2]^2 - [2(2 - \beta)(2\beta - 1)]^2}\rho \equiv \\ &\equiv m_n\sigma + q_n + \mu_n\rho\end{aligned}\tag{3.14}$$

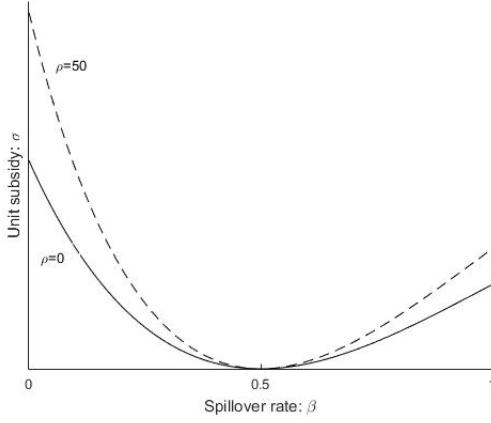


Figure 3.11: Comparison between symmetry- and asymmetry-subsidy levels.

and:

$$\begin{aligned}
 x_n^* &= \frac{18b(2-\beta)(2\beta-1)}{[18b - 2(2-\beta)^2]^2 - [2(2-\beta)(2\beta-1)]^2} \sigma + \\
 &\quad + \frac{2(2-\beta)(a-\bar{c})}{18b - 2(2-\beta)^2 - 2(2-\beta)(2\beta-1)} - \\
 &\quad - \frac{4(2-\beta)[18b - 2(2-\beta)^2 - (2-\beta)(2\beta-1)]}{[18b - 2(2-\beta)^2]^2 - [2(2-\beta)(2\beta-1)]^2} \rho = \\
 &\equiv m_n^* \sigma + q_n + \mu_n^* \rho
 \end{aligned} \tag{3.15}$$

First, let us remark that equations (3.5) and (3.6), which describe investment in the symmetric-firms case, may be derived from equations (3.14) and (3.15) once set $\rho = 0$, being the former a special case of the latter. Second, let us focus on the impact of asymmetry on investment. In particular, it is possible to show that $\mu_n > 0$ and $\mu_n^* < 0$, that is, the validity of the following statement:

Lemma 13. *Given the level of subsidy granted by the government, in an asymmetric industry the more efficient firm is willing to invest greater resources in R&D than the less efficient one.*

The proof follows directly from the definition of μ_n and μ_n^* , as shown in the appendix. In an asymmetric industry, the larger firm is less sensitive to the behaviour of the smaller one and, thus, is constrained by the best response of the rival to a minor extent. Such constraint slackness allows the larger firm to increase the investment in research and development. For a symmetric argument, the best response of the larger firm is particularly binding to the smaller firm, which accordingly reduces the level of investment.

The government of country H , unlike country F 's, desires to grant a subsidy to R&D activity to influence the private incentive to invest and achieve maximal

welfare. From the optimisation problem of the government we get the following expression for the research subsidy:

$$\begin{aligned} \sigma_n = & \frac{2[(2-\beta)m_n + (2\beta-1)m_n^*][a - \bar{c} + \rho + (2-\beta)(q_n + \mu_n\rho) + (2\beta-1)(q_n + \mu_n^*\rho)]}{18bm_n(m_n - 1 + \delta) - 2[(2-\beta)m_n + (2\beta-1)m_n^*]^2} - \\ & - \frac{9b(q_n + \mu_n\rho)(2m_n - 1 + \delta)}{18bm_n(m_n - 1 + \delta) - 2[(2-\beta)m_n + (2\beta-1)m_n^*]^2} \end{aligned} \quad (3.16)$$

which is similar to the symmetric-case one (equation 3.8), but also captures the effect of ρ . Figure (3.11) shows the impact of the asymmetry parameter on the subsidy σ_n . The dashed line represents the case of positive asymmetry, where the domestic firm is the more efficient and ρ is larger than zero, whereas the continuous line depicts the benchmark case of symmetry seen in the previous chapters. As it can be seen, the former lies above the latter, which means that the social planner should grant higher subsidies when the domestic firm is more efficient than the foreign competitor. Oppositely, the subsidy should be lower for those industries where the domestic firm is less competitive than the foreign rivals.

Proposition 14. *Under R&D competition, an increase in the asymmetry coefficient ρ determines a rise in the research unit subsidy σ .*

In other words, the government should help ‘winners’ to a greater extent than ‘losers’, trying to address public funds towards those sector where the domestic economy has a competitive advantage over the foreign one. The same conclusion is achieved in the literature of output subsidy in open economies; see for example Neary (1994), in which the author points out that the profit-shifting argument of output subsidies is enhanced in the presence of comparative advantages.

3.6.2 Cooperation

The two firms may decide to develop a new technology in a cooperative manner by forming a research joint venture; if so, they select the amount of R&D investment so as to maximise profits at the cartel-level. It can be shown that the equilibrium

value of investment is equal to:

$$\begin{aligned}
x_c &= \frac{9b[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]}{[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]^2 - [4(2 - \beta)(2\beta - 1)]^2} \sigma + \\
&+ \frac{2(a - \bar{c})(1 + \beta)}{18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2 - 4(2 - \beta)(2\beta - 1)} + \\
&+ 2 \frac{(4 - 5\beta)[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2] - 4(5 - 4\beta)(2 - \beta)(2\beta - 1)}{[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]^2 - [4(2 - \beta)(2\beta - 1)]^2} \rho \equiv \\
&\equiv m_c \sigma + q_c + \mu_c \rho
\end{aligned} \tag{3.17}$$

for the domestic firm; and equal to:

$$\begin{aligned}
x_c^* &= \frac{36b(2 - \beta)(2\beta - 1)}{[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]^2 - [4(2 - \beta)(2\beta - 1)]^2} \sigma + \\
&+ \frac{2(a - \bar{c})(1 + \beta)}{18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2 - 4(2 - \beta)(2\beta - 1)} - \\
&- 2 \frac{(5 - 4\beta)[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2] - 4(4 - 5\beta)(2 - \beta)(2\beta - 1)}{[18b - 2(2 - \beta)^2 - 2(2\beta - 1)^2]^2 - [4(2 - \beta)(2\beta - 1)]^2} \rho \equiv \\
&\equiv m_c^* \sigma + q_c + \mu_c^* \rho
\end{aligned} \tag{3.18}$$

for the foreign firm. Not surprisingly, by setting $\rho = 0$ these investment functions are equivalent to those stated in equations (3.10) and (3.11) - describing the symmetric-firms case. Given the level of subsidy σ , the impact of firm asymmetry on investment depends on the terms μ_c for the domestic firm and μ_c^* for its foreign competitor. It is possible to prove the following (see the appendix):

Lemma 14. *Let the two firms form a research joint venture. Given the level of subsidy σ , there exists a critical value $\bar{\beta} \in (1/2, 1)$ such that asymmetry does not impact the investment of the domestic firm ($\mu_c(\bar{\beta}) = 0$). Moreover, such impact is positive for $\beta = 0$ (private knowledge) and $\beta = 1/2$, and negative for $\beta = 1$ (common knowledge). On the other hand, the impact of an increasing ρ on foreign investment x_c^* is always negative.*

