UNIVERSITY OF SIENA
DEPARTMENT OF ECONOMICS

Essays on Markets and Networks

Anna Papaccio

SUPERVISOR

Prof. Stefano Vannucci

PHD COORDINATOR

Prof. Ugo Pagano

Abstract

This thesis addresses two main arguments that have in common the study of strategic interaction of firms. To start with, firms play in an international environment and they are aggregated in order to represent a country. Each country in turn takes decision on the formation of free trade agreements (Fta). Secondly, firms, are treated as single strategic unites, and play in an upstream and downstream market in order to choose among vertical and horizontal strategies. Essentially, the first issue is developed in two essays. The focus of the first part of the thesis is on the well known bilateralism versus multilateralism controversy. In most of the literature addressing the free trade agreements formation issue the arguments have been analysed in a political-economy setting. Using the political-support approach, Krishna (1994) and Grossman and Helpman (1995) show that Ftas are more likely to be adopted if they are trade diverting, suggesting that politically successful Ftas are likely to be of the harmful type. Recently, the formation of Ftas has been modeled as a network formation game. The main finding of this approach is that the global free trade network is a stable and efficient outcome. In the first chapter, the formation of free trade agreements has been studied as a network formation game in order to address two main issues: first, to find out if bilateralism is consistent with global free trade; second, to investigate on the connection between bilateralism and multilateralism. We made use of different stability concepts such as *pairwise stability* and *strong stability* in order to better evaluate the impact of regionalism on the multilateral process.

In the second chapter, the dynamic formation of bilateral agreements between countries of different size and characteristics and the tension between stability and efficiency have been studied by means of the network formation theory. We focused on the efficiency of the potential configurations and on the relationship between stability and efficiency with symmetric and asymmetric countries. We found out that whenever we assume that countries are symmetric both in market sizes and number of firms we always obtain that the stable network configuration is also the efficient one. On the other hand, if we assume a degree of asymmetry among countries we end up with a network configuration that is not the efficient one and that leads to a network architecture in which bigger countries are better off and small countries are worse off.

In chapter three we draw away from the international environment. Due to market concentration concern and the increase of market power by retailers in both vertical and horizontal dimension, in recent years, public institutions in the EU have performed studies focused on the potential for "bad" price transmission in food-markets. According to studies on competition in food retail distribution sector developed for the European Commission the consolidation in food retailing emerged clearly as general feature across most EU countries. However, even if this tendency is sometimes associated with benefits deriving from improved efficiency and service, the increasing concentration brings concerns about the exercise of market power by retailers (buyers and/or sellers) and its consequence on welfare. Therefore, in the final part of the thesis the strategic interaction among retailers embedded in an upstream and a downstream market has been studied using game theory and network theory analysis. We modelled retailers' strategic behaviour considering the vertical relation between retailers and suppliers in the food industry whereby retailers exercise seller power in their relation with consumers and buyer power in their relation with producers. Our main purpose was to show that when strategic interaction among agents shapes the power structure of the market (described by means of a graph structure) it results in equilibrium architectures in which retailers have both market power and buyer power (countervailing power) driving to a "bad" price transmission mechanism.

Contents

Introduction and outline of the thesis

This thesis consists of three essays in which two main topics have been addressed. Mainly we deal with the strategic interaction in international network environment and in markets in which the structure - where players are going to take final optimal decisions - is the resulting outcome of horizontal and vertical linking games.

In July 2008, the Doha Development Round, the multilateral trade-negotiation round of the World Trade Organization have stalled over a divide on major issues, such as agriculture, industrial tariffs and non-tariff barriers, services, and trade remedies. Its objective was to lower trade barriers around the world, allowing countries to increase trade globally. The most significant divergences, arising during the multilateral talks, have been between developed nations and the major developing countries. It is evident nowadays, that countries differences and needs are the main source of disagreement in every trade field.

It was said that the suspension of negotiations might not imply an increase in protectionism over the short term, or a collapse of the multilateral trade system. It might encourage new regional trade agreements favouring the more advanced countries and damaging the poorest ones in particular (F. Steinberg)[1].

Following the deadlock of multilateralism talks and the beginning of a new wave of regionalism arrangements, different issues about free trade negotiations arise. For instance, the prospect of a new wave of preferential trade agreements has given rise to a new interest about the potential relationship between regionalism and multilateralism. According to the economic literature about the effect of preferential trade agreements (Ptas) on the multilateral process, under particular conditions regionalism may represent an important field test of the new deep integration among countries. If the interests supported by Ptas are compatible with the multilateral ones, this conjecture can be fulfilled. In this respect Ptas may represent a bridge between national and global system. Otherwise the different integration features may become rivals leading to losses for each country, especially for the poorest and less developed ones.

The issue on the relationship between regionalism and multilateralism has taken the form of a specific question to be answered. Scholars investigated on whether

[1] Federico Steinberg .(2007) (Analyst in International Economy at the Elcano Royal Institute and Professor at the Autonomous University of Madrid).*The Future of World Trade: Doha or Regionalism and Bilateralism? (ARI).*

the outcome of a trade agreements process would be a global free trade scenario when we keep increasing the number of trade agreements (i.e. bilateral). Alternatively, the process would stop before to reach the complete free trade outcome.

This puzzle has been investigated in two different ways. In some works, regionalism and multilateralism have been described as two different processes that may or not interact. In other words, it has been assumed that the time-path of the multilateral trade negotiation and the one of preferential agreements may influence each other or not. In the first case, as Krishna (1995)[2] and Levy (1994)[3] noted, a negative outcome emerges. Like Grossman and Helpman (1995)[4], Krishna found that the greater the degree of "trade diversion"[5], the more likely the agreement will be accepted. Moreover, Krishna shows that with a sufficient degree of trade diversion, the option to form a trade agreement can render a previously (feasible) multilateral liberalization not feasible.

In the second approach, scholars like Baldwin (1993)[6] have analyzed the incentives facing outsiders of a preferential agreement to seek entry. He found out that the preferential agreements expansion may create a sort of "Domino Effect".

A recent approach to this controversy originates from the network formation theory literature. The prediction of the network formation approach in Goyal-Joshi (2006)[7] is that the Global Free trade network is a *stable*[8] and *efficient*[9] outcome.

Therefore, by means of network theory it has been shown that an increasing number of agreements lead to a Complete Network configuration, that is, the global free trade scenario. Accordingly, an initial scenario containing a number of bilateral agreements may lead to the above outcome, which in turn implies that bilateral agreements don't prevent multilateral liberalization.

Furthermore, together with the evolution and the stability of bilateral agreements between countries of different size and characteristics, the efficiency of the resulting free trade scenario - for each group of countries – should be studied, for instance, by means of the well known network formation theory. Indeed, attention has been given to the welfare effects of regional free trade associations and customs unions (see for example Krugman (1991)).

The first Chapter of the thesis has two aims. First, the formation of free trade agreements amongst symmetric and asymmetric countries has been studied by means of the network formation theory. Second, we have focused on the connection between bilateralism and multilateralism and we have evaluated the efficiency of the equilibrium configurations. In order to characterize and

---

[2] Krishna, P. *Regionalism and Multilateralism: A Political Economy Approach*. Mimeo, Economics Department, Columbia University, December 1995.

[3] Levy, P. *A Political Economic Analysis of Free Trade Agreements*. Economic growth Center Discussion Paper n.718, Yale university, 1994.

[4] Grossman, G.; Helpman, E. 1995.*The Politics of Free Trade Agreements*. American Economic Review. 85:4. pp.667-690.

[5] A measure of trade diverted away from non member countries.

[6] Baldwin, R. *A Domino Theory of Regionalism.* Centre for Economic Policy research (London). Working paper n.857, November 1993.

[7] Goyal, S. and Joshi, S., 2006, *Bilateralism and Free Trade.* International Economic Review.

[8] Global free trade s*tability* (*pairwise stability*) is a lack of incentives to deviate from a global free trade scenario (a complete network). That is, starting from a network of global free trade agreements no one has incentive to cut or to form a new link.

[9] The concept of *Efficiency* is linked to the World Welfare. To satisfy this property, the welfare generated by a global free trade network should be greater than any other generated by a non-complete network (i.e. every trade network not including the set of all countries).

distinguish between bilateral and multilateral outcomes we have used two different network stability notions, that is, *pairwise stability* and *strong stability*. Moreover, from the analysis of pairwise and strong stability we have also obtained insights on the relationship between bilateralism and multilateralism.

A Preferential trade agreement (i.e. a free trade agreement in our model) between two countries is described by a link. A link is an unordered pair of two countries. A link between two countries implies that the goods produced in both countries can be traded without any tariffs. In the absence of links, goods are traded with some tariffs. A Ptas architecture has been represented by an "undirected graph"[10] consisting on a set of countries and a network (i.e., a collection of links). Countries interaction shapes network configurations. We developed a free trade network formation game in a setting with three countries (nodes). By taking into account the profits-feasibility of the agreement, we used an imperfect competition model to investigate the global free trade outcome. Represented by firm profits in each country, the payoff functions depend on the equilibrium quantities and tariffs of the oligopolistic intra-industry trade model as in Krishna (1998). Also, profits depend crucially on the number of firms competing in the country (i.e., belonging to the arrangements) and on the market size of countries. In each country the decision to form a link depends on the balance between the profits generated by the proposed agreement and those generated by the *status-quo* solution. The total welfare is represented by the value generated by the emerging network architecture. We examined the direction of country incentives to the agreement formation and the total welfare associated with the increase in the number of links. In other words, we investigated if, by allowing countries to build up links, countries interaction would lead to a further liberalization (global free trade) and to an increase in welfare (efficiency). In our model the value function is not component additive and the allocation rule consequently not component balanced. For this reason, externalities across and inside components exist, therefore we cannot use the concept of constrained efficiency. The allocation rule is fixed and the value of each network configuration cannot be freely distributed among nodes since, even if transfers could be implemented, the promise of transfers may not be credible in this free trade agreements environment. For this reason, in order to evaluate the outcomes of the network formation process, we use also the notion of Pareto efficiency.

In general, when countries are symmetric and large the global free trade outcome may emerge in equilibrium both via bilateral agreements expansion and multilateral trade rounds. This result is consistent with recent findings on the empirical properties of world trade web by Fagiolo, Reyes and Schiavo (2009). They show that the world trade web is an extremely symmetric network, where almost all trade relationships tend to be reciprocated with similar intensities. Therefore, they studied its characteristics as if it were a weighted[11] undirected network. They pointed out that richer countries tend to be more clustered (and increasingly so over the years) supporting with their finding the "rich club phenomenon".

On the other hand, with medium-size countries we see a potential adverse effect of bilateralism on multilateralism. In other words, even though global free trade agreement may still be reached through bilaterals, multilateral negotiation can be

[10] The formation of a link (i.e., the agreement) is mutual, not unilateral.

[11] Each directed link from node i to j is weighted by total exports of country i to country j and then divided by the country i's GDP (i.e., the exporter country). Such a weighting setup allows one to measure how much economy i depends on economy j as a buyer (Fagiolo et al, 2009).

blocked by the existence of profitable deviations to the one link network (i.e. bilateralism). Moreover, because of the trade diverting feature of Ptas, when emerging only through bilateral agreements, the complete network, is not the efficient configuration.
When countries are asymmetric we obtain different results on efficiency and stability depending both on the degree of asymmetry and the market size intervals.
In the asymmetric case the global free trade outcome (also efficient for large countries) is never obtained as result of a multilateral trade round since the complete network is not strongly stable and the one link network is immune to multilateral deviations (except in the case of small countries). Moreover, when countries are asymmetric, another form of link architecture emerges in equilibrium: the hub and spoke configuration. Hub and spoke agreements take place more often among a developed country as a hub and more smaller spokes. Thus, the presence of such agreements in equilibrium is justified by the extent of asymmetry among countries. Indeed, empirical recent studies in which the relative weight of trade flows towards and from countries is considered, describe the world trade web as a very high density network but with the average strength of nodes being rather poor (Fagiolo et al, 2009). In other words, most countries hold mainly weak relationships, whereas only a selected core on nodes combine high degree and high strength. Indeed, the world trade web, according to Fagiolo, Reyes and Schiavo has a dis-assortative nature. That is, countries holding many (and more intense) trade relationships preferably trade with poorly connected countries.
Chapter 2 completes the analysis on the network formation of free trade agreements addressing this issue with a dynamic framework. The principal purpose of the second essay is to shed light on the relationship between stability and efficiency of stable networks of free trade agreements with symmetric and asymmetric countries.
The evolution of free trade agreements process among countries and the efficiency of the resulting free trade scenario have been studied by means of the network formation theory. The number of countries, the structure of the payoff functions and other assumptions of the model follow the one presented in the first chapter. On the other hand, we draw away from the previous model for what concerns the network formation process. Indeed we employed a dynamic framework in which networks are formed over time as the one used by Watts (2001) and Jackson and Watts (2002), in the context of the symmetric connections model.
The total welfare is represented by a value function depending on the network architecture. We found out that whenever we assume that countries are symmetric both in market sizes and number of firms we always obtain that the stable network configuration is also the Pareto-efficient one. On the other hand, if we assume a degree of asymmetry among countries we end up with a network configuration that is not the efficient one and that leads to a network architecture in which bigger countries are better off and small countries are worse off.
The third essay relies on the issue of market power and bad price transmission mechanism. It drew from the concern about the consolidation in food retailing emerged clearly as general feature across most EU countries. Usually the grocery market was composed by a very large number of buyers and/or sellers operating as small price takers, whereas, large buyers and sellers, each with significant power, drive the current market. However, even if this tendency is sometimes associated with benefits deriving from improved efficiency and service, the increasing concentration brings concerns about the exercise of market power by retailers (buyers and/or sellers) and its consequence on welfare.

For instance, in Dobson (1999) it has been argued that, when efficiency benefits are not able to off-set the negative effect of concentration, seller power by retailers could result in higher prices for consumers and perhaps reduced choice. In general, the economic welfare effects arising from the exploitation of buyer power are ambiguous. In fact, suppliers will generally suffer if the prices obtained for their goods are reduced while consumers might gain if lower upstream market prices result in retailers setting lower final consumers' prices. Although the net effect is not clear, there seems to be a general belief which maintains that retail concentration is bad for social welfare.
Moreover, recent literature emphasises the role of market structure in determining the degree of price transmission along the marketing chain. Some studies, as Weldegebriel (2004), model vertical price transmission allowing for both oligopoly (seller) power in the retail sector and oligopsony (buyer) power in the supply sector. They suggest that the exercise of market power by retailers does not totally explain why suppliers' price changes are not fully reflected as retail price changes. Due to market concentration and the increase of market power by retailers in both vertical and horizontal dimension, in recent years, public institutions in the EU have performed studies focused on the potential for asymmetric price transmission in food-markets. The main concern is that consumers may not benefit as much as expected from liberal agricultural policy reforms if suppliers and retailers do not fully transform it into the proposed price reductions. In summary, many authors have suggested that market power can lead to a not symmetric price transmission. However most scholars predict that in the common oligopoly context, both positive and negative price transmission are likely to emerge, depending on market structure and strategic behaviour. For instance when the wholesale price raises, retailers try to save their normal profit margin, while they try to widen it, at least temporarily, when the input price falls. Because of consumers search costs, eventually profits go down and prices tend to the competitive levels. However, another situation may happen. When wholesale prices grow, each firm increase in short time its selling price as signal to competitors to agree to a tacit agreement; in the opposite case in order to save the tacit agreement it slowly decreases the selling price. The most important conclusion to draw from the literature summarised above might be that more research is needed to shed light on the increasingly complicated relationships among behaviour of agents along the supply chain and the resulting prices and quantities.
In Chapter 3 we modelled retailers' strategic behaviour considering the vertical relation between retailers and suppliers in the food industry whereby retailers exercise seller power in their relation with consumers and buyer power in their relation with producers. In order to describe the vertical structure of the supplier-retailer relation we distinguish between upstream firms (producers or suppliers) and the downstream firms (processors, manufacturers or retailers). Due to the not competitive nature of the downstream level of the market we can observe that the reduction for the purchasing prices received by the suppliers is often not proportional[12] to the reduction of the final prices set by retailers. In other words, a countervailing power[13] may lead to social welfare benefits but a downstream market power may be expected to have an adverse effect. This evidence suggests

[12] We can define this situation as a *bad price transmission mechanism* or as previously quoted *"a positive asymmetric price transmission"*.
[13] It has been coined by Galbraith (1952) to describe the ability of large buyers in concentrated downstream markets to extract price concessions from suppliers.

that when both forms of power are simultaneously present, for consumers to gain (obtaining lower prices), the selling power of buyers (which increases final prices) needs to be more than off-set by lower suppliers' prices (resulting from buyer power).The result is that retailers derive monopoly profits and consumers bear higher prices and less retail products variety.

The strategic interaction among retailers embedded in an upstream and a downstream market has been studied using game theory and network theory analysis. Game theory and Network theory capture strategic interactions among players aware that their payoffs depend on other players' decisions. Our main purpose was to show how the strategic interaction among agents shape the power structure of the market (described by means of a graph structure) allowing for equilibrium architectures in which retailers have both market power and buyer power (countervailing power). Furthermore, strategic market interaction in turn may lead to a bad price transmission causing the worst outcome for consumers. The Upstream-Downstream game (U-D game) has been represented by a three stages game. Each stage of the game is represented by a simultaneous game among retailers and at the end of every stage is full aware of the strategies chosen by the other retailers, so each stage represents a proper subgame of the U-D game. In the first stage, retailers choose the number of suppliers are going to provide them wholesale products (we assume that each producers sells the same good) creating a vertical link with them.

In the second stage each retailer chooses whether to create a collaboration link with others retailers in order to exploit market power in the downstream market and/or buying power in the upstream market. In the third stage retailers maximize their profits (the strategic variable is the quantity) given the upstream and downstream market structure resulted from retailers strategic interactions in the previous stages of the game.

The equilibrium outcome is characterized by a graph architecture and a payoffs structure associated to it (deriving by strategic interaction among nodes). A semi-bipartite graph represents, in our model, the outcome, in terms of links architecture, of the equilibrium strategies in the first two stages. The payoffs structure corresponds to the monempory payoff for every retailer and in turn consumer prices are at their maximum level.

1

Bilateralism and Multilateralism: A network approach

1.1 Introduction

The Doha Development Round, the current (multilateral) trade-negotiation round of the World Trade Organization (WTO), started in November 2001. Its objective was to lower trade barriers throughout the world, allowing countries to increase trade globally. In July 2008, talks have stalled over a divide on major issues, such as agriculture, industrial tariffs and non-tariff barriers, services, and trade remedies. The most significant divergences have been between developed nations and the major developing countries. It is evident nowadays, that differences and needs amongst countries are the main source of disagreement in every trade field. The deadlock in the Doha Round and the proliferation of *regional* and *bilateral trade agreements* (Ptas)[14] has given rise to a new interest in the potential relationship between regionalism and multilateralism. According to the recent economic literature about the effect of Ptas on the multilateral process, under particular conditions, regionalism may represent an important field to test the new deep integration among countries. If the interests supported by Ptas are compatible with the multilateral ones, Ptas may represent a bridge between national and global system. On the contrary, the different integration features may become rivals leading to losses for each country, especially for the poorest and less developed ones.