Firts, it is convenient to observe the fact that under research cooperation the best-response argument seen in no-cooperation, by which the larger firm is less sensitive to the best response of the smaller one, is weakened because the goal of each firm is to cooperate as to attain cartelwide profit maximisation. Second, let us recall the result of lemma (9): there exists a strategic relationship between investments

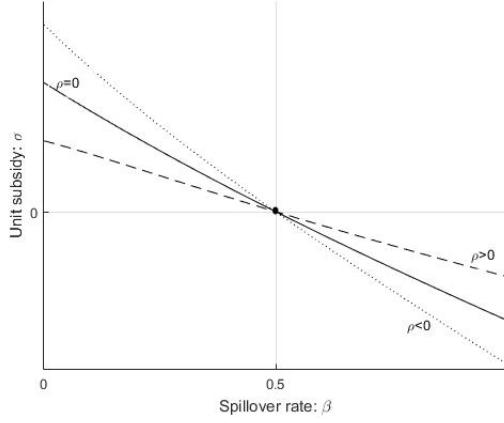


Figure 3.12: Comparison between symmetry- and asymmetry-subsidy levels.

of the firms; more specifically, low spillovers determines strategic substitutability and high spillovers produces strategic complementarity. Having recalled that we can move to interpreting the result of lemma (14) by considering an increase in the sector asymmetry in favour of the domestic firm. Like the no-cooperation case, the equilibrium output of the foreign firm falls (see equation 3.13), and so does its capacity to invest in R&D; which explains the fact that μ_c^* be lower than zero. The output capacity of the domestic firm instead rises, but this does not engulf directly an increase in investment. In particular its behaviour is mainly determined by the working of lemma (9) and marginally by the weakened best response argument: when spillovers are low the fall in foreign investment produces a rise in domestic investment due to investment substitutability; if $\beta = 1/2$ lemma (9) does not produce any effect and the weakened best response argument contributes alone to an increase in domestic investment; when spillovers are large investment complementarity entails a fall in domestic investment.

To attain the social optimum, the government selects the level of subsidy according to the following:

$$\sigma_c = \frac{2[(2 - \beta)m_c + (2\beta - 1)m_c^*][a - \bar{c} + \rho + (2 - \beta)(q_c + \mu_c\rho) + (2\beta - 1)(q_c + \mu_c^*\rho)]}{18bm_c(m_c - 1 + \delta) - 2[(2 - \beta)m_c + (2\beta - 1)m_c^*]^2} - \frac{9b(q_c + \mu_c\rho)(2m_c - 1 + \delta)}{18bm_c(m_c - 1 + \delta) - 2[(2 - \beta)m_c + (2\beta - 1)m_c^*]^2} \quad (3.19)$$

We can conduct a comparative statics exercise based on graphical representation to evaluate the impact of the asymmetry coefficient ρ on the amount of subsidy. Figure (3.12) serves this purpose. The continuous line indicates the firm-symmetry case ($\rho = 0$), whereas the dashed and the dotted lines describe the level of subsidy when the domestic firm is, respectively, more and less efficient than the foreign

rival ($\rho > 0$ and $\rho < 0$). As it can be seen, positive asymmetry has the effect to reduce the degree of government intervention; and negative asymmetry requires greater intervention by the government: higher subsidies when spillovers are low and higher taxes when spillovers are large.

Proposition 15. *If the two firms cooperate at the R&D level, an increase in the asymmetry coefficient ρ determines a fall in the amount of subsidy σ (in absolute terms).*

This result carries important policy implications, since it overturns the statement of proposition (14) holding for R&D competition: within an R&D cartel, asymmetry has the effect of *weakening* the profit-shifting argument of subsidies, which means that the social benefit of government intervention is smaller, the larger the domestic firm. The reason of this result may be attributed to the divergence of objectives between the domestic firm and the government, since the former aims at the maximisation of profits *at the cartel-level* and the latter is constrained in domestic profit (minus the subsidy bill) maximisation by the firm behaviour. An increase in the size of the firm tightens such constraint and reduces the capability of the government to pursue welfare improvements through subsidy policy.

3.7 Subsidy social cost

Government needs to collect monetary resources from private agents to fund the distribution of R&D subsidies to firms. The amount of subsidies constitutes the subsidy bill, which affects directly the welfare function of the government. The collecting activity, on the other hand, may be socially costly and affect negatively the welfare level of the country. For instance, it may require the application of a distortionary tax producing a reduction in Pareto-efficiency; further interpretations are discussed in the literature, see Lahiri *et al* (2000). In this case, the government is not willing to trade off one dollar of extra profits with one dollar of subsidy payments. The presence of social costs is thus likely to affect the analysis of subsidy policy. In this section I shall shed light on this topic in both R&D regimes by investigating the impact of the social cost coefficient δ on the determination of optimal subsidy.

If the two firms compete at the R&D level, the expression for optimal subsidy of the home government is given by condition (3.8). As observed, a rise in the social cost parameter δ has the unambiguous effect of lowering the amount of subsidy granted by the government. Figures (3.13) and (3.14) depicts how optimal subsidy is sensitive to variations of the social cost parameter δ . The figures represent, respectively, the case of $\beta = 0$ and $\beta = 1$. As it can be seen, graphical simulation suggests that the optimal subsidy may become negative if the social

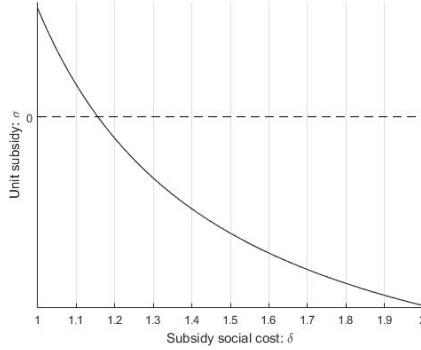


Figure 3.13: Optimal unit subsidy in no-cooperation as subsidy social cost varies ($\beta = 0$)

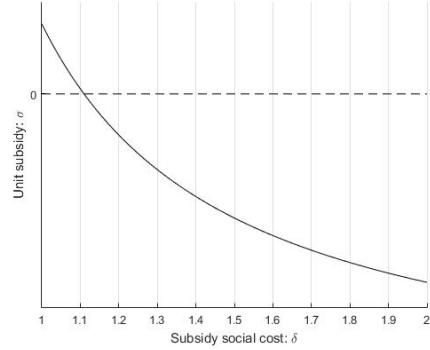


Figure 3.14: Optimal unit subsidy in no-cooperation as subsidy social cost varies ($\beta = 1$)

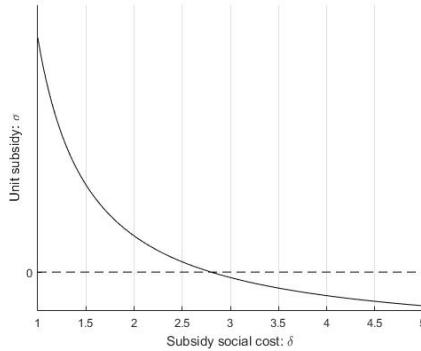


Figure 3.15: Optimal unit subsidy in cooperation as subsidy social cost varies ($\beta = 0$)

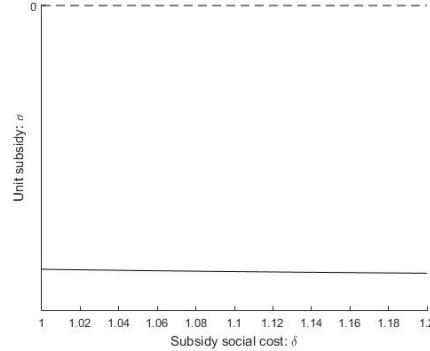


Figure 3.16: Optimal unit subsidy in cooperation as subsidy social cost varies ($\beta = 1$)

cost is sufficiently high (at a value lying between 1.1 and 1.2). This is no surprise and is in fact intuitive, because an increase in the social cost of subsidy payments is expected to reduce their optimal value and, eventually, turn them into R&D taxes.

The same sort of graphical inspection is offered also when the two firms cooperate at the R&D level; the expression for optimal subsidy is given by equation (3.12). Figures (3.15) and (3.16) shows how a change in the social cost of subsidy affects the optimal amount of subsidy payments. The former figure represents the case $\beta = 0$, at which the optimal subsidy is positive, whereas the latter depicts the case $\beta = 1$, where an R&D tax is justified. Again, the graphs confirm that δ as a negative impact on σ , reducing its optimal amount. It may be noteworthy

to observe that in the latter figure the subsidy is negative even for $\delta = 1$ and that increases in δ do not produce a significant decrease in the level of subsidy. This is due to the fact that there exists a critical level of taxation at which the amount of investment hits its zero lower-bound, thus any increase above such critical level is redundant.

3.8 Concluding remarks

This is a paper of partial equilibrium in which two firms belonging to different countries export to a third-country market. The firms may decide both the output level and the amount of investment in research activities, able to lower their marginal cost. Moreover, they may decide either competition or cooperation at the R&D level. Finally, the home government may concede a research subsidy to stimulate investment of the domestic firm. The goal of this paper is to explore the implication of such a subsidy in an international framework.

An R&D subsidy may impact the level of investment carried by the foreign firm, since it reacts to the behaviour of the domestic one. In particular, the amount of spillovers in the industry determines the sign of the reaction. An R&D subsidy is able to decrease foreign investment when spillovers are small, or to increase it when spillovers are large. This result is independent of the research regime.

Unlike previous contributions on output subsidy, this analysis shows that research subsidies need not be always positive. Under R&D cartels, high levels of the spillover rate are not associated to free-riding issues on technological development. Since any positive subsidy from the home government benefits largely the foreign firm, a tax to R&D cartels is justified. However, when spillovers are low or firms compete at the R&D level, a positive subsidy may increase domestic profits and welfare.

Simulations suggest that R&D cooperation is the welfare-maximising regime for every value of the spillover rate. For this reason the government should always encourage its creation. No active intervention is required when spillovers are low, because domestic firms are spontaneously willing to cooperate due to market dynamics. Instead, government intervention is needed when spillovers are large, because in this case R&D cooperation does not represent the more profitable regime. One way of intervention may be the application of taxes on R&D activities of non-cooperative firms, which thus could be motivated even out of research cartels.

Further, the paper explores the implication of cost heterogeneity. The main finding is that asymmetry has opposed implications on research subsidy according to the research regime: the more efficient firm requires either greater intervention

when firms compete at the R&D level or looser intervention when they cooperate. The former result moves along the lines of output subsidy literature, where the main contributions highlight the same kind of relationship. The latter, instead, is a novelty to the literature. Cooperation has the effect of creating a gap between the firm's goal (cartel-wide profits) and the government's (domestic profits), and as a result the effectiveness of public action is weakened. Moreover, the paper sheds light on the effect of asymmetry on R&D investment in absence of government intervention. In particular, the less efficient firm always desires to reduce its investment level. The more efficient firm, on the other hand, may respond differently according to the research regime and the amount of the spillover rate.

Finally, I address the issue of social cost of subsidy payments. Its value may exceeds the private revenue of the recipient firm due to the costly activity associated to the collection of public funds, like the raise of a distortionary tax for instance. An exercise of comparative statics shows how the presence of such social cost δ affects the level of subsidy granted by the home government. Graphical inspection suggests that an increase in δ always reduces the amount of subsidy. In no-cooperation the value of δ at which the subsidy turns negative seems to be close to 1.1-1.2; whereas in cartels it strongly depends on the amount of spillovers. For instance, with large spillover an R&D tax is justified even in absence of social cost ($\delta = 1$).

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Appendix A

Appendix

A.1 Appendix

A.1.1 Proposition 1

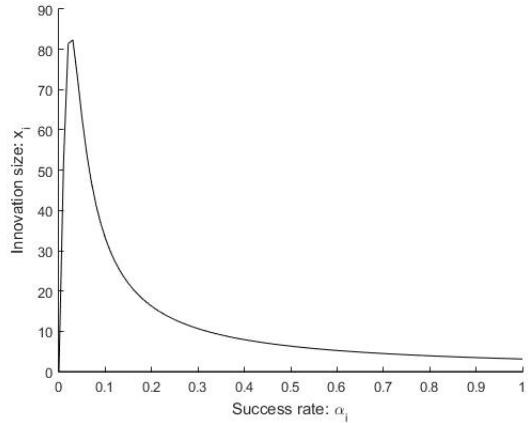


Figure A.1: Relationship between α_i and x_i

I want to show that in no-cooperation a rise in the risk rate determines a rise in the optimal amount of innovation unless the risk rate is very close to unity, that is, that the function $x_i(\alpha_i)$ is decreasing for most values of the risk parameter.

Such function is given by equation 1.19. The derivative with respect to α_i gives:

$$\begin{aligned}
D_{\alpha_i}[x_i] &= \\
&= - \frac{\left(-\alpha_j(1-2\beta)^2 + \left(-\frac{18(0.025+\alpha_i)b}{\alpha_i(-2+\beta)} + \frac{9(0.025+\alpha_i)^2b}{\alpha_i^2(-2+\beta)} \right) \left(-2 - \frac{9(0.025+\alpha_j)^2b}{\alpha_j(-2+\beta)} + \beta \right) \right)}{\left(-\alpha_i\alpha_j(1-2\beta)^2 + \left(-2 - \frac{9(0.025+\alpha_i)^2b}{\alpha_i(-2+\beta)} + \beta \right) \left(-2 - \frac{9(0.025+\alpha_j)^2b}{\alpha_j(-2+\beta)} + \beta \right) \right)^2} \times \\
&\quad \times \frac{\left(-2 - \frac{9(0.025+\alpha_j)^2b}{\alpha_j(-2+\beta)} + \beta + \alpha_j(-1+2\beta) \right)}{\left(-\alpha_i\alpha_j(1-2\beta)^2 + \left(-2 - \frac{9(0.025+\alpha_i)^2b}{\alpha_i(-2+\beta)} + \beta \right) \left(-2 - \frac{9(0.025+\alpha_j)^2b}{\alpha_j(-2+\beta)} + \beta \right) \right)^2}
\end{aligned}$$

which is equal to zero if $\alpha \approx 0.025$. As shown in figure A.1 this represents a maximum: the function is increasing to the left-hand side and decreasing to the right-hand side as we wanted to check.