It was said that the suspension of the Doha Round of WTO negotiations might not imply an increase in protectionism over the short term, or a collapse of the multilateral trade system. It might encourage new regional trade agreements

[14] Agreements (bilateral or regional) among a group of countries that reduce barriers to trade on a reciprocal and preferential basis for those in the group.

favouring the more advanced countries and damaging the poorest ones (Steinberg, 2007).
Indeed, according to the WTO regional trade agreements database, around 400 Ptas are scheduled to be implemented by 2010. Free trade agreements and partial scope agreements account for over 90% of the total amount of Ptas[15] while customs unions account for less than 10 % (WTO) . Moreover, many countries are now members of more than one Pta. In particular, a country which belongs to distinct Ptas and whose partners are not linked to each other has been defined as the "hub" and its partners as "spokes" (Wonnacott, 1996). "In a hub-and-spoke trading system, the largest markets sign individual agreements with a wide range of peripheral countries among which market access remains restricted. Such agreements can marginalize the spokes, where market access conditions are usually less advantageous than in the hub, which enjoys improved access to all of the spokes" (GEP 2005, World Bank). Moreover, a hub or a spoke may itself be a multi-country regional agreement. These kind of Ptas are present in all geographical areas of the world economy and it has been argued that a *hub-and-spoke* structure in world trade is emerging (GEP 2005, World Bank).
According to Lloyd and Maclaren (2003) one possible outcome of the Ptas formation process is a tripolar world with one large bloc of freely trading countries in Europe, one in the Americas and one in the emerging Asian nations. Since most of the hubs are developed countries, there is a high possibility that some developing countries would be left outside the major regional arrangements. "When the larger size of the markets in developed countries and especially the US and the EU is taken into account, there is no doubt that the increase in market access resulting from RTAs has gone overwhelmingly to developed countries and not to developing countries" (Lloyd and Maclaren, 2003) . If we assume that Ptas (in the form of free trade agreements) increase trade flows among its members, as it has already been shown in the empirical work by Baier and Bergstrand (2007), then the empirical analysis based on the intensity and the shape of trade in the world trade network provides significant insights to the study of free trade network formation. Recent findings on the empirical properties of world trade web by Fagiolo, Reyes and Schiavo (2009) show that the world trade web is an extremely symmetric network, where almost all trade relationships tend to be reciprocated with similar intensities. Therefore, they studied the world trade web features as if it were a weighted[16] undirected network. Fagiolo et al. pointed out that richer countries tended to be more clustered (and increasingly so over the years) supporting the "rich club phenomenon"[17]. Furthermore, they found out that, in spite of a very high density, the average strength of nodes was rather poor. In other words, according to Fagiolo et al., the majority of countries holds mainly weak relationships (in terms of relative dependence on the exports and imports of partner countries), whereas only a selected core combines high degree and high strength. Indeed, they also maintain that the world trade network shows a "*dis-assortative* nature", that is, countries holding many (and more intense) trade relationships preferably trade with poorly-connected ones.

[15] The total number of preferential agreements considered (according to the WTO source) include also agreements in force but not notified yet, those signed but not yet in force, those currently being negotiated, and those in the proposal stage.
[16] Each directed link from node i to j is weighted by total exports of country i to country j and then divided by the country i's GDP (i.e., the exporter country). Such a weighting setup allows one to measure how much economy i depends on economy j as a buyer (Fagiolo et al, 2009).
[17] Countries that have higher trade intensities trade a lot among themselves.

The prospective of a new wave of regionalism arrangements as a result of the deadlock in multilateralism talks has given rise to new and old concerns about the evolution of bilateral agreements formation between countries different in size and characteristics and the efficiency of the resulting free trade scenario. Two main concerns about Ptas proliferations appear to be of great interest among scholars.
First, it has been questioned whether bilateral arrangements (defined as regional arrangements with two members) would lead to broader liberalization (e.g., Bhagwati (1993) and Levy (1997)). Moreover, attention has also been given to the welfare effects of regional free trade associations and customs unions (see for example Krugman (1991)).
The issue regarding the relationship between regionalism and multilateralism has given rise to a specific question. Some scholars investigated the outcome of the trade agreement formation process with a view to the possibility of reaching global free trade by increasing the number of trade agreements (i.e. bilateral).
This puzzle has been examined in two different ways. In some works, regionalism and multilateralism have been described as two different processes that might or might not interact. In other words, it has been assumed that the time-paths of the multilateral trade negotiation and that of preferential agreements may or may not influence each other. In the first case, as Krishna (1998) and Levy (1997) noted, a negative outcome emerges. As in Grossman and Helpman (1995), Krishna finds that greater is the degree of "trade diversion"[18], higher is the possibility that the agreement will be accepted. Moreover, Krishna shows that, with a sufficient degree of trade diversion, the option to form a bilateral trade agreement can turn a previously (feasible) multilateral agreement into a not feasible one.
In the second investigation, scholars like Baldwin (1993) have analyzed which incentives outsiders of a preferential agreement have in order to seek entry and he proved that the preferential agreements expansion may create a sort of "Domino Effect".
A recent approach to this controversy belongs to the network formation theory literature. Goyal and Joshi (2006) have investigated the formation of free trade agreements as a network formation game. They argued that the process of bilateral trade agreements may lead to a global free trade configuration. In particular, the prediction of the network formation approach in Goyal-Joshi is that the global free trade network represents a *stable*[19] and *efficient*[20] outcome of the free trade network formation game. Therefore, by means of the network theory it has been shown that an increasing number of agreements lead to a complete network configuration, that is, the global free trade scenario. Accordingly, this result in turn implies that bilateral agreements do not prevent the multilateral liberalization. On the other hand, according to Furusawa and Konishi (2002) with symmetric countries, the global free trade network is stable. On the contrary, if countries are asymmetric in the market size and/or in the size of the industrial goods sector, the global free trade network may not be reached.
This chapter has two main aims. First, the formation of free trade agreements amongst symmetric and asymmetric countries has been studied by means of the

[18] A measure of trade diverted away from non member countries.
[19] Global free trade s*tability* (*pairwise stability*) represents the absence of unilateral incentives to deviate from a global free trade scenario (a complete network) to any other configuration achievable through the unilateral deviation. That is, starting from a network of global free trade agreements no one has incentive to cut or to form a new link.
[20] The concept of *Efficiency* is linked to the World Welfare. To satisfy this property, the welfare generated by a global free trade network should be greater than any other generated by a non-complete network (i.e. every trade network not including the set of all countries).

network formation theory. Second, we have focused on the connection between bilateralism and multilateralism and we have evaluated the efficiency of the equilibrium configurations. In order to characterize and distinguish between bilateral and multilateral outcomes we have used two different network stability notions, that is, *pairwise stability* and *strong stability*. Moreover, from the analysis of pairwise and strong stability we have also obtained insights on the relationship between bilateralism and multilateralism.

A Preferential trade agreement (i.e. a free trade agreement in our model) between two countries is described by a link. A link is an unordered pair of two countries. A link between two countries implies that the goods produced in both countries can be traded without any tariffs. In the absence of links, goods are traded with some tariffs. A Ptas architecture has been represented by an "undirected graph"[21] consisting on a set of countries and a network (i.e., a collection of links). Countries interaction shapes network configurations. We developed a free trade network formation game in a setting with three countries (nodes). By taking into account the profits-feasibility of the agreement, we used an imperfect competition model to investigate the global free trade outcome. Represented by firm profits in each country, the payoff functions depend on the equilibrium quantities and tariffs of the oligopolistic intra-industry trade model as in Krishna (1998). Also, profits depend crucially on the number of firms competing in the country (i.e., belonging to the arrangements) and on the market size of countries. In each country the decision to form a link depends on the balance between the profits generated by the proposed agreement and those generated by the *status-quo* solution. The total welfare is represented by the value generated by the emerging network architecture. We examined the direction of country incentives to the agreement formation and the total welfare associated with the increase in the number of links. In other words, we investigated if, by allowing countries to build up links, countries interaction would lead to a further liberalization (global free trade) and to an increase in welfare (efficiency). In our model the value function is not component additive and the allocation rule consequently not component balanced. For this reason, externalities across and inside components exist, therefore we cannot use the concept of constrained efficiency. The allocation rule is fixed and the value of each network configuration cannot be freely distributed among nodes since, even if transfers could be implemented, the promise of transfers may not be credible in this free trade agreements environment. For this reason, in order to evaluate the outcomes of the network formation process, we use also the notion of Pareto efficiency.

The chapter proceeds as follows. In the next section we introduce some important definitions on stability and efficiency. In section 1.3 we develop the model, describing the free trade agreements formation and the oligopolistic trade model. In section 1.4 we show the results on network stability and efficiency in the symmetric and asymmetric settings. In section 1.5 we conclude with some remarks.

[21] The formation of a link (i.e., the agreement) is mutual, not unilateral.

1.2 Definitions

1.2.1 Networks and Network games

Two main research threads have been carried out on networks. The first approach on network theory originated from the *Random Graph* literature According to this literature, network formation is modelled by specifying a stochastic process or by an algorithmic process through which links are formed (Barabasi, 2003; Vega-Redondo, 2007; Watts, 2001)..

The second approach uses the *Game Theory* and originated from the economic literature (Bala and Goyal, 2000; Brandes and Erlebach, 2005; Currarini and Morelli 2000; Demange and Wooders, 2005; Dutta and Mutuswami, 1997; Goyal, 2007; Jackson, 2008; Myerson, 1977; Wasserman, Faust and Iacobucci, 1994;). The agents are represented by nodes (vertices) and the links among agents by the edges of a graph. The formation of links depends upon the nodes. Agents derive benefits and bear costs of connection depending on the network configuration.

Thus, the economic approach to the network formation model focuses on the resulting equilibrium networks in which links are formed by agents controlling nodes or represented by them. This approach intends to study the possibility that an efficient network[22] can be formed and explains also the reasons why a particular network is formed. Indeed, it takes in to account agents' incentives and the costs of connections. Furthermore, the economic approach intends also to describe the natural tension between the incentives and efficiency concept. The network strategic formation is modelled according to the theory of games. There are two main characteristics of these kinds of models:

- Agents derive utilities from the network. This means that for every possible emerging network there is a different total welfare.
- The links, formed by agents' choices and the resulting networks, can be predicted through equilibrium concepts or through dynamic stochastic processes.

The first contribution to this literature derives from Myerson (1977). He characterized the cooperative game theoretic solution concept, the Shapely value, without directly imposing the additivity assumption. He analyzed a class of cooperative games using a graph structure and introduced the Myerson Value for splitting a surplus specifically dependent on the shape of the network. Systematic game theoretical treatment of network formation starts with Jackson and Wolinsky (1996) where the utilities derive directly from the networks rather than from the coalitions. In network formation games, as in every game theoretic model, there are different equilibrium solution concepts. The standard Nash equilibrium concept is not suitable for the network framework because, in the case of undirected graph, for the formation of a link it is necessary the willing of both the agents[23]. A network g is a list of unordered pairs of players linked to each other. A network game is

[22]The efficient network can be interpreted as the network that maximizes the total benefits of the society.

[23] Since we need the common consent of both agents to the formation of a link, we need a coalitional equilibrium concept or we have to study the game in extensive form where each agent chooses to accept or refuse the links in sequence.

represented by a pair (N, v, a) where N represents the set of players, v is the value function that generates value for each network created linking players in N and a is the allocation rule that specifies the way this value is distributed among players in the network. Countries are represented by nodes of a graph and links indicate bilateral agreements between the countries. For instance, if $N = \{i, j, k\}$ then $g = \{ij\}$ is the network in which just nodes i and j are linked. Let $g + jk$ be the network derived by adding the link jk to network g ; and $g - jk$ obtained by deleting it. Let $N(g)$ be the set of players who have at least one link in the network g. For every network g, $N(g) = \{i \mid \exists j \text{ s.t. } ij \in g\}$. A simple way to deal with the network equilibria is to define a stability notion directly over the networks.

1.2.2 Value functions and allocation rules

Network structure determines the overall utility or productivity of the society and of players. The study of network efficiency is based on the comparison of different *values* that different network architectures create. The *value function* and the *allocation rule* represent respectively the value generated by a specific network and the way in which this value is distributed among players. The value function derives directly from the characteristic function and the allocation rule from the imputation rule in cooperative game theory. In cooperative game theory those two concepts depend only by the set of player in the coalition. In the network approach these concepts depend on the global structure of the network rather than on a coalition. The possible valuations are identified through the use of the function $v : G \rightarrow \mathbb{R}$ where $G = \{g \mid g \subseteq g^N\}$ is the set of all possible networks on N and g^N represents the set of all subsets of N of size 2. The value function can include costs and benefits to links. The network can be characterized by externalities within and across component[24] of a network. A value function is *component additive* if $v(g) = \sum_{g' \in C(g)} v(g')$[25] for all $g \in G$. With a component additive value function the value of a network is simply the sum of the value of its components. In this way the possibility of externalities across components is ruled out because this characteristic implies that the value of one component doesn't depend on the structure of the other components. A value function is *anonymous* if, for every permutation of the set of players π we have that $v(g^\pi) = v(g)$ with $g^\pi = \{\pi(i)\pi(j) \mid ij \in g\}$. The network g^π is a network with the same structure as g but in which the set of nodes is relabelled according to π. *An allocation rule* is a function $Y : G \times v \rightarrow \mathbb{R}^N$ s.t. $\sum_i y_i(g, v) = v(g)$ for all v and g. When a value function is component additive the allocation rule is often component balanced, Y can be arbitrary otherwise. An allocation rule is *component balanced* if for any component additive $v, g \in G$, and $g' \in C(g)$ $\sum_{i \in N(g')} Y_i(g', v) = v(g')$. This concept implies that the value generated by any component is allocated to the players among that component. An allocation rule is

[24] See the Appendix A.1 for the definition of components of a network.

[25] $C(g)$ represents the set of components of a network g.

a*nonymous* if for every permutation of the set of players $\pi, v \in V,\ and\ g \in G$: $Y_{\pi(i)}\left(g^{\pi}, v^{\pi}\right) = Y_i\left(g, v\right)$. That is, the allocation only changes following the relabeling.

1.2.3 The Pairwise stability and Strong stability concepts

In network theory every player individually can withdraw a connection (undirected link) and every pair of player can build it. Let $\mathrm{N} = \{1, 2, ..., N\}$ denote a finite set of players. The network relations among players are represented by graphs. The nodes represent players and the edges the relationships between any pair of players. Let $N(g) = \{i \mid \exists j\ s.t.\ ij \in g\}$ be the set of player who have at least one link in the network g and $N_i(g) = \{j \in \mathrm{N} : ij \in g\}$ be the set of players with whom player i has a link in g. A *path* in $g \in G$ connecting i_1 and i_n is a set of distinct nodes $\{i_1, i_2, ..., i_n\} \subset N(g)$ such that $\{i_1 i_2, i_2 i_3, ..., i_{n-1} i_n\} \subset g$. A graph $g' \subset g$ is a *component* of g, if for all $i \in N(g')$ and $j \in N(g')$, $i \neq j$, $\exists$ a path in g' that links i and j, and for any $i \in N(g')$ and $j \in N(g')$, $ij \in g$ implies $ij \in g'$.
The stability condition requires that, in case a link which does not belong to the network exists, and a player find it profitable, then the potential partner of the new link would not find it profitable. In other words, a network is *pairwise stable* (*PS*) if neither players want to cut a link nor any pair of players simultaneously want to add a new link. Formally,
A network g is pairwise stable with respect to v and Y *i*f

(i) $\quad$ *for every* $ij \in g$, $\mathrm{Y}_i(g, v) \geq \mathrm{Y}_i(g - ij, v)$
and
$\mathrm{Y}_j(g, v) \geq \mathrm{Y}_j(g - ij, v)$

(ii) $\quad$ *for every* $ij \notin g$, *if* $\mathrm{Y}_i(g, v) < \mathrm{Y}_i(g + ij, v)$
then
$\mathrm{Y}_j(g, v) > \mathrm{Y}_j(g + ij, v)$

Condition (i) requires that for any pair of linked players neither player has an incentive to cut the link. Condition (ii) requires that for any pair of ij (in g) disjointed players at least one of them has no incentive to form a link with the other.
This concept takes into account only one link and two players deviations at most, whereas, a profitable deviation involving *more than one link* could also exist. Thus, a pairwise stable network could still be not immune from multilateral links deviations. Moreover, also a *set of players* together can gain from a reorganization of the links. Therefore, a more suitable concept, that takes into account those deviations, is represented by the *Strong Stability* (*SS*) notion.
In this case changes in the structure of the network operating by coalitions without the consent of non-member players are allowed. Every new link has to be formed

only between members of the coalition[26]. At least one of the two players belonging to the excluded link has to belong to the coalition[27]. Formally, strong stability is defined in Jackson and van den Nouweland (2005) as follows:

A network $g' \in G$ *is obtainable from* $g \in G$ *via deviations by a coalition* $S \subseteq N$ *if*

(i) $ij \in g'$ *and* $ij \notin g$ *implies* $ij \subset S$, *and*

(ii) $ij \in g$ *and* $ij \notin g'$ *implies* $ij \cap S \neq \emptyset$

A network g is strongly stable with respect to allocation rule Y and value function v if for any $S \subseteq N$, g' *that is obtainable via deviations by S, and* $i \in S$ *such that* $Y_i(g',v) > Y_i(g,v)$, *there exists* $j \in S$ *such that* $Y_j(g',v) < Y_j(g,v)$[28].

The definition implies that a network is not strongly stable if a single player can strictly increase his payoff by deleting some or all of his links or if a coalition of two or more players can deviate to a network in which some of its members get a strictly higher payoff while none of its members get a lower payoff, Jackson and van den Nouweland (2005).

Intuitively, a network is strongly stable if (for every coalition), when a new network can be created via profitable deviations (such that the payoff associates to the new network is greater than the previous one) by a coalition, then at least one player in the coalition loses from the new arrangement.

1.2.4 The concept of efficiency

The concept of efficiency has been defined in different ways, differing mainly on the applicability to different settings and economic situations. The basic concept is that of Pareto Efficiency (*P-efficiency*). In this case, a network is Pareto efficient if there does not exist any network that leads to higher payoffs for all members of the society. Formally, a network g is *Pareto efficient* relative to v and Y if there does not exist any $g' \in G$ such that $Y_i(g',v) \geq Y_i(g,v)$ for all i with strict inequality for some i.

A second and stronger efficiency notion is based on the overall network payoff. It is assumed that value is fully transferable. A network g is *efficient* relative to v if $v(g) \geq v(g')$ for all $g' \in G$. The first definition of efficiency applies for a given allocation rule (where transfers among players are not possible). The second concept applies in situations where any Y can be redistributed in arbitrary ways. A third concept of efficiency is the *constrained efficiency*. This concept applies in

[26] That is, it is always necessary both players' consent.

[27] In other words, each player may unilaterally cut a link.

[28] ["The difference between this definition of strong stability from Jackson and van den Nouweland (2005) and that of Dutta and Mutuswami (1997) is as follows. The above definition allows for a deviation to be valid if some members are strictly better off and others are weakly better off, while the definition in Dutta and Mutuswami (1997) considers a deviation valid only if all members of a coalition are strictly better off. While the difference is fairly minor, this stronger notion implies pairwise stability while Dutta and Mutuswami's (1997) definition does not".][M. O. Jackson, (2005)]

situations where the value is not fully transferable but the allocation rule is component balanced and anonymous. For this reason it can be collocated between the two previous concepts.
A network g is *constrained efficient* relative to v if there does not exist any $g' \in G$ and a component balanced and anonymous Y such that $Y_i\left(g',v\right) \geq Y_i\left(g,v\right)$ for all i with strict inequality for some i. In our model the value function is not component additive and the allocation rule consequently not component balanced. This means that we find externalities across and inside components and that we cannot use the concept of constrained efficiency. Indeed, we are going to use the notion of Pareto efficiency because the network value is also not fully transferable among nodes and we have a specific allocation rule. As Jackson (2003) pointed out in his work on stability and efficiency, the relationship between the stability and efficiency of networks is *context dependent*. That is, they are not always compatible, but are compatible for certain classes of value functions and allocation rules. Individual incentives might not lead to overall efficiency because of the existence of externalities intra and inter networks. Sometimes individual payoff depends also on the position of each player in the network and not only on what value the network generates. For instance, in a Star network the hub node may not gain moving to another configuration but the overall efficiency may be lower than other network architectures. In this case the source of inefficiency comes from the bargaining power generated by the position in the network. In general, it has been noticed that, "there does not exist any component balanced and anonymous allocation rule (or even a component balanced rule satisfying equal treatment of equals) such that for every v there exists a constrained efficient network that is pairwise stable" (Jackson, 2003). Whereas, if there is complete control over the allocation rule and not component balance, then to guarantee that all efficient networks are also pairwise stable it is simply necessary to use an egalitarian allocation rule.

1.3. The Model

1.3.1 Firms maximization problem

Let $N = \{X, Y, Z\}$ be the set of countries in which the World is split. In each country a number of identical firms produce perfect substitute goods and sell them in imperfect competitive markets. Firms regard each country as a separate market and chooses the profit-maximizing quantity for each country separately. Each firm has a Cournot perception of the optimal quantity produced by other firms in the market. The optimal equilibrium quantity for each firm is given by the Cournot-Nash solution as the one in Krishna (1998). A freely traded numeraire good is transferred across countries to balance trade. There are no fixed costs of production and marginal costs are constant.
Assume that uniform non discriminatory tariffs are applied by all countries on imports from other countries and that the tariff is added on to marginal costs of firms.
With a linear inverse demand function, in country j:

$$P_j = A_j - Q_j \tag{1}$$

Profits for each firm *i* selling in country *j* are defined in the following way:

$$\pi_j^i = P_j\left(Q_j\right)q_j^i - \left(c + t_j^i\right)q_j^i \qquad \text{with} \qquad Q_j = \sum_i n^i q_j^i \tag{2}$$

Where q_j^i represents the quantity supplied in country *j* by each firm in country *i*; t_j^i the tariff against firms selling in country *j* belonging to country *i* and *c* represents the constant marginal costs.
Q_j indicates the total supply of firms selling in country *j* belonging to each country and n^i the number of firms from country *i*. Each firm regards each country j as a separate market and, since marginal costs are constant, chooses its optimal quantity for each country separately. Given the symmetric nature of all firms from some country *i* we focus on the symmetric equilibrium in which each firm from country *i* selling in country *j* offers in that market the same quantity q_j^i. In such an equilibrium the following must hold for each *i* and *j*, where $q_j^i{*} \geq 0$ is the quantity of the maximizing firm which assumes that each of the other $\left(n^i - 1\right)$ firms from country *i* sells q_j^i in country *j*:

$$\begin{aligned} q_j^i &= \max_{q_j^i*} \pi_j^i\left(q_j^i*, (n^i - 1)q_j^i, Q_{j-i}\right) \\ &= \max_{q_j^i*} q_j^i * \left[A_j - \left(q_j^i * + (n^i - 1)q_j^i + Q_{j-i}\right) - (c + t_j^i)\right] \\ &= \max_{q_j^i*} q_j^i * \left[A_j - \left(q_j^i * + Q_{j-i_1}\right) - (c + t_j^i)\right] \end{aligned} \tag{3}$$

Where,

$$Q_{j-i} = \sum_{k \neq i} n^k q_j^k \quad \text{and} \quad Q_{j-i_1} = \sum_{k \neq i} n^k q_j^k + (n^i - 1)q_j^i = Q_j - q_j^i *$$

As shown in Appendix A.2, the above system of equations has a unique solution, which satisfies:

$$q_j^i = \left[\frac{\gamma_j}{\left(n^T + 1\right)} + \frac{\sum_k n^k t_j^k}{\left(n^T + 1\right)} - t_j^i\right] \tag{4}$$

Where $n^T = \sum_k n^k$ (with $k = x, y, z$) represents the total number of firms in the world. Assume $\gamma_j = \left(A_j - c\right)$ as a measure of the market size (demand) of country *j*.

Moreover, firm profits[29] change in the same direction as changes in equilibrium quantities:

$$\pi_j^i\left(q_j^i *\right)=\left(q_j^i *\right)^2 \qquad (5)$$

1.3.2 Free trade agreement formation

A free trade agreement between two countries eliminate all tariffs imposed on imported goods from its members. A bilateral agreement will be supported in country X and Y only if profits will be enhanced by tariffs reduction in both countries simultaneously, that is:

$$\sum_j\left({}_A\pi_j^x\right)>\sum_j\left(\pi_j^x\right) \quad \text{and} \quad \sum_j\left({}_A\pi_j^y\right)>\sum_j\left(\pi_j^y\right) \qquad (6)$$

Where ${}_A\pi_j^x$, ${}_A\pi_j^Y$ and π_j^x, π_j^y denote respectively country X and Y firms' profits deriving from sales in each market when a bilateral agreement is ratified and when tariffs are set at the initial level.
Substituting quantities in (6) by means of expression (5) and considering that the quantity sold in the market not involved in the agreement (country Z) is the same before and after the ratification we have that equation (6) reduces to:

$$\underbrace{\left(q_x^x\right)^2-\left({}_Aq_x^x\right)^2}_{\text{negative strategic effect}}<\underbrace{\left({}_Aq_y^x\right)^2-\left(q_y^x\right)^2}_{\text{positive strategic effect}} \text{ and } \left(q_y^y\right)^2-\left({}_Aq_y^y\right)^2<\left({}_Aq_x^y\right)^2-\left(q_x^y\right)^2 \qquad (6')$$

When a free trade agreement is implemented between two countries, each country gains a better access to the market partner. The production (in the foreign market) is greater and each country gains a competitive advantage over the partner firms, and over firms outside the agreement operating in that market (positive strategic effect and a reduction in marginal costs). On the other hand, firms from the partner country gain access to the domestic market that leads to a competitive loss and a market share reduction for each country in the respective domestic market (negative strategic effect).

1.3.3 The network formation game

[29] See Appendix A.3 for the derivation of equilibrium profits (Eq. 5).

Preferential trade agreements (Ptas) are characterized by bilateral relations embedded in a network (of relations). We develop a free trade network formation game in a setting with three nodes. Bilateral agreements are easily described by means of a graph structure. A Fta-graph is an undirected graph[30] consisting in the set of countries and a network (a collection of links). A free trade agreement between two countries is described by a link. A link is defined as an unordered pair of two countries. A link between two countries implies that the goods produced in both countries can be traded without any tariff. The free trade network formation game is defined by a number of countries and a set of payoff functions. The payoff of each country depends on the network configuration. Let $g^j \in g(N)$ be the realized network architecture and g the set of all possible network configurations given the number of countries N. If we assume that countries are symmetric and $|N| = 3$, then the potential network configurations are represented by the empty network g^e, the one-link network g^1, the star network g^S and the complete network g^C.

The payoff of the node i belonging to the network g^j is represented by $\phi^i_{d(i)}(g^j)$.

$$\phi^i_{d(i)}(g^j) = n^i \left[\sum_{j=1}^{N} \pi^i_j \right] \tag{7}$$

Where $d(i)$ represents the node's degree and indicates country i position in the network .

We analyze the stability and the efficiency of the emerging network of the free trade agreement network formation game. We make a distinction between the study of bilateral and multilateral incentives to free trade agreements formation simply studying both the pairwise and the strong stability of the networks generated by the strategic interaction of countries. We distinguish between symmetric and asymmetric countries. Countries are symmetric if they have the same number of firms and the same market size. When we consider the asymmetric case we assume that countries are asymmetric only in their market size.

1.4. Results

In this section we describe the results on Stability (i.e. *PS* and *SS*) and Efficiency (and P-efficiency) of the free trade network formation game presented in section 3 and their implications on the analysis of bilateralism and multilateralism interaction. The results are organized in two subsections relative to different cases. We consider two cases. First, a symmetric case in which each country has the same market size γ and the same number of firms n. Second, an asymmetric case in which two countries have bigger market sizes γ and one smaller market size γ_z.

[30] The formation of a link is mutual, not unilateral. Thus, two countries have to agree to the formation of the agreement.

1.4.1 Bilateralism and Multilateralism: Symmetric case

Assume that $\gamma_i = \gamma_j \ \forall i,j \in N$; $t^i_j = t^i_k = t \quad \forall i,j \in N$ with $i \neq j$ and $t^j_j = 0 \quad \forall j \in N$; and $n^i = n^j = n \ \forall i,j \in N$. When countries are symmetric, in both market size and number of firms, payoffs depend only on nodes' degree in each network configuration. In this case the payoff functions are defined in the following way[31]:

$$\begin{aligned}
&\phi^i_{d(0)}\left(g^e\right) \equiv \mathrm{a}_0 = \lambda\left[3\gamma^2 - 4\gamma t + 6n^2t^2 + 4nt^2 + 2t^2\right] \ ; \qquad (8)\\
&\phi^i_{d(0)}\left(g^1\right) \equiv \mathrm{b}_0 = \lambda\left[3\gamma^2 - 4\gamma t\left(n+1\right) + 12n^2t^2 + 8nt^2 + 2t^2\right];\\
&\phi^i_{d(1)}\left(g^1\right) \equiv \mathrm{b}_1 = \lambda\left[3\gamma^2 + 2\gamma t\left(n-1\right) + 2n^2t^2 + \left(n+1\right)^2 t^2\right];\\
&\phi^i_{d(1)}\left(g^S\right) \equiv \mathrm{c}_1 = \lambda\left[3\gamma^2 - 2\gamma\left(n+1\right)t + 5n^2t^2 + 4nt^2 + t^2\right];\\
&\phi^i_{d(2)}\left(g^S\right) \equiv \mathrm{c}_2 = \lambda\left[3\gamma^2 + 4\gamma nt + 2n^2t^2\right];\\
&\phi^i_{d(2)}\left(g^C\right) \equiv \mathrm{d}_2 = \lambda\left[3\gamma^2\right] \qquad \forall i \in N \text{ and where } \lambda = 1/\left(3n+1\right)^2
\end{aligned}$$

We organize country sizes in three intervals, $\gamma < \breve{\gamma}$ (small countries)[32]; $\breve{\gamma} < \gamma < \hat{\gamma}$ (medium)[33]; $\gamma > \hat{\gamma}$ (big countries).
From the analysis of the pairwise stability (*PS*), Strong Stability (*SS*), Pareto efficiency (*P-Efficiency*) and Efficiency of each configuration in each size-interval we obtain the following results:

Proposition 1. *If countries are symmetric and* $\gamma < \breve{\gamma}$ *then, the empty network configuration is the unique PS, SS (P-Efficient) and Efficient network.*

(See the Proof in Appendix B.1.)

In other words, according to Proposition 1, when countries are small and symmetric, domestic firms protection is the most valuable choice.

---

[31] a_0 represents the payoff of each player belonging to the empty network $\left(g^e\right)$ and therefore having no links; b_0 represents the payoff of each player belonging to the one link network architecture $\left(g^1\right)$ and having no links (when the other players are linked to each other); b_1 represents the payoff of each player in the one link network architecture $\left(g^1\right)$ and having one link (and it is the only link in the network); c_1 represents the payoff of each player belonging to the star network architecture $\left(g^s\right)$ and having one link (spoke); c_2 represents the payoff of each player in the star network architecture $\left(g^s\right)$ and having two links (hub); d_2 represents the payoff of each player belonging to the complete network architecture $\left(g^C\right)$ and therefore having two links (each player is connected to each other).

[32] Where $\breve{\gamma} = \frac{(n+1)t}{2}$; $\hat{\gamma} = \frac{\left(3n^2 + 2n + 1\right)t}{2}$.

Proposition 2. *If countries are symmetric and the market size belongs to the interval* $\breve{\gamma} < \gamma < \hat{\gamma}$*, then, either the empty network is the unique PS, SS and Efficient network or the one link network is PS, SS and efficient while the complete network is only PS .*

(See the Proof in Appendix B.2.)

When we consider countries with a larger market size, both global free trade and regionalism appear immune to bilateral deviations and can emerge in equilibrium. In other words, a situation in which world trade is divided in two components (one represented by a protected and isolated country and the other composed of a bilateral agreement), due to the absence of incentives towards a further multilateralization, may emerge and crystallize. On the other hand, the global free trade scenario has still a possibility to come out by means of bilaterals.
The strong stability of the one link network predicts that once a bilateral agreement is formed it prevents the global free trade outcome to arise in two different ways. First, there are no incentives to create further bilaterals and therefore to reach the global free trade through regionalism. Second, it prevents also a further multilateral agreements formation (multilateral trade rounds).

Proposition 3. *If countries are symmetric and* $\gamma > \hat{\gamma}$*, then if* $\gamma > \hat{\hat{\gamma}}$ *(* with $\hat{\hat{\gamma}} > \hat{\gamma}$*) the unique PS, SS and Efficient network is the complete configuration while the star and the one link networks are P-efficient. If* $\hat{\gamma} < \gamma < \hat{\hat{\gamma}}$ *then the Efficient network is the star configuration.*

(See the Proof in Appendix B.3.)

In other words, for very large countries, bilateral agreements don't prevent global free trade to arise in equilibrium (i.e., the complete network is immune to bilateral deviations). Furthermore, for very large symmetric countries, bilateralism doesn't affect the multilateral process (i.e., the complete network is immune to multilateral deviations). We can interpret this result considering the role of intra industry trade and countries similarities. Most scholars found some evidence on the mutual influence of regional trade agreements on intra-industry trade and vice versa (Behar, 1991). Moreover, intra-industry trade is greater among countries with similar and large markets (Milgram–Baleix, Moro–Egido, 2005).
Furthermore, regional trade agreements among developed countries increase intra-regional trade because similar income level and similar preference increase the potential trade volume in intra-industry trade. Therefore, in a world with few symmetric and large countries the presence of intra-industry trade may foster regional trade agreements until global free trade is reached.

1.4.2 Bilateralism and Multilateralism: Asymmetric case

Assume that $\gamma_i = \gamma_j = \gamma \quad \forall i, j \neq k$ and $\gamma_k = \gamma_z$ with $\gamma > \gamma_Z$. Furthermore, $t_j^i = t_k^i = t$ $\forall i, j \in N$ with $i \neq j$ and $t_j^j = 0 \quad \forall j \in N$; and $n^i = n^j = n \; \forall i, j \in N$. When countries are not symmetric, payoffs do not depend only on nodes degree in each network configuration but also on countries' identity. Moreover, the potential network configurations are represented in this case by the empty network g^e, the one-link networks g_{xy}^1 and g_{zi}^1 (representing respectively the one link network in which the two identical countries and asymmetric countries are linked), the star networks $g_{x,y}^S$ and g_z^S (representing respectively the star network in which one of the bigger size country and the small one is the hub), and the complete network g^C.Countries payoffs in the asymmetric case are illustrated in the following way,

$$\phi_{d(0)}^{x,y}\left(g^e\right) \equiv \mathrm{a} = \lambda\left[2\gamma^2 + \gamma_z^2 + 2\gamma t\left(n-1\right) - 2\gamma_z t\left(n+1\right) + 6n^2t^2 + 4nt^2 + 2t^2\right] \; ; \qquad (9)$$

$$\phi_{d(0)}^{z}\left(g^e\right) \equiv \mathrm{a}_z = \lambda\left[2\gamma^2 + \gamma_z^2 - 4\gamma t\left(n+1\right) + 4\gamma_z nt + 6n^2t^2 + 4nt^2 + 2t^2\right] \; ;$$

$$\phi_{d(0)}^{x,y}\left(g_{zi}^1\right) \equiv \mathrm{b}_0 = \lambda\left[2\gamma^2 + \gamma_z^2 - 2\gamma t - 2\gamma_z t\left(2n+1\right) + 12n^2t^2 + 8nt^2 + 2t^2\right];$$

$$\phi_{d(0)}^{z}\left(g_{xy}^1\right) \equiv \mathrm{b}_{z_0} = \lambda\left[2\gamma^2 + \gamma_z^2 + 4\gamma_z nt - 4\gamma t\left(2n+1\right) + 12n^2t^2 + 8nt^2 + 2t^2\right]$$

$$\phi_{d(1)}^{x,y}\left(g_{zi}^1\right) \equiv \mathrm{b}_1' = \lambda\left[2\gamma^2 + \gamma_z^2 - 2\gamma t + 2\gamma_z nt + 3n^2t^2 + 2nt^2 + t^2\right];$$

$$\phi_{d(1)}^{x,y}\left(g_{xy}^1\right) \equiv b_1 = \lambda\left[2\gamma^2 + \gamma_z^2 + 4\gamma nt - 2\gamma_z\left(n+1\right)t + 2n^2t^2 + \left(n+1\right)^2 t^2\right];$$

$$\phi_{d(1)}^{z}\left(g_{zi}^1\right) \equiv b_{z_1} = \lambda\left[2\gamma^2 + \gamma_z^2 - 2\gamma t + 2\gamma_z nt + \left(n+1\right)^2 t^2 + 2n^2t^2\right];$$

$$\phi_{d(1)}^{x,y}\left(g_{x,y}^S\right) \equiv \mathrm{c}_1 = \lambda\left[2\gamma^2 + \gamma_z^2 + 2\gamma nt - 2\gamma_z\left(2n+1\right)t + 5n^2t^2 + 4nt^2 + t^2\right];$$

$$\phi_{d(2)}^{x,y}\left(g_{x,y}^S\right) \equiv \mathrm{c}_2 = \lambda\left[2\gamma^2 + \gamma_z^2 + 2\gamma nt + 2\gamma_z nt + 2n^2t^2\right];$$

$$\phi_{d(1)}^{x,y}\left(g_z^S\right) \equiv \mathrm{c}_1' = \lambda\left[2\gamma^2 + \gamma_z^2 - 2\gamma\left(n+1\right)t + 5n^2t^2 + 4nt^2 + t^2\right];$$

$$\phi_{d(1)}^{z}\left(g_{x,y}^S\right) \equiv \mathrm{c}_{z_1} = \lambda\left[2\gamma^2 + \gamma_z^2 - 2\gamma\left(2n+1\right)t + 2\gamma_z nt + 5n^2t^2 + 4nt^2 + t^2\right];$$

$$\phi_{d(2)}^{z}\left(g_z^S\right) \equiv \mathrm{c}_{z_2} = \lambda\left[2\gamma^2 + \gamma_z^2 + 4\gamma nt + 2n^2t^2\right];$$

$$\phi_{d(2)}^{i}\left(g^C\right) \equiv \mathrm{d}_2 = \lambda\left[2\gamma^2 + \gamma_z^2\right] \qquad \forall i \in N \text{ and where } \lambda = 1/\left(3n+1\right)^2$$

From the analysis of Pairwise stability, Strong stability, Pareto efficiency and Efficiency in the asymmetric case and considering that strongly stable networks are necessarily Pareto efficient we obtain the following propositions:

Proposition 4. *When countries are asymmetric and* $\gamma < \breve{\gamma}$, *the unique PS, SS (P-efficient) network is the empty configuration and the Efficient architecture is represented by the one link network* g_{xy}^1.

(The proof of Proposition 4 is described in Appendix B.4.)

According to Proposition 4, when countries are small, and the world is composed by countries with asymmetric market sizes, although results on pairwise stability and strong stability do not differ from the ones in the symmetric case (for small countries), here, the efficient network is represented by a situation in which the two bigger market size countries are linked. That is, the value generated in a network where trade is free to flow between two bigger countries is higher than the one generated by a network in which each country protects its own market. This is a standard result since when trade is liberalized trade flow among members increases as well. On the other hand, since, as in this model, the total value of a free trade network, is not equally distributed among countries, the efficiency of the one link network doesn't imply that at country level (in the empty network) incentives to liberalize exist. This mechanism generates tension between stability and efficiency. At multilateral level, since the empty network is also strongly stable, this result implies that negotiations may stuck and in turn prevent regionalism to obtain a further liberalization.

Proposition 5. *When countries are asymmetric and* $\breve{\gamma} < \gamma < \bar{\gamma}$ *, either the empty network or the one link network* g^1_{xy} *are PS and SS respectively if* $\breve{\gamma} < \gamma < \tilde{\gamma}$ *with* $\gamma_z < \gamma_z^5(\gamma, n)$*, and* $\tilde{\gamma} < \gamma < \bar{\gamma}$*. If* $\tilde{\gamma} < \gamma < \bar{\gamma}$ *then the one link network* g^1_{xy} *is PS, SS and Efficient.*

(The proof of Proposition 5 is described in Appendix B.5.)