A.1.2 Proposition 2

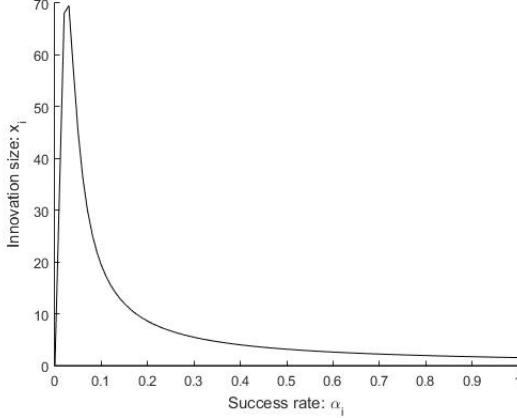


Figure A.2: Relationship between α_i and x_i

I want to show that in cooperation a rise in the risk rate determines a rise in the optimal amount of innovation unless the risk rate is very close to unity, that is, that the function $x_i(\alpha_i)$ is decreasing for most values of the risk parameter. Such function is given by equation 1.23. The derivative with respect to α_i gives:

$$\begin{aligned}
D_{\alpha_i}[x_i] &= \\
&= - \frac{\left(-4\alpha_j(2-\beta)^2(-1+2\beta)^2 + \left(\frac{18(0.025+\alpha_i)b}{\alpha_i} - \frac{9(0.025+\alpha_i)^2b}{\alpha_i^2} \right) \left(-5 + \frac{9(0.025+\alpha_j)^2b}{\alpha_j} + 8\beta - 5\beta^2 \right) \right)}{\left(-4\alpha_i\alpha_j(2-\beta)^2(-1+2\beta)^2 + \left(-5 + \frac{9(0.025+\alpha_i)^2b}{\alpha_i} + 8\beta - 5\beta^2 \right) \left(-5 + \frac{9(0.025+\alpha_j)^2b}{\alpha_j} + 8\beta - 5\beta^2 \right) \right)^2} \times \\
&\quad \times \frac{\left(2\alpha_j(2-\beta)(1+\beta)(-1+2\beta) + (1+\beta) \left(-5 + \frac{9(0.025+\alpha_j)^2b}{\alpha_j} + 8\beta - 5\beta^2 \right) \right)}{\left(-4\alpha_i\alpha_j(2-\beta)^2(-1+2\beta)^2 + \left(-5 + \frac{9(0.025+\alpha_i)^2b}{\alpha_i} + 8\beta - 5\beta^2 \right) \left(-5 + \frac{9(0.025+\alpha_j)^2b}{\alpha_j} + 8\beta - 5\beta^2 \right) \right)^2}
\end{aligned}$$

which is equal to zero if $\alpha \approx 0.025$. As shown in figure A.2 this represents a maximum: the function is increasing to the left-hand side and decreasing to the right-hand side as we wanted to check.

A.1.3 Proposition 4

We want to show that $x^C > x^N$ if and only if $\beta > 1/2$. The amount of innovation is described by equations 1.19 and 1.23, thus:

$$\begin{aligned} x^C &> x^N \\ \frac{1 + \beta}{\frac{9b\Gamma^2}{\alpha} - 2\alpha(2 - \beta)(2\beta - 1)} &> \frac{2 - \beta}{\frac{9b\Gamma^2}{\alpha} - \alpha(2\beta - 1)} \\ 3\alpha^2(2 - \beta)(\beta - 1)(1 - 2\beta) &> 9b\Gamma^2(1 - 2\beta) - \alpha(2 - \beta)(1 - 2\beta)^2 \end{aligned}$$

If $1 - 2\beta > 0$ ($\beta < 1/2$):

$$3\alpha^2(2 - \beta)(\beta - 1) > 9b\Gamma^2 - \alpha(2 - \beta)(1 - 2\beta)$$

which is clearly impossible, since the right-hand side is unambiguously positive (it is the assumption that assures optimal investment be positive). If $1 - 2\beta > 0$ ($\beta < 1/2$):

$$3\alpha^2(2 - \beta)(\beta - 1) < 9b\Gamma^2 - \alpha(2 - \beta)(1 - 2\beta)$$

which is always true since the left-hand side is unambiguously non-positive, ending the proof.

A.1.4 Parameter discussion

To have economic meaning equations 1.18 and 1.22 need deliver a positive quantity of investment, that is, their denominator must be larger than zero. As for the former, which describes the no-cooperation case, this means in symmetric equilibria:

$$b > \frac{\alpha(2 - \beta)^2}{9\Gamma^2}$$

from which $b > 4.4444$ ensures the holding of the property for any value of α and β .

As for the cooperative equation we must have in symmetric equilibria:

$$b > \frac{\alpha(2 - \beta)^2}{9\Gamma^2}$$

from which $b > 5.5555$ ensures the holding of the property for any value of α and β .

A.2 Appendix

A.2.1 Proof of lemma 1

It follows from equation 2.3.

A.2.2 Proof of lemma 2

It follows from equation 2.5

A.2.3 Proof of proposition 5

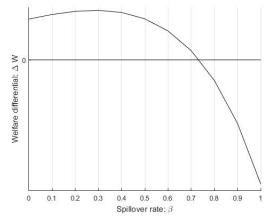


Figure A.3: $\gamma = 0.8$
and $b = 7$

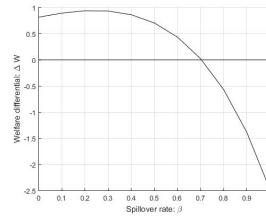


Figure A.4: $\gamma = 0.5$
and $b = 7$

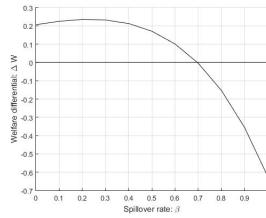


Figure A.5: $\gamma = 0.2$
and $b = 7$

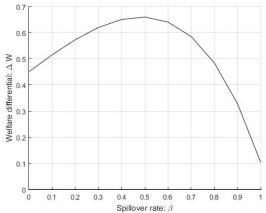


Figure A.6: $\gamma = 0.5, b = 10$

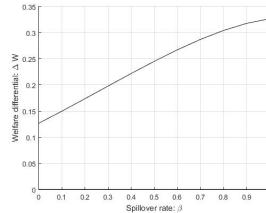


Figure A.7: $\gamma = 0.5, b = 20$

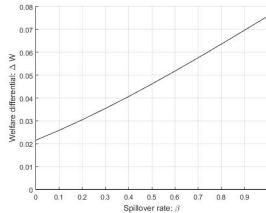


Figure A.8: $\gamma = 0.5, b = 50$

The proof is based on graphical arguments because it is not possible to compute derivatives (α is a binary variable). Figures A.3-A.5 show the welfare differential between the case $\alpha = \bar{\alpha} = 1$ and the case $\alpha = \underline{\alpha} = 1 - \gamma$ for, respectively, $\gamma \in \{0.8; 0.5; 0.2\}$ and $b = 7$, which ensures that optimal investment be positive. As it can be seen, the validity of the proposition holds for any of the considered values. As b grows larger, that is, as the demand function becomes more elastic, the risky activity is less likely to be preferred and eventually is never welfare-maximising, as shown in figure A.6-A.8 depicted for $\gamma = 0.5$ and $b \in \{10; 20; 50\}$.