When the market size of the two symmetric countries are bigger than the threshold value $\tilde{\gamma}$, the one link network, representing bilateralism between similar countries, emerges in equilibrium and represents the unique SS and Efficient network. Thus, the formation of bilateral agreements between similar countries may prevent a further liberalization through bilaterals and may block a further multilateral negotiation. Moreover, the condition on Z market size $\gamma_z^5(\gamma, n)$ is not increasing in γ, then, when $\breve{\gamma} < \gamma < \tilde{\gamma}$ it is easy to see that the greater the asymmetry, the higher the possibility that the empty network is *SS*. Second, according to Lemma 3.2 (see Appendix B.4.) the one link network g^1_{xy} is strongly stable if $\gamma > \tilde{\gamma}$ and $\gamma_z < \gamma_z^{12}(\gamma, n)$. Moreover $\gamma_z^{12}(\gamma, n) > \gamma_z^5(\gamma, n)$ with $\gamma_z^{12}(.)$ not decreasing in γ and $\gamma_z^5(.)$ not increasing in γ. Thus, SS conditions requires a stronger asymmetry for g^e to be *SS* than for the one link network g^1_{xy}. This result highlights the trade diverting feature of regional agreements. The greater the market size of the small country with respect to the two symmetric (and bigger) ones, the greater the incentives to build a link between the symmetric countries diverting trade from the asymmetric one.

Proposition 6. *When countries are asymmetric and* $\bar{\gamma} < \gamma < \gamma^B$*, then, the one link network* g^1_{xy} *is PS, SS and Efficient if* $\gamma_z < \gamma_z^{12}(\gamma, n)$*, while the complete network and the star network* $g^S_{x,y}$ *are PS (for higher values of* γ_z *).*

(The proof of Proposition 6 is described in Appendix B.6)

The stumbling block effect of bilateral agreements on the multilateral free trade process is confirmed by Proposition 6. When we increase the size of the two symmetric countries ($\bar{\gamma} < \gamma < \gamma^B$ *)* the empty network equilibrium is ruled out by the existence of profitable deviations. Moreover, when $\bar{\gamma} < \gamma < \gamma^B$ also both the global free trade scenario and a star architecture, with one of the symmetric country as hub, are pairwise stable. This result highlights the possibility to obtain the complete architecture and the hub and spoke configuration in equilibrium. In addition, the complete network is *PS* for higher values of Z market size. In other words, consistent with the symmetric case results, the less the asymmetry the higher the possibility that the complete network emerges in equilibrium as result of a bilateral process. Furthermore, it has been argued that an *hub-and-spoke* (star network) structure in world trade is emerging. Indeed, hub and spoke Ftas nowadays represents a quite common form of agreement taking place often among one developed country as hub and more developing spokes.

Proposition 7. *When countries are asymmetric and $\gamma > \gamma^B$, then, the one link network g^1_{xy} is the unique SS network. Moreover, g^1_{xy} is also the unique PS network for small values of γ_z. The complete network is the unique PS, Efficient (and P-efficient) network for large values of γ_z and the star network $g^s_{x,y}$ is PS otherwise.*

(The proof of Proposition 7 is described in Appendix B.7)

When we consider two large symmetric countries (i.e. $\gamma > \gamma^B$) and strong asymmetry (e.g., small γ_z), the bilateral process may crystallize in two components (e.g., blocs of regional agreements) preventing a further expansion both via bilaterals and via multilateral negotiations. What is more, the efficient outcome (i.e., the global free trade) can not be achieved.

1.5. Conclusions

Over the course of the negotiations at the Doha round, the influence of the groups that want to protect themselves from foreign competition (mainly farmers in rich countries and manufacturers in emerging ones) seems to have been greater than that of those who sought a higher integration. This made it impossible to strike a deal which could be acceptable for the main countries of the WTO (Steinberg, 2007).
In this chapter we have analyzed the stability and the efficiency of the bilateral free trade agreement formation in a network formation game setting in order to evaluate the impact of bilateralism on the global free trade achievement and the relationship between regionalism and multilateralism. The theory of network formation game is a valid tool to deal with the complex environment of international trade relationships defined by Bhagwati as “a messy maze of preferences as PTAs formed between two countries, with each having bilaterals with other and different countries, the latter in turn bonding with yet others, each in turn having different rules of origin for different sectors” (Bhagwati, 2002). In the current global market a large share of world trade takes place within sixty (or more) overlapping arrangements that reduce barriers to trade on a preferential basis. Thirty percent of world trade takes place within the two largest preferential trading areas: the EU and the NAFTA.
In order to evaluate the impact of regionalism on the multilateral process, two different kinds of stability concepts have been used: *pairwise stability* and *strong stability.* The

prediction of the network formation approach in the recent network formation game literature, presented by Goyal-Joshi (2006), is that the Global Free trade network is a pairwise *stable* network. World trade is described by an intra-industry trade model as in Krishna (2005). We have described the relationship between stability and efficiency in a symmetric and asymmetric countries environment. That is, we have distinguished between an asymmetric case in which countries are identical with the same market size and number of firms and an asymmetric case in which two countries have larger market size with respect to a third one.

The results, in the symmetric and in the asymmetric case depend respectively on countries market size and on the degree of asymmetry among them. We distinguish three market size intervals: small ($\gamma < \breve{\gamma}$), medium ($\breve{\gamma} < \gamma < \hat{\gamma}$) and big ($\gamma > \hat{\gamma}$) countries.

When the market size belongs to the interval $\gamma < \breve{\gamma}$ we obtain that the unique *PS*, *SS* (and then also P-efficient) and Efficient network is the empty configuration. Moreover also the one link network is P-efficient.

When the market size belongs to the interval $\breve{\gamma} < \gamma < \hat{\gamma}$ we have two different situations. Either we have that the empty network is the unique PS, SS and Efficient network or the one link network is PS, SS and efficient while the complete network is only PS .

When the market size belongs to the interval $\gamma > \hat{\gamma}$ we obtain that if $\gamma > \hat{\hat{\gamma}}$ (with $\hat{\hat{\gamma}} > \hat{\gamma}$)the unique *PS*, *SS* and Efficient network is the complete configuration while the star and the one link networks are P-efficient. If $\hat{\gamma} < \gamma < \hat{\hat{\gamma}}$ then the Efficient network is the star configuration.

In general, when countries are symmetric and large the global free trade outcome may emerge in equilibrium both via bilateral agreements expansion and multilateral trade rounds. This result is consistent with recent findings on the empirical properties of world trade web by Fagiolo, Reyes and Schiavo (2009). They show that the world trade web is an extremely symmetric network, where almost all trade relationships tend to be reciprocated with similar intensities. Therefore, they studied its characteristics as if it were a weighted[34] undirected network. They pointed out that richer countries tend to be more clustered (and increasingly so over the years) supporting with their finding the "rich club phenomenon".

On the other hand, with medium-size countries we see a potential adverse effect of bilateralism on multilateralism. In other words, even though global free trade agreement may still be reached through bilaterals, multilateral negotiation can be blocked by the existence of profitable deviations to the one link network (i.e. bilateralism). Moreover, because of the trade diverting feature of Ptas, when emerging only through bilateral agreements, the complete network, is not the efficient configuration.

When countries are asymmetric we obtain different results on efficiency and stability depending both on the degree of asymmetry and the market size intervals.

When the market size of the symmetric countries belongs to the interval $\gamma < \breve{\gamma}$ we obtain that the unique *PS*, *SS* (and then also P-efficient) is the empty network while the one link network g^1_{xy} is Efficient.

When the market size of the symmetric countries belongs to the interval $\breve{\gamma} < \gamma < \bar{\gamma}$ we have two different situations. First, if $\breve{\gamma} < \gamma < \tilde{\gamma}$ then either the empty network or the one link network g^1_{xy} are PS and SS depending on the market Z size. If $\gamma_z < \gamma_z^5(\gamma, n)$ then the

[34] Each directed link from node i to j is weighted by total exports of country i to country j and then divided by the country i's GDP (i.e., the exporter country). Such a weighting setup allows one to measure how much economy i depends on economy j as a buyer (Fagiolo et al, 2009).

empty network is SS and since $\gamma_z^5(\gamma,n)$ is not increasing in γ, when $\breve{\gamma} < \gamma < \tilde{\gamma}$ the greater is the asymmetry higher is the possibility that the empty network is SS. Second, if $\tilde{\gamma} < \gamma < \bar{\gamma}$ then the one link network g_{xy}^1 is *PS*, *SS* and Efficient.

When the market size of the symmetric countries belongs to the interval $\bar{\gamma} < \gamma < \gamma^B$ we obtain that the one link network g_{xy}^1 is *PS*, *SS* and Efficient while the complete network and the star network $g_{x,y}^S$ are *PS* (for higher values of γ_z).

When the market size of the symmetric countries is $\gamma > \gamma^B$ we obtain that the one link network g_{xy}^1 is the unique *SS* network. Moreover, g_{xy}^1 is also the unique PS network for small values of γ_z. The complete network is the unique PS, Efficient (and P-efficient) network for large values of γ_z and the star network $g_{x,y}^s$ is PS otherwise.

In the asymmetric case the global free trade outcome (also efficient for large countries) is never obtained as result of a multilateral trade round since the complete network is not strongly stable and the one link network is immune to multilateral deviations (except in the case of small countries). Moreover, when countries are asymmetric, another form of link architecture emerges in equilibrium: the hub and spoke configuration. Hub and spoke agreements take place more often among a developed country as a hub and more smaller spokes. Thus, the presence of such agreements in equilibrium is justified by the extent of asymmetry among countries. Indeed, empirical recent studies in which the relative weight of trade flows towards and from countries is considered, describe the world trade web as a very high density network but with the average strength of nodes being rather poor (Fagiolo et al, 2009). In other words, most countries hold mainly weak relationships, whereas only a selected core on nodes combine high degree and high strength. Indeed, the world trade web, according to Fagiolo, Reyes and Schiavo has a dis-assortative nature. That is, countries holding many (and more intense) trade relationships preferably trade with poorly connected countries.

The model we presented could be further extended including increasing economies of scale in order to underline the role of the intra-industry trade and its effect on regionalism and multilateralism. Moreover, many North-South preferential agreement are asymmetric in trade liberalization. That is, these types of agreements often allow "one- way" free access to developing countries in developed market such as agreement between UE and ACP countries (Africa, Caribbean, Pacific). Therefore, it would be interesting also to take into account the one-way access among asymmetric countries describing these agreements by means of digraphs.

1.6 Appendix A

1.6.1 A.1 The components of a Network

One possible representation of networks is by means of graphs: $gr = (v, e)$ with vertices (nodes) v and edges (links) e.

A sub-graph $H = (w, f)$ of a graph $gr = (v, e)$ is a graph where $w \subseteq v$ and *f* is a set of links in G such that its endpoints belong to w.

An induced sub-graph $H = (w, f)$ of a graph $gr = (v, e)$ is a graph where $w \subseteq v$ and *f* consists of *all* the links whose endpoints belong to w.

An induced sub-graph $H = (w, f)$ of a graph $gr = (v, e)$ is a *component* of G if

(i) H is connected[35];

(ii) H is a maximal connected sub-graph of G: that is, there not exists any sub-graph *K* of G with $v(H) \subseteq v(K)$ and *K* connected, except *H*.

Thus, the components of a network are the distinct connected sub-graphs of a network. The set of the components of g is represented by $C(g)$ and $g = \bigcup_{g' \in C(g)} g'$.

1.6.2 A.2 The Cournot-Nash solution

The first order conditions of the maximization problem represented by eq. (3) are:

$$\frac{\partial \pi_j^i}{\partial q_j^i *} = A_j - Q_{j-i_1} - 2q_j^i * - \left(c + t_j^i\right) = 0,$$

where $Q_{j-i_1} = \sum_{k \neq i} n^k q_j^k + (n^i - 1) q_j^i = Q_j - q_j^i *$ and $Q_{j-i} = \sum_{k \neq i} n^k q_j^k$

$$2q_j^i * = \left(A_j - c\right) - Q_{j-i_1} - t_j^i \tag{10}$$

From equation (10):

[35] A graph G is connected if there does exist a *path* between every pair of vertices in the graph. A path between two players in a graph is a finite sequence of vertices $v_1, \ldots, v_k$ where, for every $1 \le i \le (k-1)$ we have that v_i, v_{i+1} are adjacent (that is, $v_i v_{i+1} \in g$) and where edges and vertices are distinct (they are not repeated twice).

$$2q_j^i{*} = \left(A_j - c\right) - \left(Q_j - q_j^i{*}\right) - t_j^i$$
$$q_j^i{*} + t_j^i = \left(A_j - c\right) - Q_j$$

Then, for each firm

$$q_j^i{*} + t_j^i = q_j^k{*} + t_j^k = \left(A_j - c\right) - Q_j$$

Therefore,

$$Q_{j-i_1} = \sum_k n^k \left(q_j^i{*} - t_j^k + t_j^i\right) - q_j^i{*} \tag{11}$$

Substituting (11) in (10) we get:

$$\frac{\sum_k n^k \left(q_j^i{*} - t_j^k + t_j^i\right) - q_j^i{*}}{2} + q_j^i{*} = \frac{\left(A_j - c\right)}{2} - \frac{t_j^i}{2}$$
$$\frac{\sum_k n^k q_j^i{*} + 2q_j^i{*} - q_j^i{*}}{2} = \frac{\left(A_j - c\right)}{2} - \frac{t_j^i}{2} - \frac{\sum_k n^k t_j^i}{2} + \frac{\sum_k n^k t_j^k}{2}$$
$$\left(n^T + 1\right) q_j^i{*} = \left(A_j - c\right) + \sum_k n^k t_j^k - t_j^i - \left(n^T - n^i\right) t_j^i$$
$$\left(n^T + 1\right) q_j^i{*} = \left(A_j - c\right) + \sum_k n^k t_j^k - \left(1 + n^T - n^i\right) t_j^i$$
$$\left(n^T + 1\right) q_j^i{*} = \left(A_j - c\right) + \sum_k n^k t_j^k + n^i t_j^i - \left(n^T + 1\right) t_j^i$$
$$\left(n^T + 1\right) q_j^i{*} = \left(A_j - c\right) + \sum_k n^k t_j^k - \left(n^T + 1\right) t_j^i$$

The Cournot Nash solution: $q_j^i{*} = \frac{\left(A_j - c\right)}{\left(n^T + 1\right)} + \frac{\sum_k n^k t_j^k}{\left(n^T + 1\right)} - t_j^i$ (4)

1.6.3 A.3 Equilibrium Profits

Countries' payoff, in recent literature on free trade network, has been often described by the social welfare function. In this model, since we assume that producers are crucial in each country decision to accept an agreement, producers' profits changes are good indicators of the incentives to form links. In other words, countries are represented by firms because governments follow producers' pressure in shaping trade policy.
Profits of each firm *i* selling in country *j* are represented by eq. (2):

$$\pi_j^i = P_j\left(Q_j\right) q_j^i - \left(c + t_j^i\right) q_j^i \text{ where } Q_j = \sum_i n^i q_j^i$$
$$\pi_j^i = q_j^i \left[A_j - \sum_i n^i q_j^i - \left(c + t_j^i\right)\right]$$

From the Cournot-Nash solution in eq. (4):

$$q_j^i = \frac{(A_j - c)}{(n^T + 1)} + \frac{\sum_k n^k t_j^k}{(n^T + 1)} - t_j^i$$

Then, multiplying by n^i and sum each side over all i we get:

$$\sum_i n^i q_j^i = \frac{n^T}{n^T + 1}(A_j - c) + \frac{n^T}{n^T + 1}\sum_k n^k t_j^k - \sum_i n^i t_j^i$$

$$\sum_i n^i q_j^i = \frac{n^T}{n^T + 1}(A_j - c) - \frac{1}{n^T + 1}\sum_k n^k t_j^k \qquad (12)$$

Substituting (12) in (2) yields:

$$\pi_j^i = q_j^i \left[A_j - (c + t_j^i) - \left(\frac{n^T}{n^T + 1}(A_j - c) - \frac{1}{n^T + 1}\sum_k n^k t_j^k \right) \right]$$

$$\pi_j^i = q_j^i \left[\frac{(A_j - c)}{n^T + 1} + \frac{\sum_k n^k t_j^k}{n^T + 1} - t_j^i \right]$$

$$\pi_j^i (q_j^i) = (q_j^i)^2 \blacksquare \qquad (13)$$

1.7 Appendix B

1.7.1 B.1 Proof of Proposition 1.

The proof is organized in three steps. First, we prove that the empty network is the unique pairwise stable configuration. Second, we concentrate on the strong stability and then on the efficiency and Pareto efficiency.

Part 1. Pairwise Stability

When countries are symmetric and $\gamma < \breve{\gamma}$, the empty network is the unique pairwise stable network.

According to the definition of pairwise stability, a network is pairwise stable if (i) for any pair of linked players neither player has an incentive to cut the link and (ii) for any pair of disjointed players at least one of them has no incentive to form a link with the other. In particular, the empty network is pairwise stable if for every $ij \notin g$ whenever $\phi_{d(0)}^i(g^e) < \phi_{d(1)}^i(g^1)$ then $\phi_{d(0)}^j(g^e) > \phi_{d(1)}^j(g^1)$.

From the analysis of the payoffs we have that,

$$\phi_{d(0)}^i(g^e) > \phi_{d(1)}^i(g^1) \ \forall i \in N \ \text{ if } \gamma < \frac{(3n^2 + 2n + 1)t}{2(n+1)} \equiv \tilde{\gamma} \qquad (14)$$

Since $\tilde{\gamma} > \breve{\gamma}$ and (14) we obtain that when countries' market size is less than $\breve{\gamma}$, the empty network is pairwise stable.
For the other network to be stable the condition on market size are the followings:
The one link and the complete architecture are pairwise stable if respectively,

$$\tilde{\gamma} \equiv \frac{\left(3n^2 + 2n + 1\right)t}{2(n+1)} < \gamma < \frac{\left(7n^2 + 4n + 1\right)t}{2(n+1)} \equiv \tilde{\gamma}' \tag{15}$$

$$\gamma > \frac{\left(5n^2 + 4n + 1\right)t}{2(n+1)} \equiv \bar{\gamma} \tag{16}$$

It is clear from expressions (15) and (16) that both values of $\tilde{\gamma}$ and $\bar{\gamma}$ are greater than $\breve{\gamma}$, then when countries' size is lower than $\breve{\gamma}$ (15) and (16) are not satisfied . The star configuration is not pairwise stable since pairwise stability requires both $\gamma > \tilde{\gamma}'$ and $\gamma < \bar{\gamma}$ but $\bar{\gamma} < \tilde{\gamma}'$. Therefore, the empty network is the unique pairwise stable network. ■

Part 2. Strong Stability

The strong stability condition allows changes in the structure of the network operating by coalitions without the consent of non-member players . Once we focus on strongly stable network in the symmetric case with $\gamma < \breve{\gamma}$ we obtain the following proposition:

When countries are symmetric and $\gamma < \breve{\gamma}$, the empty network is the unique strongly stable network.

Before to proceed with the proof we first need to introduce the following Lemmas:

Lemma 1.1 When countries are symmetric, the empty network is strongly stable (SS) if the value of $\gamma < \gamma''$.

Proof of Lemma 1.1
According to the strong stability condition the empty network is strongly stable if for every possible coalition $S \subseteq N$ of players and for any network g obtainable[36] from g^e through deviation by S, and $i \subset S$, when $\phi^i_{d(i)}(g) > \phi^i_{d(0)}\left(g^e\right)$ then, there exists $j \subset S$ such that $\phi^j_{d(0)}\left(g^e\right) > \phi^j_{d(j)}(g)$.
Using the payoffs definition depicted in (8) we can describe the conditions, for each g obtainable from g^e via every possible $S \subseteq N$, under which the empty network is strongly stable. That is,
The one link network is obtainable from g^e via every $|S| = 2;$ or $S = N$. For such deviations to be not profitable, in the symmetric case, at least one member of the deviating coalition should not gain in the new configuration. Then, for $|S| = 2$ $b_1 < a_0$ and for $S = N$ $b_1 < a_0$ or $b_0 < a_0$

[36] A network g' is obtainable from $g \in G$ via deviations by S if (i) every new link is formed only between members of the coalition (it is always necessary both members' consent) and (ii) at least one of the two players belonging to the excluded links has to belong to the coalition (each coalition member may unilaterally cut a link).

(i) $\mathrm{b}_1 < \mathrm{a}_0$ if $\gamma < \tilde{\gamma}$

(ii) $\mathrm{b}_1 < \mathrm{a}_0$ if $\gamma < \tilde{\gamma}$ or $\mathrm{b}_0 < \mathrm{a}_0$ if $\gamma > \gamma' \ (> \tilde{\gamma})$

The star network is obtainable from g^e via $S = N$. For such deviations to be not profitable, in the symmetric case, at least one of the deviating coalition member should not gain in the new configuration. Then, for $S = N$ $c_2 < \mathrm{a}_0$ or $c_1 < \mathrm{a}_0$

(iii) $c_2 < \mathrm{a}_0$ if $\gamma < \gamma'' \ (< \tilde{\gamma})$[37] or $c_1 < \mathrm{a}_0$ is not satisfied for $n > 1$

The complete network is obtainable from g^e via $S = N$. For such deviations to be not profitable, in the symmetric case, at least one of the deviating coalition member should not gain in the new configuration. Then, for $S = N$ $d_2 < \mathrm{a}_0$

(iv) $d_2 < \mathrm{a}_0$ if $\gamma < \hat{\gamma} \ (> \tilde{\gamma})$[38]

It is easy to verify that conditions (i)-(iv) are simultaneously satisfied for $\gamma < \gamma''$ since $\hat{\gamma} > \tilde{\gamma} > \gamma''$.

Lemma 1.2. When countries are symmetric, the one link network is strongly stable if $\tilde{\gamma} < \gamma < \tilde{\gamma}'$.

Proof of Lemma 1.2.

The empty network is obtainable from g^1 via several deviating coalitions, such as, $S = \{i\}$, $S = \{j\}$ with $ij \in g^1$ or via $S = \{i, j\}$ with $ij \in g^1$ or via $S = \{i, j\}$ with $ij \notin g^1$ and $i, j \in N$ or $S = N$. For such deviations to be not profitable, in the symmetric case, at least one member of the deviating coalition should not gain in the new configuration. Then,

(i), (ii) $\mathrm{b}_1 > \mathrm{a}_0$ if $\gamma > \tilde{\gamma}$

(iii) $\mathrm{b}_1 > \mathrm{a}_0$ if $\gamma > \tilde{\gamma}$ or $\mathrm{b}_0 > \mathrm{a}_0$ if $\gamma < \gamma' \ (> \tilde{\gamma})$

(iv) $\mathrm{b}_1 > \mathrm{a}_0$ if $\gamma > \tilde{\gamma}$ or $\mathrm{b}_0 > \mathrm{a}_0$ if $\gamma < \gamma' \ (> \tilde{\gamma})$

The star network is obtainable from g^1 via $S = \{i, j\}$ with $ij \notin g^1$ and $i, j \in N$ or $S = N$.
For such deviations to be not profitable, in the symmetric case, at least one member of the deviating coalition should not gain in the new configuration. Then,

(v) $\mathrm{b}_1 > \mathrm{c}_2$ if $\gamma < \breve{\gamma}$ or $\mathrm{b}_0 > \mathrm{c}_1$ if $\gamma < \tilde{\gamma}'$

(vi) $\mathrm{b}_1 > \mathrm{c}_2$ if $\gamma < \breve{\gamma}$ or $\mathrm{b}_0 > \mathrm{c}_1$ if $\gamma < \tilde{\gamma}'$ or $\mathrm{b}_1 > \mathrm{c}_1$ if $\gamma > \breve{\gamma}$

The complete network is obtainable from g^1 $S = N$. For such deviations to be not profitable, in the symmetric case, at least one of the deviating coalition member should not gain in the new configuration. Then,

(vii) $d_2 < \mathrm{b}_1$ for $\gamma > 0$ and $n > 1$ or $d_2 < \mathrm{b}_0$ if $\gamma < \bar{\gamma}'$

---

[37] $\gamma' = \frac{(3n+2)t}{2} > \gamma'' = \frac{(2n^2+2n+1)t}{2(n+1)}$

[38] $\hat{\gamma} = \frac{(3n^2+2n+1)t}{2} > \gamma' > \tilde{\gamma} > \gamma''$

For the one link network to be strongly stable conditions (i) to (vii) require that $\gamma > \tilde{\gamma}$ and $\gamma < \tilde{\gamma}''$[39]. Since $\tilde{\gamma} < \tilde{\gamma}''$ all conditions are satisfied for $\tilde{\gamma} < \gamma < \tilde{\gamma}''$. ■

Lemma 1.3. When countries are symmetric the star network is not strongly stable

Proof of Lemma 1.3.

The empty network is obtainable from g^S via $S = \{i\}$ with $ij, ik \in g$ or via $S = \{i, j\}$ with $ij \notin g^1$ or $S = N$. For such deviations to be not profitable, in the symmetric case, at least one of the deviating coalition member should not gain in the new configuration. Then,

(i) $c_2 > a_0$ if $\gamma > \gamma''$

(ii) $c_1 > a_0$ for $\gamma > 0$ and $n > 1$

(iii) $c_2 > a_0$ if $\gamma > \gamma''$ or $c_1 > a_0$ for $\gamma > 0$ and $n > 1$

The one link network is obtainable from g^S via $S = \{i\}$ with $ij, ik \in g$; $S = \{i\}$ with $ij \in g$ or $S = N$. For such deviations to be not profitable, in the symmetric case, at least one member of the deviating coalition should not gain in the new configuration. Then, for $|S| = 1$ $b_1 < c_2$ and $b_0 < c_1$ for $S = N$ $b_1 < c_2$ or $b_0 < c_1$

(iv) $b_1 < c_2$ if $\gamma > \breve{\gamma}$

(v) $b_0 < c_1$ if $\gamma > \tilde{\gamma}'$

(vi) $b_1 < c_2$ if $\gamma > \breve{\gamma}$ or $b_0 < c_1$ if $\gamma > \tilde{\gamma}'$

The complete network is obtainable from g^S via $S = \{i, j\}$ with $ij \notin g$ or $S = N$. For such deviations to be not profitable, in the symmetric case, at least one member of the deviating coalition should not gain in the new configuration. Then,

(vii) $d_2 < c_1$ if $\gamma < \bar{\gamma}$

(viii) $d_2 < c_1$ if $\gamma < \bar{\gamma}$ or $d_2 < c_2$ satisfied for $\gamma > 0$

Conditions (i) to (viii) require both $\gamma > \tilde{\gamma}'$ and $\gamma < \bar{\gamma}$. Since $\bar{\gamma} < \tilde{\gamma}'$, the two requirements cannot be simultaneously satisfied and the star network is not strongly stable.■

Lemma 1.4. When countries are symmetric, the complete network is strongly stable if $\gamma > \hat{\gamma}$.

Proof of Lemma 1.4.

The empty network is obtainable from g^C via $|S| = 2$ or $S = N$. For such deviations to be not profitable, in the symmetric case, at least one of the deviating coalition member should not gain in the new configuration. Then,

(i) $d_2 > a_0$ if $\gamma > \hat{\gamma}$

[39] $\breve{\gamma} = \frac{(n+1)^2 t}{2} < \tilde{\gamma}$; $\bar{\gamma}' = \frac{(6n^2 + 4n + 1)t}{2(n+1)}$; $\tilde{\gamma}' = \frac{(7n^2 + 4n + 1)t}{2(n+1)}$

The one link network is obtainable from g^C via $|S|=1$, $|S|=2$ or $S=N$. For such deviations to be not profitable, in the symmetric case, at least one member of the deviating coalition should not gain in the new configuration. Then,

(ii) $d_2 > b_0$ if $\gamma > \bar{\gamma}'$

(iii) $d_2 > b_1$ for $\gamma > 0$ and $n > 1$ or $d_2 > b_0$ if $\gamma > \bar{\gamma}'$

(iv) $d_2 > b_1$ for $\gamma > 0$ and $n > 1$ or $d_2 > b_0$ if $\gamma > \bar{\gamma}'$

The star network is obtainable from g^C via $|S|=1$, $|S|=2$ or $S=N$. For such deviations to be not profitable, in the symmetric case, at least one member of the deviating coalition should not gain in the new configuration. Then,

(v) $d_2 > c_1$ if $\gamma > \bar{\gamma}$

(vi) $d_2 > c_2$ is not satisfied for $\gamma > 0$ or $d_2 > c_1$ if $\gamma > \bar{\gamma}$

(vii) $d_2 > c_2$ is not satisfied for $\gamma > 0$ or $d_2 > c_1$ if $\gamma > \bar{\gamma}$

From the analysis of conditions (i) to (vii), the complete network is strongly stable if $\gamma > \hat{\gamma}$ since $\hat{\gamma}$ is both greater than $\bar{\gamma}'$ and $\bar{\gamma}$.■

Proof of Part 2.

We know from Lemma 1.1 that the empty network is strongly stable if the market size of the symmetric countries is lower than γ''. Therefore, it is easy to show that when the market size of countries is $\gamma < \breve{\gamma}$ it implies that is also lower than γ'' since $\breve{\gamma} < \gamma''$. The empty network is the unique strongly stable configuration because if $\gamma < \breve{\gamma}$ then the condition $\tilde{\gamma} < \gamma < \tilde{\gamma}'$ in Lemma 1.2 cannot hold. That is, the one link is not strongly stable because the market size cannot be simultaneously greater than $\tilde{\gamma}$ and lower than $\breve{\gamma}$ given $\breve{\gamma} < \tilde{\gamma}$. According to Lemma 1.3 the star network is not strongly stable. Finally, the condition $\gamma > \hat{\gamma}$ in Lemma 1.4 cannot hold since $\breve{\gamma} < \hat{\gamma}$. ■

Part 3. Efficiency and Pareto efficiency

Part 3.1

When countries are symmetric and $\gamma < \breve{\gamma}$ *the empty network and the one link network are Pareto efficient.*

Proof of Part 3.1

A network g is Pareto efficient relative to, a value function, v and, a payoff function, ϕ if there does not exist any $g' \in G$ such that $\phi^i(g', v) \geq \phi^i(g, v)$ for all i with strict inequality for some i. Given the condition $\gamma < \breve{\gamma}$ we look for the network that are not Pareto dominated (P-dominated) by any other. Simply, for $\gamma < \breve{\gamma}$ the empty network is not Pareto dominated by any other configuration since $a_0 > b_1; a_0 > c_2; a_0 > d_2$. On the other hand also the one link network is not Pareto dominated since $b_0 > a_0; b_0 > c_1; b_0 > d_2; b_1 > c_2; b_1 > d_2$.■

Part 3.2

If countries are symmetric and $\gamma < \hat{\gamma}$ *either the empty network or the one link network are efficient. If* $\gamma > \hat{\gamma}$ *either the complete network or the star network are efficient.*

Proof of Part 3.2

A network g is *efficient* relative to a value function v if $v(g) \geq v(g^{'})$ for all $g^{'} \in G$. From the analysis of the value functions of each network configuration we obtain the following lemmas:

Lemma 1.5 If countries are symmetric and $\gamma < \breve{\gamma}$ the empty network is the efficient network architecture.
Lemma 1.6 If countries are symmetric and $\breve{\gamma} < \gamma < \hat{\gamma}$ the one link network is the efficient network architecture.
Lemma 1.7 If countries are symmetric and $\gamma > \hat{\gamma}$ the complete network is the efficient network configuration if $\gamma > \hat{\hat{\gamma}}$ (with $\hat{\hat{\gamma}} > \hat{\gamma}$) and the star network is the efficient network when $\hat{\gamma} < \gamma < \hat{\hat{\gamma}}$[40].

Proofs.
Proofs of Lemma 1.5, 1.6 and 1.7 follow directly from the analysis of the values generated by each network in the three different γ intervals. It can be easily shown that the value generated by the empty network when $\gamma < \breve{\gamma}$ is greater than any other among the potential network architectures in the model. That is, $v(g^e) > v(g^1) > v(g^S) > v(g^C)$ where $v(g^i) = \sum_{k=1}^{N} \phi_{d(j)}^{k}(g^i)$. On the other hand, when $\breve{\gamma} < \gamma < \hat{\gamma}$, $v(g^1) > v(g^e) > v(g^S) > v(g^C)$. That is, the one link network is the efficient configuration. When $\hat{\gamma} < \gamma < \hat{\hat{\gamma}}$ we obtain $v(g^S) > v(g^1) > v(g^C) > v(g^e)$ and finally, when $\gamma > \hat{\hat{\gamma}}$, we have $v(g^C) > v(g^S) > v(g^1) > v(g^e)$.■

1.7.2 B.2 Proof of Proposition 2.

The proof is organized in three steps. First, we prove the pairwise stability. Second, we concentrate on the strong stability and then on the efficiency and Pareto efficiency.

Part 1. Pairwise Stability

If countries are symmetric and $\breve{\gamma} < \gamma < \hat{\gamma}$ *then, either the empty network is the unique pairwise stable architecture if* $\breve{\gamma} < \gamma < \tilde{\gamma}$ *or the one link network and the complete network are the only pairwise stable configurations if* $\bar{\gamma} < \gamma < \tilde{\gamma}'$.

[40] $\hat{\hat{\gamma}} = \frac{(6n^2 + 4n + 1)t}{2}$

Proof of Part 1.
When $\breve{\gamma} < \gamma < \hat{\gamma}$ we can have three situations. First if $\breve{\gamma} < \gamma < \tilde{\gamma}$ from condition (14) we have that the empty network is pairwise stable. Second, since we know from (15) and (16) that both values of $\tilde{\gamma}$ and $\bar{\gamma}$ are greater than $\breve{\gamma}$ and that $\tilde{\gamma}' < \hat{\gamma}$, then we have that the one link configuration is pairwise stable if $\tilde{\gamma} < \gamma < \tilde{\gamma}'$. Moreover, since $\tilde{\gamma} < \bar{\gamma} < \tilde{\gamma}'$ we have that both one link and the complete networks are pairwise stable if $\bar{\gamma} < \gamma < \tilde{\gamma}'$. Finally, if $\tilde{\gamma}' < \gamma < \hat{\gamma}$ no pairwise stable network exists. Therefore, either the empty network is the unique pairwise stable architecture if $\breve{\gamma} < \gamma < \tilde{\gamma}$ or the one link and the complete architectures are the only pairwise stable networks if $\bar{\gamma} < \gamma < \tilde{\gamma}'$.■

Part 2. Strong Stability

If countries are symmetric and $\breve{\gamma} < \gamma < \hat{\gamma}$ then, either the empty network is the unique strongly stable configuration with $\breve{\gamma} < \gamma < \tilde{\gamma}$ or the one link network is the unique strongly stable architecture with $\tilde{\gamma} < \gamma < \tilde{\gamma}'$.

Proof of Part 2.

It is straightforward to notice that condition $\breve{\gamma} < \gamma < \hat{\gamma}$ does not imply neither condition in Lemma 1.1 nor Lemma 1.2. On the other hand it excludes condition in Lemma 1.4. Thus, when countries size belongs to $\breve{\gamma} < \gamma < \hat{\gamma}$ we can have three possible cases. No strongly stable networks exist (if $\tilde{\gamma}' < \gamma < \hat{\gamma}$), the empty network is strongly stable (if $\breve{\gamma} < \gamma < \tilde{\gamma}$) and the one link network is strongly stable (if $\tilde{\gamma} < \gamma < \tilde{\gamma}'$). Since each previous case rules outs the other and since from Lemma 1.3 and Lemma 1.4 we know that the complete and the star networks are not strongly stable, therefore, we also have that the empty and the one link network are, if the condition of the specific case is satisfied, also the unique *SS* configurations.■

Part 3. Efficiency and Pareto efficiency

Part 3.1
When countries are symmetric and $\breve{\gamma} < \gamma < \hat{\gamma}$, then either the empty network is P-efficient if $\breve{\gamma} < \gamma < \tilde{\gamma}$, or *the one link network is P-Efficient if either $\tilde{\gamma} < \gamma < \hat{\gamma}$.*

Proof of Part 3.1

The empty network is not P-dominated by any other if $a_0 > b_1(\gamma < \tilde{\gamma}); a_0 > c_2(\gamma < \gamma''); a_0 > d_2(\gamma < \hat{\gamma})$. Therefore we have that the empty network is P-efficient for $\breve{\gamma} < \gamma < \tilde{\gamma}$ since $\breve{\gamma} < \gamma'' < \tilde{\gamma} < \hat{\gamma}$. The one link network is not P-dominated if $a_0 < b_1(\gamma > \tilde{\gamma})$ *or* $a_0 < b_0(\gamma < \gamma'); b_1 > c_1(\gamma > \breve{\gamma})$ *or* $b_0 > c_1(\gamma < \tilde{\gamma}')$ *or* $b_1 > c_2(\gamma < \breve{\gamma}); b_1 > d_2(\gamma > 0)$.
Therefore, when $\breve{\gamma} < \gamma < \hat{\gamma}$ the one link network is P-efficient if $\tilde{\gamma} < \gamma < \hat{\gamma}$. ■

Part 3.2
See Part 3.2 of proposition 1 in Appendix 1.

1.7.3 B.3 Proof of Proposition 3.

The proof is organized in three steps. First, we prove the pairwise stability. Second, we concentrate on the strong stability and then on the efficiency and Pareto efficiency.

Part 1. Pairwise Stability

If countries are symmetric and $\gamma > \hat{\gamma}$ *then, the complete network configuration is the unique pairwise stable network.*

Proof of Part 1.
From (16) we have that $\gamma > \hat{\gamma}$ implies that $\gamma > \overline{\gamma}$ since $\hat{\gamma} > \overline{\gamma}$. Therefore, the complete network is pairwise stable. The condition $\gamma < \tilde{\gamma}$ in the expression (14) and $\gamma < \tilde{\gamma}'$ in the expression (15) are not satisfied because respectively $\hat{\gamma} > \tilde{\gamma}$ and $\hat{\gamma} > \tilde{\gamma}'$. Then, the empty and the one link network are not pairwise stable. Therefore, the complete network is the unique pairwise stable architecture. ■

Part 2. Strong Stability

If countries are symmetric and $\gamma > \hat{\gamma}$ *then, the complete network configuration is the unique strongly stable network.*

Proof of Part 2.

When $\gamma > \hat{\gamma}$ it is easy to see from Lemma 1.1, 1.2, 1.3 that the conditions for the empty, one link and star network to be SS are not satisfied since $\hat{\gamma} > \tilde{\gamma}' > \tilde{\gamma}$. On the other hand, from Lemma 4 we know that the complete configuration is *SS* if $\gamma > \hat{\gamma}$.■

Part 3. Efficiency and Pareto efficiency

Part 3.1
When countries are symmetric and $\gamma > \hat{\gamma}$ *the star network, the one link and complete network are Pareto efficient.*

Proof of Part 3.1
For $\gamma > \hat{\gamma}$ the one link network is not P-dominated since $b_1 > a_0; b_1 > c_1; b_1 > d_2$**.** The Star network is not P-dominated by any other configuration since $c_2 > a_0; c_1 > b_0; c_2 > b_1; c_2 > d_2$. Finally, the complete network configuration is not P-dominated since $d_2 > a_0; d_2 > b_0; d_2 > c_1$.■

Part 3.2
See Part 3.2 of proposition 1 in Appendix 1.

1.7.4 B.4 Proof of Proposition 4.

The proof is organized in three steps. First, we prove the pairwise stability. Second, we concentrate on the strong stability and then on the efficiency and Pareto efficiency.

Part 1. Pairwise Stability

When countries are asymmetric and $\gamma < \tilde{\gamma}$, the empty network g^e is the unique Pairwise Stable (PS) configuration if $\gamma_z^2(\gamma,n) < \gamma_z < \gamma_z^1(\gamma,n)$.