A.2.4 Proof of proposition 6

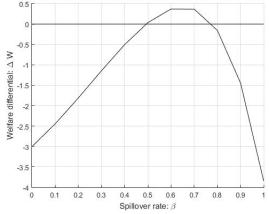


Figure A.9: $\gamma = 0.8, b = 7$

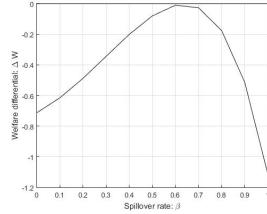


Figure A.10: $\gamma = 0.5, b = 7$

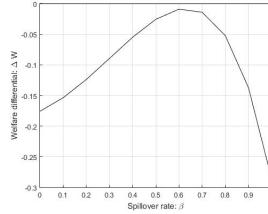


Figure A.11: $\gamma = 0.2, b = 7$

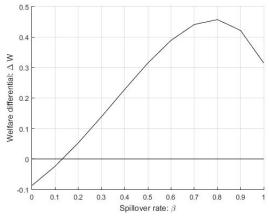


Figure A.12: $\gamma = 0.5, b = 10$

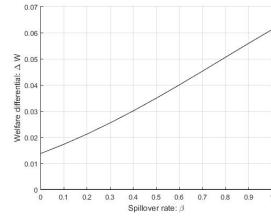


Figure A.13: $\gamma = 0.5, b = 20$

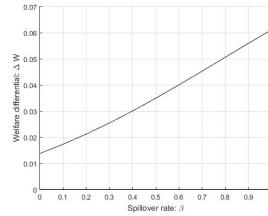


Figure A.14: $\gamma = 0.5, b = 50$

The proof is based on graphical arguments because it is not possible to compute derivatives (α is a binary variable). Figures A.9-A.11 show the welfare differential between the case $\alpha = \bar{\alpha} = 1$ and the case $\alpha = \underline{\alpha} = 1 - \gamma$ for, respectively, $\gamma \in \{0.8; 0.5; 0.2\}$ and $b = 7$, which ensures that optimal investment be positive. As it can be seen, the validity of the proposition holds for any of the considered values. As b grows larger, that is, as the demand function becomes more elastic, the risky activity is less likely to be preferred and eventually is never welfare-maximising, as shown in figure A.12-A.14 depicted for $\gamma = 0.5$ and $b \in \{10; 20; 50\}$.

A.2.5 Proof of lemma 4

The proof consists of two parts. First, I need to show that $x^C > x^N$ if and only if $\beta > 1/2$. Let us observe that $x^C = x^N$ for $\beta = 1/2$:

$$x^N(\beta = 1/2) = \frac{(a - \bar{c})\frac{3}{2}}{\frac{9b\Gamma(\alpha)^2}{\alpha} - \left(\frac{3}{2}\right)^2}$$

$$x^C(\beta = 1/2) = \frac{(a - \bar{c})\frac{3}{2}}{\frac{9b\Gamma(\alpha)^2}{\alpha} - \left(\frac{3}{2}\right)^2}$$

Consider now a level of β larger than $1/2$. The denominator of x^N is unambiguously larger than the denominator of x^C and the numerator of the former is smaller than the numerator of the latter. For this reason $\beta > 1/2$ implies $x^C > x^N$. Let us consider now a level of β less than $1/2$. The denominator of x^N is unambiguously less than the denominator of x^C and the numerator of the former is larger than the numerator of the latter. For this reason $\beta < 1/2$ implies $x^C < x^N$. This ends the proof of the first part. Second, I need to show that $\beta = \gamma = 0$ implies $x^N = x''$:

$$x^N(\beta = \gamma = 0) = \frac{2(a - \bar{c})}{9b\Gamma(1)^2 - 2}$$

$$x''(\beta = \gamma = 0) = \frac{a - \bar{c}}{\frac{9b\Gamma(1)^2}{2} - 1}$$

which ends the proof.

A.2.6 Proof of proposition 7

I need to show that in second-best we have $E[W^C] > E[W^N]$ if and only if $\beta > 1/2$. The level of welfare depends on two main parameters: the spillover rate β and the risk rate γ . I shall present an algebraic proof for $\gamma = 1/2$ and deliver graphical proofs for other values of the risk parameter.

If $\gamma = 1/2$ we have:

$$\begin{aligned} E[W^C] &= \frac{1}{4}W_{I,I}^C + \frac{1}{2}W_{I,O}^C + \frac{1}{4}W_{O,O}^C \\ E[W^N] &= \frac{1}{4}W_{I,I}^N + \frac{1}{2}W_{I,O}^N + \frac{1}{4}W_{O,O}^N \end{aligned}$$

from which:

$$E[W^C] - E[W^N] = \frac{1}{4}(W_{I,I}^C - W_{I,I}^N) + \frac{1}{2}(W_{I,O}^C - W_{I,O}^N) + \frac{1}{4}(W_{O,O}^C - W_{O,O}^N)$$

For instance, when $b = 7$, ensuring that optimal investment is positive, we get:

$$\begin{aligned} E[W^C] - E[W^N] &= \\ &= -\frac{0.535714(-7.48428 + \beta)(-6.33761 + \beta)(-4.14341 + \beta)(-0.5 + \beta)}{(1006.71 + 142.779\beta - 90.6863\beta^2 - 4.5\beta^3 + 0.\beta^4)^2} \times \\ &\quad \times \frac{(0.270951 + \beta)(2.83812 + \beta)(3.33761 + \beta)(4.01862 + \beta)}{(1006.71 + 142.779\beta - 90.6863\beta^2 - 4.5\beta^3 + 0.\beta^4)^2} \end{aligned}$$

which is clearly positive if and only if $\beta > 1/2$.

Figures A.15 and A.16 show the difference $E[W^C] - E[W^N]$ for $\gamma \in \{0.8; 0.2\}$. As it can be seen the function always lies above the horizontal axis if and only if $\beta > 1/2$ confirming the validity of the proposition.