Before to proceed with the proofs of the previous proposition we need to introduce the following lemmas:

Lemma 2.1. When countries are asymmetric the empty network g^e is *PS* if $\gamma < \tilde{\gamma}$ and $\gamma_z^2(\gamma,n) < \gamma_z < \gamma_z^1(\gamma,n)$.
Proof.
The empty network g^e is pairwise stable if for any pair of disjointed players at least one of them has no incentive to form a link with the other. In particular, the empty network is *PS* if for every $ij \notin g$ whenever $\phi^i_{d(0)}(g^e) < \phi^i_{d(1)}(g^1)$ then $\phi^j_{d(0)}(g^e) > \phi^j_{d(1)}(g^1)$. That is,

(i) $b_1 < a$ (for $\gamma < \tilde{\gamma}$) and
if $b_1' > a$ (for $\gamma_z > \gamma_z^1(\gamma,n)$) then $b_{z_1} < a_z$ (for $\gamma_z > \gamma_z^2(\gamma,n)$) and
if $b_{z_1} > a_z$ (for $\gamma_z < \gamma_z^2(\gamma,n)$) then $b_1' < a$ (for $\gamma_z < \gamma_z^1(\gamma,n)$)

Conditions that prevent each country to sign an agreement and that depend on country Z market size, are simultaneously satisfied for $\gamma_z^2(\gamma,n) < \gamma_z < \gamma_z^1(\gamma,n)$[41]. Where $\gamma_z^2(\gamma,n) < \gamma_z^1(\gamma,n)$ for $\gamma < \tilde{\gamma}$. Since we need that at least one country doesn't find profitable an agreement that represents a gain for the potential partner, we have also that (i) is satisfied for $\gamma < \tilde{\gamma}$ and $\gamma_z > \gamma_z^2(\gamma,n)$, or $\gamma < \tilde{\gamma}$ and $\gamma_z < \gamma_z^1(\gamma,n)$.■

Lemma 2.2. When countries are asymmetric the one link network g^1_{xy} is *PS* if $\gamma > \tilde{\gamma}$ and either $\gamma_z > \gamma_z^{13}(\gamma,n)$ or $\gamma_z < \gamma_z^{12}(\gamma,n)$
Proof.
The one link network g^1_{xy} is pairwise stable if (i) for any pair of linked players neither player has an incentive to cut the link and (ii) for any pair of disjointed players at least one of them has no incentive to form a link with the other.

(i) $b_1 > a$ (for $\gamma > \tilde{\gamma}$)

[41] $\gamma_z^1 = \gamma\dfrac{n}{(2n+1)} + \dfrac{(3n^2+2n+1)t}{2(2n+1)}$; $\gamma_z^2 = \dfrac{\gamma(2n+1)}{n} - \dfrac{(3n^2+2n+1)t}{2n}$.

(ii) if $c_2 > b_1$ (for $\gamma_z > \gamma_z^{12}(\gamma,n)$) then $c_{z_1} < b_{z_0}$ (for $\gamma_z > \gamma_z^{13}(\gamma,n)$) and
if $c_{z_1} > b_{z_0}$ (for $\gamma_z < \gamma_z^{13}(\gamma,n)$) then $c_2 < b_1$ (for $\gamma_z < \gamma_z^{12}(\gamma,n)$)

For (i) and (ii) to be simultaneously satisfied , $\gamma > \tilde{\gamma}$ and either $\gamma_z > \gamma_z^{13}(\gamma,n)$ or $\gamma_z < \gamma_z^{12}(\gamma,n)$ [42] .■

Lemma 2.3. When countries are asymmetric the one link network g_{zi}^1 is not *PS*.

Proof.

The one link network g_{zi}^1 is pairwise stable if (i) for any pair of linked players neither player has an incentive to cut the link and (ii) for any pair of disjointed players at least one of them has no incentive to form a link with the other.

(i) $b_1' > a$ (for $\gamma_z > \gamma_z^{l}(\gamma,n)$) and $b_{z_1} > a_z$ (for $\gamma_z < \gamma_z^{h}(\gamma,n)$)

(ii) if $c_2 > b_1'$ (for $\gamma > \breve{\gamma}$) then $c_1 < b_0$ (for $\gamma < \bar{\bar{\gamma}}$ [43]), if $c_{z_2} > b_{z_1}$ (for $\gamma_z < \gamma_z^{22}(\gamma,n)$) then $c_1' < b_0$ (for $\gamma_z < \gamma_z^{21}(\gamma,n)$) and
if $c_1 > b_0$ (for $\gamma > \bar{\bar{\gamma}}$), then $c_2 < b_1'$ (for $\gamma < \breve{\gamma}$) if $c_1' > b_0$ (for $\gamma_z > \gamma_z^{21}(\gamma,n)$) then $c_{z_2} < b_{z_1}$ (for $\gamma_z > \gamma_z^{22}(\gamma,n)$)

Conditions (i) and conditions on γ_z in (ii) are satisfied for $\gamma_z^{l}(\gamma,n) < \gamma_z < \gamma_z^{h}(\gamma,n)$. Indeed, since $\gamma_z^{l}(\gamma,n) > \gamma_z^{22}(\gamma,n)$, then $\gamma_z > \gamma_z^{l}(\gamma,n)$ implies $\gamma_z > \gamma_z^{22}(\gamma,n)$. Since $\bar{\bar{\gamma}} > \breve{\gamma}$, we have that g_{zi}^1 is pairwise stable for $\gamma < \breve{\gamma}$ and $\gamma_z^{l}(\gamma,n) < \gamma_z < \gamma_z^{h}(\gamma,n)$. Moreover, $\gamma_z^{l}(\gamma,n) < \gamma_z^{h}(\gamma,n)$ verifies only for $\gamma > \tilde{\gamma}$ since both $\gamma_z^{l}(\gamma,n)$ and $\gamma_z^{h}(\gamma,n)$ are not decreasing in γ but the effect of its reduction is stronger in $\gamma_z^{h}(\gamma,n)$ than in $\gamma_z^{l}(\gamma,n)$. Therefore, since $\breve{\gamma} < \tilde{\gamma}$, for $\gamma < \breve{\gamma}$ g_{zi}^1 is not pairwise stable. ■

Lemma 2.4. When countries are asymmetric the star network $g_{x,y}^S$ is *PS* if $\gamma > \bar{\bar{\gamma}}$ and $\gamma_z^{i}(\gamma,n) < \gamma_z < \gamma_z^{l}(\gamma,n)$.

Proof.

The star network $g_{x,y}^S$ is pairwise stable if (i) for any pair of linked players neither player has an incentive to cut the link and (ii) for any pair of disjointed players at least one of them has no incentive to form a link with the other.

(i) $c_2 > b_1$ (for $\gamma_z > \gamma_z^{i}(\gamma,n)$); $c_2 > b_1'$ (for $\gamma > \breve{\gamma}$); $c_1 > b_0$ (for $\gamma > \bar{\bar{\gamma}}$); $c_{z_1} > b_{z_0}$ (for $\gamma_z < \gamma_z^{13}(\gamma,n)$

(ii) if $d_2 > c_{z_1}$ (for $\gamma_z < \gamma_z^{l'}(\gamma,n)$) then $d_2 < c_1$ (for $\gamma_z < \gamma_z^{l}(\gamma,n)$) and
if $d_2 > c_1$ (for $\gamma_z > \gamma_z^{l}(\gamma,n)$) then $d_2 < c_{z_1}$ (for $\gamma_z > \gamma_z^{l'}(\gamma,n)$)

[42] $\gamma_z^{12} = \frac{(n^2+2n+1)t}{2(2n+1)} + \gamma\frac{n}{(2n+1)}$; $\gamma_z^{13} = \gamma\frac{(2n+1)}{n} - \frac{(7n^2+4n+1)t}{2n}$.

[43] $\bar{\bar{\gamma}} = \frac{(7n^2+4n+1)t}{2(n+1)}$

Conditions (i) are satisfied for $\gamma > \bar{\bar{\gamma}}$ and $\gamma_z^i(\gamma,n) < \gamma_z < \gamma_z^{13}(\gamma,n)$. Condition (ii) are satisfied either if $\gamma_z < \gamma_z^l(\gamma,n)$ or $\gamma_z > \gamma_z^{l'}(\gamma,n)$. Since $\gamma_z^l(\gamma,n) < \gamma_z^{13}(\gamma,n)$ then for $\gamma_z^i(\gamma,n) < \gamma_z < \gamma_z^l(\gamma,n)$ [44] both condition (i) and (ii) on γ_z are satisfied. ■

Lemma 2.5. When countries are asymmetric the star network g_z^S is *PS* if $\gamma < \bar{\gamma}$ and $\gamma_z^{21}(\gamma,n) < \gamma_z < \gamma_z^{22}(\gamma,n)$.

Proof.

The star network g_z^S is pairwise stable if (i) for any pair of linked players neither player has an incentive to cut the link and (ii) for any pair of disjointed players at least one of them has no incentive to form a link with the other.

(i) $c_{z_2} > b_{z_1}$ (for $\gamma_z < \gamma_z^{22}(\gamma,n)$) and $c_1' > b_0$ (for $\gamma_z > \gamma_z^{21}(\gamma,n)$)

(ii) $d_2 < c_1'$ (if $\gamma < \bar{\gamma}$)

Conditions (i) and (ii) are satisfied for $\gamma < \bar{\gamma}$ and $\gamma_z^{21}(\gamma,n) < \gamma_z < \gamma_z^{22}(\gamma,n)$. ■

Lemma 2.6. When countries are asymmetric the complete network g^C is *PS* if $\gamma > \bar{\gamma}$ and $\gamma_z^l(\gamma,n) < \gamma_z < \gamma_z^{l'}(\gamma,n)$.

Proof.

The complete network is *PS* if (i) for any pair of linked players neither player has an incentive to cut the link .

(i) $d_2 > c_1'$ (if $\gamma > \bar{\gamma}$), $d_2 > c_1$ (if $\gamma_z > \gamma_z^l(\gamma,n)$), $d_2 > c_{z_1}$ (if $\gamma_z < \gamma_z^{l'}(\gamma,n)$)

Conditions (i) and (ii) are satisfied for $\gamma > \bar{\gamma}$ and $\gamma_z^l(\gamma,n) < \gamma_z < \gamma_z^{l'}(\gamma,n)$.■

Proof of part 1.

When $\gamma < \breve{\gamma}$, from the analysis of pairwise conditions in Lemma 2.1 to 2.6 it is easy to verify that the only two potential pairwise stable networks are g^e and g_z^S since $\breve{\gamma} < \tilde{\gamma} < \bar{\gamma}$. Moreover, from the analysis of the conditions on the star network (g_z^S) stability we notice that when $\gamma < \breve{\gamma}$ $\gamma_z^{21}(\gamma,n) > \gamma_z^{22}(\gamma,n)$ [45]. Therefore the condition $\gamma_z^{21}(\gamma,n) < \gamma_z < \gamma_z^{22}(\gamma,n)$ is not satisfied and g_z^S is not pairwise stable. ■

Part 2. Strong Stability

When countries are asymmetric and $\gamma < \breve{\gamma}$ the empty network is the unique SS network.

The following lemmas will simplify afterwards the proof of the previous proposition.

Lemma 3.1. The empty network is strongly stable (*SS*) if $\gamma < \tilde{\gamma}$ and $\gamma_z < \gamma_z^5(\gamma,n)$.

---

44 $\gamma_z^i = \gamma\frac{n}{(2n+1)} + \frac{(n^2+2n+1)t}{2(2n+1)}$; $\gamma_z^l = \gamma\frac{n}{(2n+1)} + \frac{(5n^2+4n+1)t}{2(2n+1)}$; $\gamma_z^{l'} = \gamma\frac{(2n+1)}{n} - \frac{(5n^2+4n+1)t}{2n}$.

45 Both $\gamma_z^{21}(\gamma,n)$ and $\gamma_z^{22}(\gamma,n)$ are not decreasing in γ but the impact of a reduction of γ on $\gamma_z^{22}(\gamma,n)$ is greater than on $\gamma_z^{21}(\gamma,n)$. In particular, when $\gamma < \tilde{\gamma}$, $\gamma_z^{21}(\gamma,n) > \gamma_z^{22}(\gamma,n)$. Since $\breve{\gamma} < \tilde{\gamma}$, if $\gamma < \breve{\gamma}$ $\gamma_z^{21}(\gamma,n) > \gamma_z^{22}(\gamma,n)$.

Proof.

According to the strong stability condition the empty network is strongly stable if for every possible coalition $S \subseteq N$ of players and for any network g obtainable from g^e through deviation by S, and $i \subset S$, when $\phi^i_{d(i)}(g) > \phi^i_{d(0)}(g^e)$ then, there exists $j \subset S$ such that $\phi^j_{d(0)}(g^e) > \phi^j_{d(j)}(g)$. Using the payoffs definition described in (11) we analyze the conditions, for each g obtainable from g^e via every possible $S \subseteq N$, under which the empty network is strongly stable.

The one link network g^1_{xy} is obtainable from g^e only via $S = \{X, Y\}$ or $S = N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S = \{X, Y\}$ this is true if $\mathrm{b}_1 < \mathrm{a}$ and for $S = N$ if $\mathrm{b}_1 < \mathrm{a}$ or $\mathrm{b}_{z_0} < \mathrm{a}_z$

(i) $\mathrm{b}_1 < \mathrm{a}$ if $\gamma < \tilde{\gamma}$

(ii) $\mathrm{b}_1 < \mathrm{a}$ if $\gamma < \tilde{\gamma}$ or $\mathrm{b}_{z_0} < \mathrm{a}_z$ if $\gamma > \gamma^* \ (> \tilde{\gamma})$[46]

The one link network g^1_{zi} is obtainable from g^e only via $S = \{X, Z\}$; $S = \{Y, Z\}$ *or* $S = N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S = \{i, Z\}$ either $\mathrm{b}'_1 < \mathrm{a}$ or $\mathrm{b}_{z_1} < \mathrm{a}_z$. For $S = N$ either $\mathrm{b}_0 < \mathrm{a}$ or $\mathrm{b}'_1 < \mathrm{a}$, $\mathrm{b}_{z_1} < \mathrm{a}_z$.

(iii) $\mathrm{b}'_1 < \mathrm{a}$ if $\gamma_z < \gamma^1_z(\gamma, n)$ or $\mathrm{b}_{z_1} < \mathrm{a}_z$ if $\gamma_z > \gamma^h_z(\gamma, n)$

(iv) $\mathrm{b}'_1 < \mathrm{a}$ if $\gamma_z < \gamma^1_z(\gamma, n)$ or $\mathrm{b}_{z_1} < \mathrm{a}_z$ if $\gamma_z > \gamma^h_z(\gamma, n)$, or $\mathrm{b}_0 < \mathrm{a}$ if $\gamma_z > \gamma^3_z(\gamma, n)$[47]

The star network $g^S_{x,y}$ is obtainable from g^e via $S = N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S = N$ $c_{z_1} < \mathrm{a}_z$ or $c_1 < \mathrm{a}$ $c_2 < \mathrm{a}$

(v) $c_{z_1} < \mathrm{a}_z$ if $\gamma_z > \gamma^4_z(\gamma, n)$ or $c_1 < \mathrm{a}$ if $\gamma_z > \gamma^4_z(\gamma, n)$; or $c_2 < \mathrm{a}$ if $\gamma_z < \gamma^5_z(\gamma, n)$[48]

The star network g^S_z is obtainable from g^e via $S = N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S = N$ $c_{z_2} < \mathrm{a}_z$ or $c'_1 < \mathrm{a}$.

(vi) $c_{z_2} < \mathrm{a}_z$ if $\gamma_z > \gamma^6_z(\gamma, n)$[49] or $c'_1 < \mathrm{a}$ $\gamma_z < \gamma^7_z(\gamma, n)$ (satisfied for $\gamma_z < \gamma$)

The complete network is obtainable from g^e via $S = N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating coalition member should not gain in the new configuration. Then, for $S = N$ $d_2 < \mathrm{a}$ or $d_2 < \mathrm{a}_z$

---

46 $\gamma^* = \frac{(3n+2)t}{2}$.

47 $\gamma^h_z = \frac{\gamma(2n+1)}{n} - \frac{(3n^2+2n+1)t}{2n}$; $\gamma^3_z = (3n+2)t - \gamma$.

48 $\gamma^4_z = \frac{\gamma}{n} - \frac{(n^2+1)t}{2n}$; $\gamma^5_z = \frac{(2n^2+2n+1)t}{(2n+1)} - \frac{\gamma}{(2n+1)}$.

49 $\gamma^6_z = \frac{\gamma(2n+1)}{n} - \frac{(2n^2+2n+1)t}{2n}$; $\gamma^7_z = \frac{(n^2+1)t}{2} + 2\gamma n$.

(vii) $d_2 < \text{a}$ if $\gamma_z < \gamma_z^8(\gamma,n)$ or $d_2 < \text{a}_z$ if $\gamma_z < \gamma_z^9(\gamma,n)$ [50]

Conditions (i) and (ii) are satisfied for $\gamma < \tilde{\gamma}$. All the conditions on Z market size are satisfied for $\gamma_z < \gamma_z^5(\gamma,n)$ (with $\gamma_z^5(.)_y < 0$) since $\gamma_z^5(\gamma,n)$ is smaller than $\gamma_z^8(\gamma,n), \gamma_z^7(\gamma,n)$, $\gamma_z^1(\gamma,n)$ for $\gamma > 0$ and it implies them all. ■

Lemma 3.2. The one link network g_{xy}^1 is *SS* if $\gamma > \tilde{\gamma}$ and $\gamma_z < \gamma_z^{12}(\gamma,n)$.

Proof.

The empty network g^e is obtainable from the one link network g_{xy}^1 via $S=\{X\}$, $S=\{Y\}$ $S=\{X,Y\}$ or $S=N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S=\{X\}$ or $S=\{Y\}$ $\text{b}_1 > \text{a}$; for $S=\{X,Y\}$ $\text{b}_1 > \text{a}$ and for $S=N$ $\text{b}_1 > \text{a}$ or $\text{b}_{z_0} > \text{a}_z$.

(i) $\text{b}_1 > \text{a}$ if $\gamma > \tilde{\gamma}$

(ii) $\text{b}_1 > \text{a}$ if $\gamma > \tilde{\gamma}$ or $\text{b}_{z_0} > \text{a}_z$ if $\gamma < \gamma^*$

The one link network g_{zi}^1 is obtainable from g_{xy}^1 via $S=\{X,Z\}$; $S=\{Y,Z\}$ *or* $S=N$. For such deviations to be not profitable, for the deviating coalitions $S=\{i,Z\}$ either $\text{b}_1' < \text{b}_1$ or $\text{b}_{z_1} < \text{b}_{z_0}$. For $S=N$ either $\text{b}_1' < \text{b}_1$ or $\text{b}_{z_1} < \text{b}_{z_0}$ and $\text{b}_0 < \text{b}_1$.

(iii) $\text{b}_1' < \text{b}_1$ (satisfied $\forall \gamma, n > 0$) or $\text{b}_{z_1} < \text{b}_{z_0}$ if $\gamma_z > \gamma_z^{10}(\gamma,n)$ [51]

(iv) $\text{b}_1' < \text{b}_1$ (satisfied $\forall \gamma, n > 0$) or $\text{b}_{z_1} < \text{b}_{z_0}$ if $\gamma_z > \gamma_z^{10}(\gamma,n)$ or $\text{b}_0 < \text{b}_1$ if $\gamma_z > \gamma_z^{11}(\gamma,n)$

The star network $g_{x,y}^S$ is obtainable from g_{xy}^1 via $S=\{X,Z\}$; $S=\{Y,Z\}$ *or* $S=N$. For such deviations to be not profitable, for the deviating coalitions $S=\{i,Z\}$ either $c_2 < \text{b}_1$ or $c_{z_1} < \text{b}_{z_0}$. For $S=N$ $c_2 < \text{b}_1$ or $c_{z_1} < \text{b}_{z_0}$ and $c_1 < \text{b}_1$.