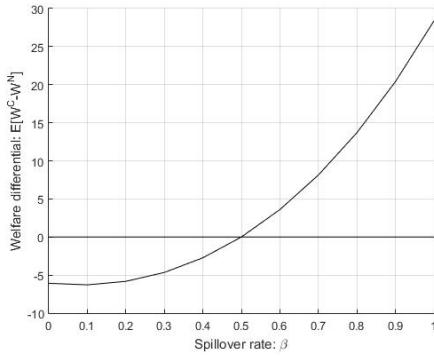


Figure A.15: $\gamma = 0.8$

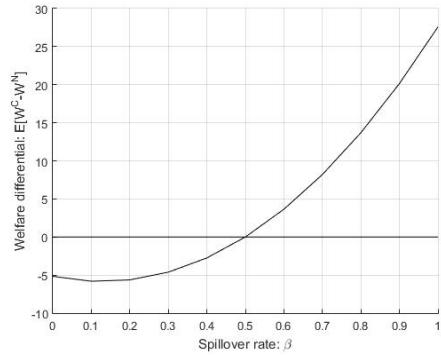


Figure A.16: $\gamma = 0.2$

A.2.7 Proof of proposition 9

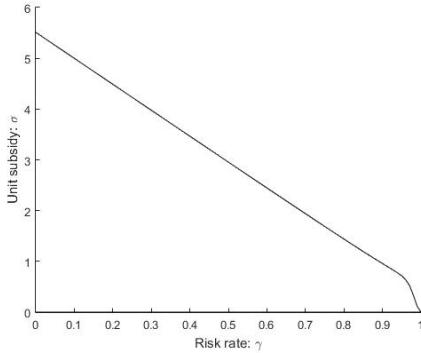


Figure A.17: Relationship between unit subsidy and risk

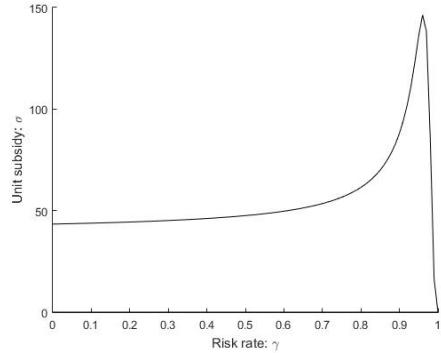


Figure A.18: Relationship between total subsidy and risk

The proof is graphical. Figures A.17 and A.18 show respectively the relationship between unit subsidy and risk and between total subsidy and risk. The former relationship is decreasing and the latter is increasing for most values of the risk parameter. In particular there exists a critical value, close to unity, after which the latter relationship reverses downwards and quickly approaches zero.

A.2.8 Proof of proposition 10

I provide many simulations (figures A.19-A.27) suggesting that σ and $\sigma x''$ are respectively decreasing and increasing in the risk rate for most values of the parameters (some exceptions in fact occurs for $\beta < 0.05$).

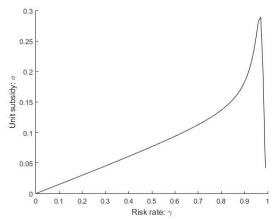


Figure A.19: $\beta = 0$

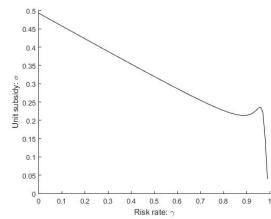


Figure A.20: $\beta = 0.05$

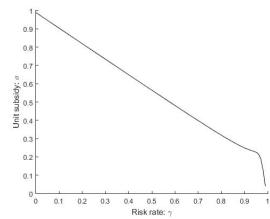


Figure A.21: $\beta = 0.1$

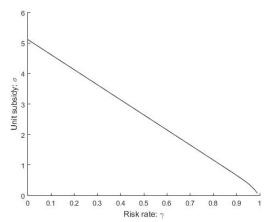


Figure A.22: $\beta = 0.5$

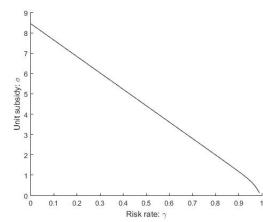


Figure A.23: $\beta = 0.8$

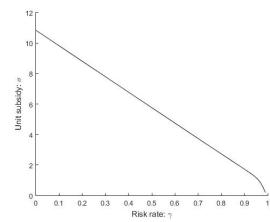


Figure A.24: $\beta = 1$

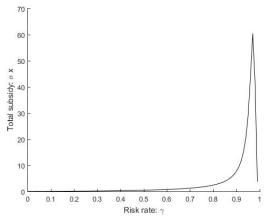


Figure A.25: $\beta = 0$

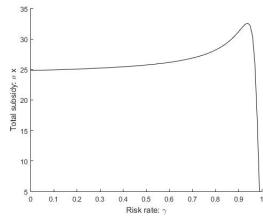


Figure A.26: $\beta = 0.5$

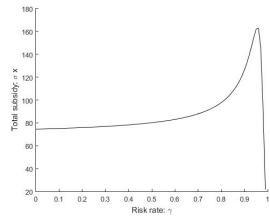


Figure A.27: $\beta = 1$

A.2.9 Proof of proposition 11

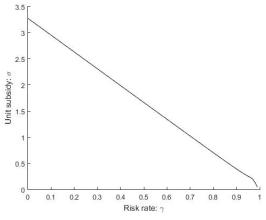


Figure A.28: $\beta = 0$

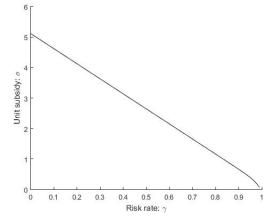


Figure A.29: $\beta = 0.5$

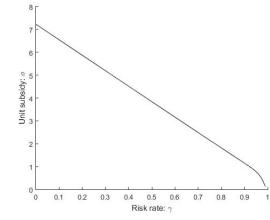


Figure A.30: $\beta = 1$

Figures A.28-A.33 depict the relationship between unit and total subsidy and the risk rate, and show the validity of the proposition for all the values of the parameter: as the risk rate grows large the amount of unit and total subsidy, respectively, falls and rises.

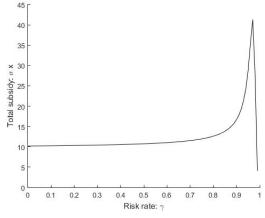


Figure A.31: $\beta = 0$

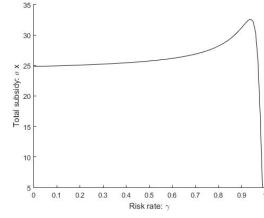


Figure A.32: $\beta = 0.5$

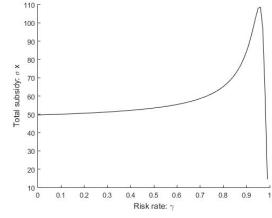


Figure A.33: $\beta = 1$

A.2.10 Equation 2.13

In each state of the world s output determination for firm i is as follows:

$$\max_{y_i} \Pi_i = (p^s - c_i^s)y_i^s - K_i$$

where the state of the world affects the value of marginal cost c_i . Solving for both firms we can obtain the following equilibrium output levels:

$$\begin{aligned} y_i^{I,I} &= \frac{a - \bar{c} + x_i(2 - \beta) + x_j(2\beta - 1)}{3b} \\ y_i^{I,O} &= \frac{a - \bar{c} + x_i(2 - \beta)}{3b} \\ y_i^{O,I} &= \frac{a - \bar{c} + x_j(2\beta - 1)}{3b} \\ y_i^{O,O} &= \frac{a - \bar{c}}{3b} \end{aligned}$$

A.2.11 Equation 2.19

Investment determination for firm i is as follows:

$$\max_{x_i} E[\Pi_i] \text{ s.t. } \Pi_y^s = 0$$

that is:

$$\max_{x_i} E[by_i^2 - K_i + \sigma_i x_i]$$

where s denotes the state of the world. The derivative with respect to x_i is:

$$\alpha_i \alpha_j \left[2by_i^{I,I} \frac{2 - \beta}{3b} \right] + \alpha_i (1 - \alpha_j) \left[2by_i^{I,O} \frac{2 - \beta}{3b} \right] = D_x[K_i] - \sigma_i$$

In the first best symmetric equilibrium we have $\alpha_i = \alpha_j$, $x_i = x_j$ and $y_i^{(I,I)}, y_i^{(I,O)}$ are given by equations 2.3 and 2.7. Thus:

$$2\frac{2-\beta}{3b}\alpha\left[\alpha\frac{a-\bar{c}+x(1+\beta)}{2b}+(1-\alpha)\frac{a-\bar{c}+x(1+\beta)/2}{2b}\right]=K_x-\sigma$$

and finally:

$$\frac{\alpha(2-\beta)}{3b}\left[a-\bar{c}+\frac{x(1+\beta)(1+\alpha)}{2}\right]=K_x-\sigma$$

A.2.12 Second-order conditions

For the first-order conditions to deliver a maximising path, it is necessary that the second-order conditions hold for every value of the parameter β at the equilibrium point; we need in particular that the hessian matrix H be negative definite; that is:

$$\begin{cases} W_{yy} < 0 \\ \det H > 0 \end{cases}$$

where W_{yy} is the second derivative of welfare with respect to output and the hessian matrix H has the following form:

$$H = \begin{bmatrix} W_{yy} & W_{yx} \\ W_{xy} & W_{xx} \end{bmatrix}$$

A.3 Appendix

A.3.1 Proof of lemma 7

The proof follows directly from equation (3.4).

A.3.2 Proof of lemma 9

The proof follows directly from equation (3.9).

A.3.3 Proof of lemma 8

An increase in σ , by construction, affects positively domestic investment. When spillovers are large, by lemma 7, a rise in domestic investment determines a rise in foreign investment.

A.3.4 Proof of proposition 12

The proof consists of showing that $\sigma_n \geq 0$, where σ_n is given by equation 3.8 once set $\delta = 1$; and that $\sigma_n = 0$ if $\beta = 1/2$. The latter can be proved by considering the following:

$$\begin{aligned} m_n(\beta = 1/2) &= \frac{2b}{4b - 1} \\ m_n^*(\beta = 1/2) &= 0 \\ q_n(\beta = 1/2) &= \frac{2(a - \bar{c})}{12b - 4} \text{ and thus} \\ \sigma_n(\beta = 1/2) &= \frac{\frac{6b}{4b-1}(a - \bar{c}) \left[1 + \frac{1}{4b-1} \right] - \frac{24b^2(a-\bar{c})}{(4b-1)^2}}{\frac{18b^2}{4b-1}} = 0 \end{aligned}$$

By setting $a - \bar{c} = 1$ to state simpler results, the former can be shown to develop into:

$$\sigma_n = \frac{2(2\beta - 1)^2(2 - \beta)[3b - 2 + 3\beta - \beta^2]}{3[81b^3 - 27b^2(2 - \beta)^2 - (2 - \beta - 2\beta^2 + \beta^3)^2 - b(2 - \beta)^2(5\beta^2 + 4\beta - 10)]} \geq 0$$

Let us observe that the denominator is always positive if b is sufficiently large, in fact $b = 1$ can be easily shown to be enough; and that the numerator is always non negative and equal to zero if and only if $\beta = 1/2$, as I wanted to show. I present also a graphical proof. Figures 17-19 are drawn at different values of the parameter b . To increase the robustness of the graphical argument I consider a very wide range of the parameters, in particular: $b = 1$, $b = 10$ and $b = 50$. The only condition is $b > 1$, by which the term m_n assumes positive values (which means that positive domestic subsidies ensures greater domestic investment). The other parameters, a and \bar{c} , only have additive relevance and thus do not affect the shape of the graph.

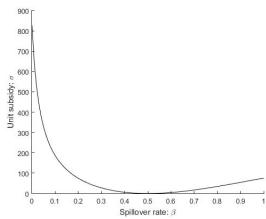


Figure A.34: $b=1$

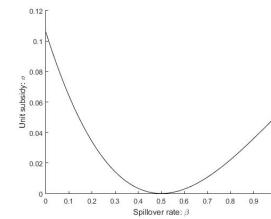


Figure A.35: $b=10$

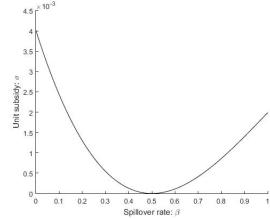


Figure A.36: $b=50$

As it can be seen, in all figures the function lies above the horizontal axis, which suggests that the subsidy σ is always non-negative, as required; and the function crosses the horizontal axis at $\beta = 1/2$, as expected.

A.3.5 Proof of lemma 10

The proof follows directly from lemma 9.

A.3.6 Proof of proposition 13

The proof consists of showing that $\sigma_c \geq 0$ if and only if $\beta \leq 1/2$, where σ_c is given by equation 3.12 once set $\delta = 1$; and that $\sigma_c = 0$ if $\beta = 1/2$. The latter can be proved by considering the following:

$$\begin{aligned} m_c(\beta = 1/2) &= \frac{2b}{4b - 1} \\ m_c^*(\beta = 1/2) &= 0 \\ q_c(\beta = 1/2) &= \frac{2(a - \bar{c})}{12b - 4} \text{ and thus} \\ \sigma_c(\beta = 1/2) &= \frac{\frac{6b}{4b-1}(a - \bar{c}) \left[1 + \frac{1}{4b-1} \right] - \frac{24b^2(a-\bar{c})}{(4b-1)^2}}{\frac{18b^2}{4b-1}} = 0 \end{aligned}$$

By setting $a - \bar{c} = 1$ to state simpler results, the former can be shown to develop into:

$$\sigma_c = \frac{4 - 2\beta}{9b - (2 - \beta)^2} \geq 0$$

which is clearly positive if and only if $\beta < 1/2$, as we wanted to show. I present also a graphical proof. Figures 20-22 are drawn at different values of the parameter b . To increase the robustness of the graphical argument I consider a very wide range of the parameters, in particular: $b = 2$, $b = 10$ and $b = 50$. The only condition is $b > 1$, by which the term m_c assumes positive values (which means that positive domestic subsidies ensures greater domestic investment). The other parameters, a and \bar{c} , only have additive relevance and thus do not affect the shape of the graph.