(v) $c_2 < \text{b}_1$ if $\gamma_z < \gamma_z^{12}(\gamma,n)$ or $c_{z_1} < \text{b}_{z_0}$ if $\gamma_z > \gamma_z^{13}(\gamma,n)$

(vi) $c_2 < \text{b}_1$ if $\gamma_z < \gamma_z^{12}(\gamma,n)$ or $c_{z_1} < \text{b}_{z_0}$ if $\gamma_z > \gamma_z^{13}(\gamma,n)$, $c_1 < \text{b}_1$ if $\gamma > (n+1)t$

The star network g_z^S is obtainable from g_{xy}^1 via $S=N$. For $S=N$ to be not a profitable deviating coalition, either $c_1' < \text{b}_1$ or $c_{z_2} < \text{b}_{z_0}$ have to hold.

(vii) $c_1' < \text{b}_1$ if $\gamma_z < \gamma_z^{14}(\gamma,n)$ [52] or $c_{z_2} < \text{b}_{z_0}$ $\gamma_z > \gamma_z^{15}(\gamma,n)$

The complete network is obtainable from g_{xy}^1 via $S=N$. The conditions for $S=N$ are either $d_2 < \text{b}_1$ or $d_2 < \text{b}_{z_0}$.

---

[50] $\gamma_z^8 = \frac{(3n^2+2n+1)t}{(n+1)} + \frac{\gamma(n-1)}{(n+1)}$; $\gamma_z^9 = \frac{\gamma(n+1)}{n} - \frac{(n^2+2n+1)t}{2n}$.

[51] $\gamma_z^{10} = \frac{\gamma(4n+1)}{n} - \frac{(8n^2+6n+1)t}{2n}$; $\gamma_z^{11} = \frac{(9n^2+6n+1)t}{2n} - \frac{\gamma(2n+1)}{n}$.

[52] $\gamma_z^{14} = \gamma\frac{(3n+1)}{(n+1)} - nt$; $\gamma_z^{15} = \frac{\gamma(3n+1)}{n} - \frac{(5n^2+4n+1)t}{2n}$.

(viii) $d_2 < \mathrm{b}_1$ if $\gamma_z < \gamma_z^{16}(\gamma,n)$ or $d_2 < \mathrm{b}_{z_0}$ if $\gamma_z > \gamma_z^{17}(\gamma,n)$ [53]

Conditions (i) and (ii) are satisfied for $\gamma > \tilde{\gamma}$. Conditions on Z market size are simultaneously satisfied for $\gamma_z < \gamma_z^{12}(\gamma,n)$ (with $\gamma_z^{12}(.)_y < 0$), indeed, since γ_z^{12} it is smaller than γ_z^{16} for every $n > 0$ and it is smaller than γ_z^{14} (for $\gamma > \breve{\gamma}$) . ■

Lemma 3.3. The one link network g_{zi}^1 is not *SS*.

Proof.

The empty network g^e is obtainable from the one link network g_{zi}^1 via $S=\{X\}$, $S=\{Z\}$ $S=\{X,Z\}$ or $S=N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S=\{X\}$ $\mathrm{b}_1' > \mathrm{a}$; for $S=\{Z\}$ $\mathrm{b}_{z_1} > \mathrm{a}_z$; for $S=\{X,Z\}$ either $\mathrm{b}_1' > \mathrm{a}$ or $\mathrm{b}_{z_1} > \mathrm{a}_z$ and for $S=N$ either $\mathrm{b}_1' > \mathrm{a}$ or $\mathrm{b}_{z_1} > \mathrm{a}_z$ and $\mathrm{b}_0 > \mathrm{a}$.

(i) $\mathrm{b}_1' > \mathrm{a}$ if $\gamma_z > \gamma_z^1(\gamma,n)$

(ii) $\mathrm{b}_{z_1} > \mathrm{a}_z$ if $\gamma_z < \gamma_z^h(\gamma,n)$

(iii) $\mathrm{b}_1' > \mathrm{a}$ if $\gamma_z > \gamma_z^1(\gamma,n)$ or $\mathrm{b}_{z_1} > \mathrm{a}_z$ if $\gamma_z < \gamma_z^h(\gamma,n)$

(iv) $\mathrm{b}_1' > \mathrm{a}$ if $\gamma_z > \gamma_z^1(\gamma,n)$ or $\mathrm{b}_{z_1} > \mathrm{a}_z$ if $\gamma_z < \gamma_z^h(\gamma,n)$ or $\mathrm{b}_0 > \mathrm{a}$ if $\gamma_z < \gamma_z^3(\gamma,n)$

The one link network g_{xy}^1 is obtainable from g_{zi}^1 via $S=\{X,Y\}$ or $S=N$. For such deviations to be not profitable, for the deviating coalitions $S=\{X,Y\}$ either $\mathrm{b}_1 < \mathrm{b}_1'$ or $\mathrm{b}_1 < \mathrm{b}_0$. For $S=N$ either $\mathrm{b}_1 < \mathrm{b}_1'$ or $\mathrm{b}_{z_0} < \mathrm{b}_{z_1}$, or $\mathrm{b}_1 < \mathrm{b}_0$.

(v) $\mathrm{b}_1 < \mathrm{b}_1'$ (is not satisfied for every $\gamma, n > 0$) or $\mathrm{b}_1 < \mathrm{b}_0$ if $\gamma_z < \gamma_z^{11}(\gamma,n)$

(vi) $\mathrm{b}_1 < \mathrm{b}_1'$ (is not satisfied for every $\gamma, n > 0$) or $\mathrm{b}_{z_1} < \mathrm{b}_{z_0}$ if $\gamma_z < \gamma_z^{10}(\gamma,n)$ or $\mathrm{b}_1 < \mathrm{b}_0$ if $\gamma_z < \gamma_z^{11}(\gamma,n)$

The star network $g_{x,y}^S$ is obtainable from g_{zi}^1 via $S=\{X,Y\}$ or $S=N$. For such deviations to be not profitable, for the deviating coalition $S=\{X,Y\}$ either $c_2 < \mathrm{b}_1'$ or $c_1 < \mathrm{b}_0$. For $S=N$ $c_2 < \mathrm{b}_1'$ or $c_{z_1} < \mathrm{b}_{z_1}$ and $c_1 < \mathrm{b}_0$.

(vii) $c_2 < \mathrm{b}_1'$ if $\gamma < \breve{\gamma}$ or $c_1 < \mathrm{b}_0$ if $\gamma < \bar{\bar{\gamma}}$

(viii) $c_2 < \mathrm{b}_1'$ if $\gamma < \breve{\gamma}$ or $c_1 < \mathrm{b}_0$ if $\gamma < \bar{\bar{\gamma}}$, or $c_{z_1} < \mathrm{b}_{z_1}$ $\gamma > \breve{\gamma}$

The star network g_z^S is obtainable from g_{zi}^1 via $S=\{Y,Z\}$ or $S=N$. For $S=\{Y,Z\}$ to be not a profitable deviating coalition, either $c_1' < \mathrm{b}_0$ or $c_{z_2} < \mathrm{b}_{z_1}$ have to hold. For $S=N$ either $c_1' < \mathrm{b}_0$ or $c_{z_2} < \mathrm{b}_{z_1}$, or $c_1' < \mathrm{b}_1'$.

(ix) $c_1' < \mathrm{b}_0$ if $\gamma_z < \gamma_z^{21}(\gamma,n)$ or $c_{z_2} < \mathrm{b}_{z_1}$ if $\gamma_z > \gamma_z^{22}(\gamma,n)$

(x) $c_1' < \mathrm{b}_0$ if $\gamma_z < \gamma_z^{21}(\gamma,n)$ or $c_{z_2} < \mathrm{b}_{z_1}$ if $\gamma_z > \gamma_z^{22}(\gamma,n)$, or $c_1' < \mathrm{b}_1'$ if $\gamma > (n+1)t$

---

[53] $\gamma_z^{16} = \frac{(3n^2+2n+1)t}{2(n+1)} + \frac{2\gamma n}{(n+1)}$; $\gamma_z^{17} = \frac{\gamma(2n+1)}{n} - \frac{(6n^2+4n+1)t}{2n}$.

The complete network is obtainable from g^1_{zi} via $S=N$. The conditions for $S=N$ are either $d_2 < b_1'$ or $d_2 < b_{z_1}$, or $d_2 < b_0$

(xi) $d_2 < b_1'$ if $\gamma_z > \gamma_z^{23}(\gamma,n)$ or $d_2 < b_{z_1}$ if $\gamma_z > \gamma_z^{23}(\gamma,n)$, or $d_2 < b_0$ if $\gamma_z < \gamma_z^{24}(\gamma,n)$[54]

Conditions (vii) and (viii) are satisfied for $\gamma < \breve{\gamma}$. Conditions from (i) to (iv) are satisfied if $\gamma_z^1(\gamma,n) < \gamma_z < \gamma_z^h(\gamma,n)$. When $\gamma < \breve{\gamma}$ $\gamma_z^1(\gamma,n) > \gamma_z^h(\gamma,n)$, since $\gamma_z^1(.) < \gamma_z^h(.)$ only for $\gamma > \tilde{\gamma}$. Therefore, conditions (i)-(iv) and (vii), (viii) cannot be simultaneously satisfied. In other words, the one link network g^1_{zi} is not *SS* because either the empty network or the star network $g^S_{x,y}$ represent profitable deviations. ■

Lemma 3.4. The star network $g^S_{x,y}$ is not *SS*.

Proof.

The empty network g^e is obtainable from the star network $g^S_{x,y}$ via $S=\{X\}$ or $S=\{Y\}$, $S=\{Y,Z\}$ or $S=\{X,Z\}$; $S=N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S=\{X\}$ or $S=\{Y\}$ $c_2 > a$; for $S=\{X,Z\}$ or $S=\{Y,Z\}$ $c_1 > a$ and $c_{z_1} > a_z$; for $S=N$ $c_2 > a$, $c_1 > a$ or $c_{z_1} > a_z$.

(i) $c_2 > a$ if $\gamma_z > \gamma_z^5$

(ii) $c_1 > a$ if $\gamma_z < \gamma_z^4$ or $c_{z_1} > a_z$ if $\gamma_z < \gamma_z^4$

(iii) $c_2 > a$ if $\gamma_z > \gamma_z^5$ or $c_1 > a$ if $\gamma_z < \gamma_z^4$ and $c_{z_1} > a_z$ if $\gamma_z < \gamma_z^4$

The one link network g^1_{xy} is obtainable from $g^S_{x,y}$ via $S=\{X\}$; $S=\{Y\}$; $S=\{X,Z\}$; $S=\{X,Y\}$ *or* $S=N$. For such deviations to be not profitable, for the deviating coalition $S=\{X\}$, $c_2 > b_1$ must hold; for $S=\{Y\}$, $c_1 > b_1$; $S=\{X,Z\}$; $c_2 > b_1$ or $c_{z_1} > b_{z_0}$. For $S=\{X,Y\}$; $c_2 > b_1$ or $c_1 > b_1$ and for $S=N$; $c_2 > b_1$ or $c_1 > b_1$ and $c_{z_1} > b_{z_0}$.

(iv) $c_2 > b_1$ if $\gamma_z > \gamma_z^{12}(\gamma,n)$

(v) $c_1 > b_1$ if $\gamma < (n+1)t$

(vi) $c_2 > b_1$ if $\gamma_z > \gamma_z^{12}(\gamma,n)$ or $c_{z_1} > b_{z_0}$ if $\gamma_z < \gamma_z^{13}(\gamma,n)$

(vii) $c_2 > b_1$ if $\gamma_z > \gamma_z^{12}(\gamma,n)$ or $c_1 > b_1$ if $\gamma < (n+1)t$

(viii) $c_2 > b_1$ if $\gamma_z > \gamma_z^{12}(\gamma,n)$ or $c_1 > b_1$ if $\gamma < (n+1)t$ and $c_{z_1} > b_{z_0}$ if $\gamma_z < \gamma_z^{13}(\gamma,n)$

The one link network g^1_{zi} is obtainable from $g^S_{x,y}$ via $S=\{X\}$; $S=\{Y\}$ *or* $S=N$. For such deviations to be not profitable, for the deviating coalition $S=\{X\}$ $c_2 > b_1'$ must hold. For $S=\{Y\}$; $c_1 > b_0$. For $S=N$ either $c_2 > b_1'$ or $c_1 > b_0$ and $c_{z_1} > b_{z_1}$.

(ix) $c_2 > b_1'$ if $\gamma > \tilde{\gamma}$

(x) $c_1 > b_0$ if $\gamma > \bar{\bar{\gamma}}$

[54] $\gamma_z^{23} = \frac{\gamma}{n} - \frac{(3n^2+2n+1)t}{2n}$; $\gamma_z^{24} = \frac{(6n^2+4n+1)t}{(2n+1)} - \frac{\gamma}{(2n+1)}$

(xi) $c_2 > b_1^{'}$ if $\gamma > \tilde{\gamma}$ or $c_1 > b_0$ if $\gamma > \bar{\bar{\gamma}}$ and $c_{z_1} > b_{z_1}$ if $\gamma < \tilde{\gamma}$

The star network g_z^S is obtainable from $g_{x,y}^S$ via $S = \{Y, Z\}$ and $S = N$. For $S = \{Y, Z\}$ to be not a profitable deviating coalition, $c_1 > c_1^{'}$ or $c_{z_1} > c_{z_2}$ have to hold. For $S = N$ either $c_1 > c_1^{'}$ or $c_{z_1} > c_{z_2}$ and $c_2 > c_1^{'}$ have to hold.

(xii) $c_1 > c_1^{'}$ (satisfied for every $\gamma, n > 0$) or $c_{z_1} > c_{z_2}$

(xiii) $c_1 > c_1^{'}$ (satisfied for every $\gamma, n > 0$) or $c_{z_1} > c_{z_2}$ and $c_2 > c_1^{'}$

The complete network is obtainable from $g_{x,y}^S$ via $S = \{Y, Z\}$ and $S = N$. For $S = \{Y, Z\}$ to be not a profitable deviating coalition, $c_1 > d_2$ or $c_{z_1} > d_2$ have to hold. For $S = N$ either $c_1 > d_2$ or $c_{z_1} > d_2$ and $c_2 > d_2$ have to hold.

(xiv) $c_1 > d_2$ if $\gamma_z < \gamma_z^J(\gamma, n)$, or $c_{z_1} > d_2$ if $\gamma_z > \gamma_z^{J'}(\gamma, n)$

(xv) $c_1 > d_2$ if $\gamma_z < \gamma_z^J(\gamma, n)$ or $c_{z_1} > d_2$ if $\gamma_z > \gamma_z^{J'}(\gamma, n)$ and $c_2 > d_2$ (satisfied for every $\gamma, n > 0$)

Conditions (i) to (iii) are satisfied for $\gamma_z > \gamma_z^5$ and $\gamma_z < \gamma_z^4$. Conditions (xiv)-(xv) are satisfied for $\gamma_z < \gamma_z^J(\gamma, n)$. Conditions (iv) to (viii) and (ix) to (xi) cannot be simultaneously satisfied since the former are satisfied for $\gamma_z > \gamma_z^{12}$ and $\gamma < (n+1)t$; the latter for $\gamma > \bar{\bar{\gamma}}$ with $\bar{\bar{\gamma}} > (n+1)t$. Therefore, the star network $g_{x,y}^S$ is not *SS*.■

Lemma 3.5. The star network g_z^S is not *SS*.

Proof.

The empty network g^e is obtainable from the star network g_z^S via $S = \{Z\}$, $S = \{Y, Z\}$, $S = \{X, Z\}$, $S = \{X, Y\}$ and $S = N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S = \{Z\}$ $c_{z_2} > a_z$; for $S = \{X, Z\}$ or $S = \{Y, Z\}$ $c_1^{'} > a$ and $c_{z_2} > a_z$; for $S = \{X, Y\}$ $c_1^{'} > a$; for $S = N$ $c_1^{'} > a$ and $c_{z_2} > a_z$.

(i) $c_{z_2} > a_z$ if $\gamma_z < \gamma_z^6(\gamma, n)$

(ii) $c_1^{'} > a$ (is not satisfied for $\gamma_z < \gamma$) or $c_{z_2} > a_z$ if $\gamma_z < \gamma_z^6(\gamma, n)$

(iii) $c_1^{'} > a$ (is not satisfied for $\gamma_z < \gamma$)

(iv) $c_1^{'} > a$ (is not satisfied for $\gamma_z < \gamma$) or $c_{z_2} > a_z$ if $\gamma_z < \gamma_z^6(\gamma, n)$

Conditions (i) to (iv) cannot be satisfied simultaneously since $c_1^{'} < a$ (in condition (iii)). In other words, the empty network represents a profitable deviation for each component of the deviating coalition $S = \{X, Y\}$. Therefore, the star network is not *SS*.■

Lemma 3.6. The complete network is not *SS*.

Proof.

The empty network g^e is obtainable from the complete network via $S = \{X, Y\}$, $S = \{X, Z\}$ $S = \{Y, Z\}$ or $S = N$. For such deviations to be not profitable, in the asymmetric case, at least one of the deviating players should not gain in the new configuration. Then, for $S=\{X,Y\}$ $d_2 > a$; for $S=\{X,Z\}$ and $S=\{Y,Z\}$ either $d_2 > a_z$ or $d_2 > a$; for $S=N$ either $d_2 > a_z$ or $d_2 > a$.

(i) $d_2 > a$ for $\gamma_z > \gamma_z^8(\gamma,n)$

(ii) $d_2 > a_z$ for $\gamma_z > \gamma_z^9(\gamma,n)$ or $d_2 > a$ for $\gamma_z > \gamma_z^8(\gamma,n)$

(iii) $d_2 > a_z$ for $\gamma_z > \gamma_z^9(\gamma,n)$ or $d_2 > a$ for $\gamma_z > \gamma_z^8(\gamma,n)$

The one link network g_{xy}^1 is obtainable from g^C via $S=\{Z\}$ $S=\{X,Y\}$ or $S=N$. For such deviations to be not profitable, for the deviating coalitions $S=\{Z\}$ $d_2 > b_{z_0}$; $S=\{X,Y\}$ $d_2 > b_1$. For $S=N$ either $d_2 > b_{z_0}$ or $d_2 > b_1$.

(iv) $d_2 > b_{z_0}$ for $\gamma_z < \gamma_z^{17}(\gamma,n)$

(v) $d_2 > b_1$ for $\gamma_z > \gamma_z^{16}(\gamma,n)$

(vi) $d_2 > b_{z_0}$ for $\gamma_z < \gamma_z^{17}(\gamma,n)$ or $d_2 > b_1$ for $\gamma_z > \gamma_z^{16}(\gamma,n)$

The one link network g_{zi}^1 is obtainable from g^C via $S=\{X\}$, $S=\{Y\}$, $S=\{X,Z\}$, $S=\{Y,Z\}$ and $S=N$. For such deviations to be not profitable, for the deviating coalition $S=\{X\}$ and $S=\{Y\}$ $d_2 > b_0$. For $S=\{X,Z\}$ and $S=\{Y,Z\}$ either $d_2 > b_{z_1}$ or $d_2 > b_1'$. For $S=N$ $d_2 > b_0$ or $d_2 > b_{z_1}$ and $d_2 > b_1'$.

(vii) $d_2 > b_0$ for $\gamma_z > \gamma_z^{24}(\gamma,n)$

(viii) $d_2 > b_{z_1}$ for $\gamma_z < \gamma_z^{23}(\gamma,n)$ or $d_2 > b_1'$ for $\gamma_z < \gamma_z^{23}(\gamma,n)$

(ix) $d_2 > b_0$ for $\gamma_z > \gamma_z^{24}(\gamma,n)$ or $d_2 > b_{z_1}$ and $d_2 > b_1'$ for $\gamma_z < \gamma_z^{23}(\gamma,n)$

The star network $g_{x,y}^S$ is obtainable from g^C via $S=\{X\}$, $S=\{Z\}$, $S=\{Y,Z\}$, $S=\{X,Z\}$, $S=\{X,Y\}$ and $S=N$. For $S=\{Y\}$ to be not a profitable deviating coalition, $d_2 > c_1$ has to hold. For $S=\{Z\}$ $d_2 > c_{z_1}$; for $S=\{Y,Z\}$ either $d_2 > c_1$ or $d_2 > c_{z_1}$; for $S=\{X,Z\}$ either $d_2 > c_2$ or $d_2 > c_{z_1}$; for $S=\{X,Y\}$ either $d_2 > c_2$ or $d_2 > c_1$; for $S=N$ either $d_2 > c_2$ or $d_2 > c_1$ and $d_2 > c_{z_1}$.