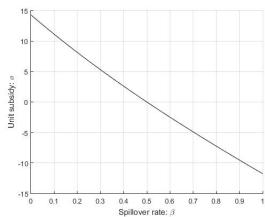


Figure A.37: $b=2$

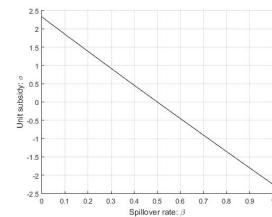


Figure A.38: $b=10$

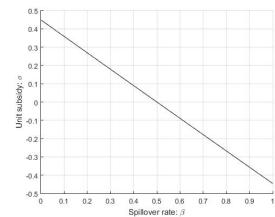


Figure A.39: $b=50$

As it can be seen, in all figures the function lies above the horizontal axis only when the spillover rate is sufficiently low, which suggests that the subsidy σ is

positive if and only if $\beta \leq 1/2$, as required; and the function crosses the horizontal axis at $\beta = 1/2$, as expected.

A.3.7 Proof of lemma 11

Without government intervention the proof is equivalent to show that $q_n \geq q_c$ if and only if $\beta \leq 1/2$, where q_n, q_c are given by equations 3.5 and 3.10. After some algebraic manipulation we get:

$$[18b - 2(2 - \beta)^2 - 2(2 - \beta) - 2(2 - \beta)(2\beta - 1)](1 - 2\beta) \geq -6(2 - \beta)(1 - 2\beta)$$

At the LHS we know that the term between square brackets is positive due to condition $b > 1$; whereas the sign of $(1 - 2\beta)$ depends on the spillover parameter. It is convenient to distinguish the two cases. If $\beta \geq 1/2$ we get:

$$[18b - 2(2 - \beta)^2 - 2(2 - \beta) - 2(2 - \beta)(2\beta - 1)] \geq -6(2 - \beta)$$

which is always true by construction. Instead, if $\beta > 1/2$ we obtain:

$$[18b - 2(2 - \beta)^2 - 2(2 - \beta) - 2(2 - \beta)(2\beta - 1)] \leq -6(2 - \beta)$$

which is always false since right the hand side amounts to a negative value. This concludes the proof.

A.3.8 Proof of conjecture 1

The proof consists of showing that $x_n(\sigma = 0)/x_n(\sigma = \sigma_n) = x_c(\sigma = 0)/x_c(\sigma = \sigma_c) = 1$; that only the cooperative ratio $x_c(\sigma = 0)/x_c(\sigma = \sigma_c)$ can exceed unity, and that this happens when $\beta > 1/2$. Again, the proof represented here is graphical. To increase the robustness of the graphical argument I consider the following range of parameters: $b = 2$, $b = 5$ and $b = 10$. The continuous and the dashed lines represent, respectively, no-cooperation and cooperation.

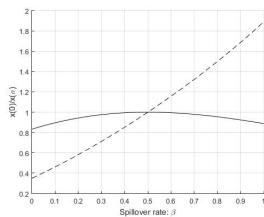


Figure A.40: $b=2$

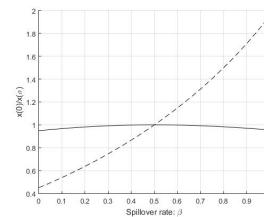


Figure A.41: $b=5$

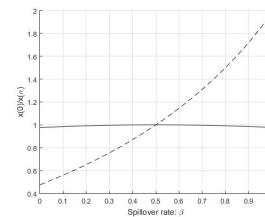


Figure A.42: $b=10$

As it can be seen, all figures 23-25 possess the properties required, suggesting the validity of the proposition. A rising b has the graphical effect to flatten out the continuous function towards unity, which means that the non-cooperative regime requires progressively a lower degree of government intervention.

A.3.9 Proof of conjecture 2

To increase the robustness of the graphical argument I consider the following range of parameters: $b = 2$, $b = 5$ and $b = 10$. The continuous and the dashed lines represent, respectively, no-cooperation and cooperation.

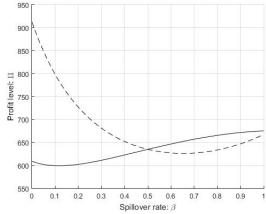


Figure A.43: $b=2$

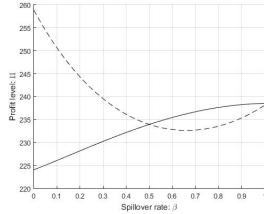


Figure A.44: $b=5$

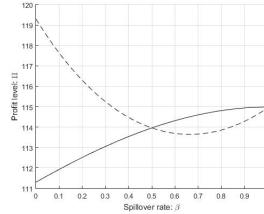


Figure A.45: $b=10$

As it can be seen, all figures 26-28 suggest that no-cooperation may be more profitable than cooperation when spillovers are large.

A.3.10 Proof of conjecture 3

See figures 29-31 for a graphical confirm.

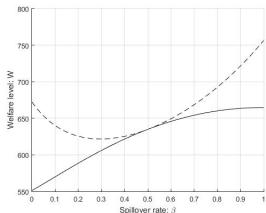


Figure A.46: $b=2$

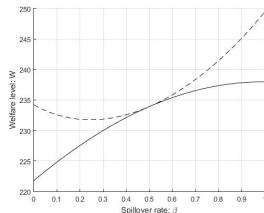


Figure A.47: $b=5$

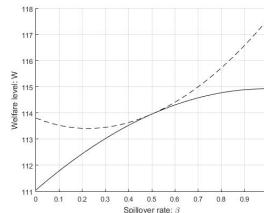


Figure A.48: $b=10$

A.3.11 Proof of lemma 13

Provide the fact that b must exceed $2/3$, showing that $\mu_n > 0$ and $\mu_n^* < 0$ can straightforwardly obtained by elementary calculations.

A.3.12 Proof of lemma 14

Provided that b must exceed unity, I need to show that $\mu_c^*(\beta) < 0$ and that the sign of $\mu_c(\beta)$ depends on spillovers. The former proof is straightforward as thus

omitted. The latter consists of showing that $\mu_c(0)$ and $\mu_c(1/2)$ are positive, and $\mu_c(1)$ is negative:

$$\mu_c(0) = 2 \frac{72b}{(18b - 10)^2 - 64} > 0 \text{ for } b > 1$$

$$\mu_c(1/2) = 2 \frac{3/2(18b - 9/2)}{(18b - 9/2)^2} > 0 \text{ for } b > 1$$

$$\mu_c(1) = 2 \frac{-18b}{(18b - 10)^2 - 64} < 0 \text{ for } b > 1$$

By continuity of the function $\mu_c(\beta)$ we can apply the intermediate value theorem, by which there exists a $\bar{\beta} \in (1/2, 1)$ such that $\mu_c(\bar{\beta}) = 0$. This ends the proof.