(x) $d_2 > c_{z_1}$ for $\gamma_z < \gamma_z^{J'}(\gamma,n)$

(xi) $d_2 > c_1$ for $\gamma_z > \gamma_z^{J}(\gamma,n)$ or $d_2 > c_{z_1}$ for $\gamma_z < \gamma_z^{J'}(\gamma,n)$

(xii) $d_2 > c_2$ (is not satisfied for $\gamma,n>0$) or $d_2 > c_{z_1}$ for $\gamma_z < \gamma_z^{J'}(\gamma,n)$

(xiii) $d_2 > c_2$ (is not satisfied for $\gamma,n>0$) or $d_2 > c_1$ for $\gamma_z > \gamma_z^{J}(\gamma,n)$

(xiv) $d_2 > c_2$ (is not satisfied for $\gamma,n>0$) or $d_2 > c_1$ for $\gamma_z > \gamma_z^{J}(\gamma,n)$ and $d_2 > c_{z_1}$ for $\gamma_z < \gamma_z^{J'}(\gamma,n)$

The star network g_z^S is obtainable from g^C via $S=\{X\}$ or $S=\{Y\}$, $S=\{Y,Z\}$, $S=\{X,Z\}$, $S=\{X,Y\}$ and $S=N$. For $S=\{X\}$ or $S=\{Y\}$ $d_2 > c_1'$; for $S=\{X,Y\}$ $d_2 > c_1'$; for $S=\{Y,Z\}$ and $S=\{X,Z\}$ either $d_2 > c_1'$ or $d_2 > c_{z_2}$. For $S=N$ are either $d_2 > c_1'$ or $d_2 > c_{z_2}$.

(xv) $d_2 > c_1'$ for $\gamma > \overline{\gamma}$

(xvi) $d_2 > c_1^{'}$ for $\gamma > \bar{\gamma}$

(xvii) $d_2 > c_1^{'}$ for $\gamma > \bar{\gamma}$ or $d_2 > c_{z_2}$ (is not satisfied for $\gamma > 0$)

(xviii) $d_2 > c_1^{'}$ for $\gamma > \bar{\gamma}$ or $d_2 > c_{z_2}$ (is not satisfied for $\gamma > 0$)

Conditions (i) to (xiv) are simultaneously satisfied for $\gamma_z^{16}(\gamma,n) < \gamma_z < \gamma_z^{23}(\gamma,n)$. Conditions (xv) to (xviii) are satisfied for $\gamma > \bar{\gamma}$. However for $n > 1$ $\gamma_z^{16}(\gamma,n) > \gamma_z^{23}(\gamma,n)$, therefore the complete network is not *SS*.■

Proof of part 2

From Lemma 3.1 we know that the empty network is strongly stable if $\gamma < \tilde{\gamma}$ and $\gamma_z < \gamma_z^5(\gamma,n)$. Since $\breve{\gamma} < \tilde{\gamma}$, it follows that $\gamma < \breve{\gamma}$ implies $\gamma < \tilde{\gamma}$. Moreover, it is easy to verify that for $\gamma < \breve{\gamma}$ and with the assumption that $\gamma_z < \gamma$ it follows that $\gamma_z < \gamma_z^5(\gamma,n)$ since $\gamma_z^5(.) > \breve{\gamma}$ ($\forall n \geq 0$ and $\gamma < \breve{\gamma}$). For $\gamma < \breve{\gamma}$ the empty network is also the unique strongly stable network because all the other networks fail to fulfil the *SS* conditions on γ.■

Part 3. Efficiency and Pareto efficiency

Part 3.1

When countries are asymmetric and $\gamma < \breve{\gamma}$ under some conditions on γ_z, the network g^e and g_{zi}^1 are Pareto efficient. Furthermore g_{xy}^1, $g_{x,y}^S$ and g_z^S are also Pareto efficient only when g_{zi}^1 is Pareto dominated by the empty network and there is more asymmetry among countries.

Proof of part 3.1

A network g is Pareto efficient (P-efficient) relative to v and ϕ if there does not exist any $g^{'} \in G$ such that $\phi^i(g^{'},v) \geq \phi^i(g,v)$ for all i with strict inequality for some i. In other words, given the condition $\gamma < \breve{\gamma}$ we look for the network that are not Pareto dominated by any other. Simply, for $\gamma < \breve{\gamma}$ the empty network is not Pareto dominated by any other configuration for $\gamma < \tilde{\gamma}$ and $\gamma_z < \gamma_z^5(\gamma,n)$ (the SS condition), g_{zi}^1 is not Pareto dominated for $\gamma < \breve{\gamma}$[55] and $\gamma_z^1(\gamma,n) < \gamma_z < \gamma_z^{24}(\gamma,n)$. Moreover, g_{xy}^1, $g_{x,y}^S$ and g_z^S are not Pareto dominated by any other network, when $\gamma < \breve{\gamma}$ (or $\gamma > \breve{\gamma}$ for g_{xy}^1 and $g_{x,y}^S$), if $\gamma_z < \gamma_z^{15}(\gamma,n)$. On the other hand, when g_{zi}^1 is Pareto efficient, since $\gamma_z^{15}(\gamma,n) < \gamma_z^1(\gamma,n)$ (for $\gamma < \breve{\gamma}$) then g_{xy}^1, g_z^S and $g_{x,y}^S$ are Pareto dominated respectively by, the star network g_z^S, the star network $g_{x,y}^S$ and by the empty network. ■

[55] The one link network g_{zi}^1 is not Pareto dominated also for $\gamma > \breve{\gamma}$ or for $\gamma < \tilde{\gamma}$.

Part 3.2

If countries are asymmetric and $\gamma < \gamma^B$ *the one link network* g^1_{xy} *is efficient for* $\gamma_z < f(n)t - \gamma$*. If* $\gamma > \gamma^B$ *the complete network is efficient for any* $\gamma_z < \gamma$*.*

The proof of part 3.2 is described by the following lemmas.

Lemma 4.1 If countries are asymmetric and $\gamma < \breve{\gamma}$ the one link network g^1_{xy} is the efficient network architecture if $\gamma_z < f(n)t - \gamma$[56]. That is, $v(g^1_{xy}) > v(g^1_{zi}) > v(g^e) > v(g^S_{x,y}) > v(g^S_z) > v(g^C)$.

Lemma 4.2 If countries are asymmetric and $\breve{\gamma} < \gamma < \overline{\gamma}$ the one link network g^1_{xy} is the efficient network architecture if $\gamma_z < f(n)t - \gamma$. That is, $v(g^1_{xy}) > v(g^1_{zi}) > v(g^e) > v(g^S_{x,y}) > v(g^S_z) > v(g^C)$.

Lemma 4.3 If countries are asymmetric and $\overline{\gamma} < \gamma < \gamma^B$ the one link network g^1_{xy} is the efficient network architecture with $\gamma_z < f(n)t - \gamma$.

Lemma 4.4 If countries are asymmetric and $\gamma > \gamma^B$ the complete network is the efficient network configuration $\forall \gamma_z < \gamma$. That is, $v(g^C) > v(g^S_{x,y}) > v(g^S_z) > v(g^1_{xy}) > v(g^1_{zi}) > v(g^e)$.

Proofs.

Proofs of Lemma 4.1, 4.2 ,4.3 and 4.4 follow directly from the analysis of the values generated by each network in the γ intervals and from the order of conditions on γ_z.

The value generated by the one link network g^1_{xy} if $\gamma < \gamma^B$ [57] and $\gamma_z < f(n)t - \gamma$, is greater than any other among the potential network architectures in the model. That is, $v(g^1_{xy}) > v(g^1_{zi}) > v(g^e) > v(g^S_{x,y}) > v(g^S_z) > v(g^C)$ where $v(g^i) = \sum_{k=1}^{N} \phi^k_{d(j)}(g^i)$.

On the other hand, when $\gamma > \gamma^B$, we have $v(g^C) > v(g^S_{x,y}) > v(g^S_z) > v(g^1_{xy}) > v(g^1_{zi}) > v(g^e)$ $\forall \gamma_z < \gamma$. ■

1.7.5 B.5 Proof of Proposition 5.

The proof is organized in three steps. First, we prove the pairwise stability. Second, we concentrate on the strong stability and then on the efficiency and Pareto efficiency.

Part 1. Pairwise stability

When countries are asymmetric and $\breve{\gamma} < \gamma < \overline{\gamma}$ *then, either the empty network* g^e *or the star network* g^S_z *are PS. If* $\tilde{\gamma} < \gamma < \overline{\gamma}$ *then, the one link network* g^1_{xy} *and the empty network are PS if* $\gamma^2_z(\gamma, n) < \gamma_z < \gamma^{12}_z(\gamma, n)$*.*

---

[56] $f(n) = \dfrac{(3n^2 + 2n + 2)t}{2}$

[57] $\gamma^B = \dfrac{(15n^3 + 17n^2 + 7n + 1)t}{2}$

Proof of Part 1.

The market size interval on γ can be divided in two cases. First consider the interval $\breve{\gamma} < \gamma < \tilde{\gamma}$. From the analysis of pairwise conditions in Lemma 2.1 to 2.6 (see B.4.) it is easy to verify that the only two potential pairwise stable networks are g_z^S and g^e if respectively $\gamma < \bar{\gamma}$ and $\gamma < \tilde{\gamma}$ (with $\breve{\gamma} < \tilde{\gamma} < \bar{\gamma}$). Moreover, the conditions on market Z size requires that $\gamma_z^2(\gamma,n) < \gamma_z < \gamma_z^1(\gamma,n)$ for the stability of the empty network g^e and that $\gamma_z^{21}(\gamma,n) < \gamma_z < \gamma_z^{22}(\gamma,n)$ for the star network g_z^S. When γ belongs to $\breve{\gamma} < \gamma < \tilde{\gamma}$ we have also that $\gamma_z^{22} > \gamma_z^{21} > \gamma_z^1 > \gamma_z^2$. Thus we expect that either g_z^S or g^e are *PS* and that the empty network is stable for smaller values of γ_z. Second, if $\tilde{\gamma} < \gamma < \bar{\gamma}$ also the one link network g_{xy}^1 is pairwise stable under the condition : $\gamma_z < \gamma_z^{12}(\gamma,n)$ (described in Lemma 2.2). Since for $\tilde{\gamma} < \gamma < \bar{\gamma}$ $\gamma_z^1 > \gamma_z^{12} > \gamma_z^2$ we have that both g^e and g_{xy}^1 are *PS* when $\gamma_z^2(\gamma,n) < \gamma_z < \gamma_z^{12}(\gamma,n)$. Finally, since $\gamma_z^{22} > \gamma_z^{21} > \gamma_z^1 > \gamma_z^{12} > \gamma_z^2$, when $\tilde{\gamma} < \gamma < \bar{\gamma}$ and $\gamma_z^{21}(\gamma,n) < \gamma_z < \gamma_z^{22}(\gamma,n)$ only the star network g_z^S is *PS*. ■

Part 2. Strong Stability.

When countries are asymmetric and $\breve{\gamma} < \gamma < \bar{\gamma}$, then either the empty network or the one link network g_{xy}^1 are the strongly stable networks.

Proof of Part 2.

The market size interval $\breve{\gamma} < \gamma < \bar{\gamma}$ can be divided in two cases. First, if $\breve{\gamma} < \gamma < \tilde{\gamma}$ we know, from Lemma 3.1 to 3.3 (see B.4.), that only g^e can be strongly stable, providing also conditions on market Z's size are satisfied. The empty network g^e is SS for $\gamma_z < \gamma_z^5(\gamma,n)$. Second, if $\tilde{\gamma} < \gamma < \bar{\gamma}$ the only potential strongly stable network is the one link network g_{xy}^1. According to Lemma 3.2 (see B.4.) the one link network g_{xy}^1 is strongly stable if $\gamma > \tilde{\gamma}$ and $\gamma_z < \gamma_z^{12}(\gamma,n)$. Moreover for $\gamma > \tilde{\gamma}$, $\gamma_z^{12}(\gamma,n) > \gamma_z^5(\gamma,n)$ with $\gamma_z^{12}(.)$ not decreasing in γ and $\gamma_z^5(.)$ not increasing in γ. Thus, SS conditions requires a stronger asymmetry for g^e to be SS than for the one link network g_{xy}^1. ■

Part 3. Efficiency and Pareto efficiency

Part 3.1

When countries are asymmetric and $\breve{\gamma} < \gamma < \bar{\gamma}$, then either g_{zi}^1 is the unique Pareto efficient configuration or g^e, g_{xy}^1, g_z^S and $g_{x,y}^S$ are Pareto efficient.

Proof of Part 3.1

When the market size of the two symmetric countries belongs to $\breve{\gamma} < \gamma < \bar{\gamma}$ the one link network g_{zi}^1 is P-efficient if $\gamma > \tilde{\gamma}$ and $\gamma_z^1(\gamma,n) < \gamma_z < \gamma_z^{11}(\gamma,n)$. The empty network g^e and

$g^S_{x,y}$ are P-efficient for $\gamma > \tilde{\gamma}$ and for $\gamma_z < \gamma_z^5(\gamma, n)$ while the one link network g^1_{xy} is P-efficient for $\gamma > \tilde{\gamma}$ and $\gamma_z < \gamma_z^{12}(\gamma, n)$. Since for $\tilde{\gamma} < \gamma < \overline{\gamma}$ both $\gamma_z^5(\gamma, n)$ and $\gamma_z^{12}(\gamma, n)$ are lower than $\gamma_z^1(\gamma, n)$, then if g^1_{zi} is P-efficient, it is the unique P-efficient configuration. ■

Part 3.2
(see Part 3.2 in B.4.)

1.7.6 B.6 Proof of Proposition 6.

The proof is organized in three steps. First, we prove the pairwise stability. Second, we concentrate on the strong stability and then on the efficiency and Pareto efficiency.

Part 1. Pairwise Stability
When countries are asymmetric and $\overline{\gamma} < \gamma < \gamma^B$, *either the one link network* g^1_{xy} *is PS if* $\gamma_z < \gamma_z^{12}(\gamma, n)$ *or the complete network is if* $\gamma_z^J(\gamma, n) < \gamma_z < \gamma_z^{J'}(\gamma, n)$ *or the star network* $g^S_{x,y}$ *is PS if* $\gamma_z^i < \gamma_z < \gamma_z^J$.

Proof of Part 1.

When $\overline{\gamma} < \gamma < \gamma^B$, from Lemma 2.2, 2.4 and 2.6 (see B.4.) we have that either the one link network g^1_{xy} or the star network $g^S_{x,y}$ or the complete network are stable respectively for smaller to greater values of γ_z since $\gamma_z^{J'} > \gamma_z^J > \gamma_z^i = \gamma_z^{12}$. That is, the greater the asymmetry the higher the possibility for the one link network to be the unique stable network.■

Part 2. Strong Stability
When countries are asymmetric and $\overline{\gamma} < \gamma < \gamma^B$ *the one link network* g^1_{xy} *is the unique strongly stable network.*

Proof of Part 2.

According to Lemma 3.2, under the following conditions on market sizes (i) $\gamma > \tilde{\gamma}$ and (ii) $\gamma_z < \gamma_z^{12}(\gamma, n)$, the one link network g^1_{xy} is the unique strongly stable network. Indeed, when $\overline{\gamma} < \gamma < \gamma^B$, since $\overline{\gamma} > \tilde{\gamma}$, the condition (i) is satisfied. ■

Part 3. Efficiency and Pareto efficiency

Part 3.1
When countries are asymmetric, $\gamma > \overline{\gamma}$ *(or* $\gamma > \gamma^B$*) and* $\gamma_z < \gamma_z^5(\gamma, n)$ *the empty network* g^e, $g^S_{x,y}$ *and* g^1_{xy} *are P-efficient.*

Proof of Part 3.1.

See the proof of Part 3.1 of Proposition 5 in (B.5.). Moreover, we can show that the complete network is not P-efficient. That is, the complete network is not Pareto dominated by any other configuration only if $\gamma > \bar{\gamma}$, $\gamma_z < \gamma_z^{23}(\gamma,n)$ and $\gamma_z > \gamma_z^{9}(\gamma,n)$. However it is easy to verify that since for $\gamma > \bar{\gamma}$, $\gamma_z^{9}(\gamma,n) > \gamma_z^{23}(\gamma,n)$, then, the conditions on market Z cannot simultaneously hold and the complete network is Pareto dominated either by the empty network or by the one link network g_{zi}^{1}.■

Part 3.2
(see Part 3.2 in B.4.)

1.7.7 B.7 Proof of proposition 7.

The proof is organized in three steps. First, we prove the pairwise stability. Second, we concentrate on the strong stability and then on the efficiency and Pareto efficiency.

Part 1. Pairwise Stability
When countries are asymmetric and $\gamma > \gamma^B$, either the one link network g_{xy}^{1} is PS if $\gamma_z < \gamma_z^{12}(\gamma,n)$ or the complete network if $\gamma_z^{J}(\gamma,n) < \gamma_z < \gamma_z^{J'}(\gamma,n)$ or the star network $g_{x,y}^{S}$ $\gamma_z^{i} < \gamma_z < \gamma_z^{J}$.

Proof of Part 1.

When $\gamma > \gamma^B$, either the one link network g_{xy}^{1} if $\gamma_z < \gamma_z^{12}$ or the complete network if $\gamma_z^{J}(\gamma,n) < \gamma_z < \gamma_z^{J'}(\gamma,n)$ or the star network $g_{x,y}^{S}$ *if* $\gamma_z^{i}(\gamma,n) < \gamma_z < \gamma_z^{J}(\gamma,n)$ are pairwise stable according to Lemma 2.2, 2.4 and 2.6 (see B.4.). When $\gamma > \gamma^B$ we have that $\gamma_z^{J'} > \gamma_z^{J} > \gamma_z^{i} = \gamma_z^{12}$, then, given the value of γ in the interval, for smaller values of γ_z (i.e. stronger asymmetry) g_{xy}^{1} is the unique pairwise stable network, while, for bigger values of Z market size, the complete network is the unique pairwise stable network. That is, the greater the asymmetry the higher the possibility for the one link network to be the unique stable network.■

Part 2. Strong Stability
When countries are asymmetric and $\gamma > \gamma^B$ the one link network g_{xy}^{1} is the unique strongly stable network.

Proof of Part 2
The proof of Part 2 of Proposition 7 follows directly from Part 2 of Proposition 6 in (B.6) since $\gamma^B > \bar{\gamma}$. In other words, when $\gamma > \gamma^B$ the one link network g_{xy}^{1}, under the following conditions on market sizes (i) $\gamma > \tilde{\gamma}$ and (ii) $\gamma_z < \gamma_z^{12}(\gamma,n)$, is the unique strongly stable network. Since $\gamma^B > \tilde{\gamma}$ the condition (i) is satisfied. ■

Part 3. Efficiency and Pareto efficiency

(see Part 3. in B.6.)

2

A Dynamic Process of Free Trade Agreements Formation

2.1 Introduction

The Doha Development Round, the current (multilateral) trade-negotiation round of the World Trade Organization (WTO), started in November 2001. Its objective was to lower trade barriers around the world, which allows countries to increase trade globally. In July 2008, talks have stalled over a divide on major issues, such as agriculture, industrial tariffs and non-tariff barriers, services, and trade remedies. The most significant divergences have been between developed nations and the major developing countries. It is evident nowadays, that countries differences and needs are the main source of disagreement in every trade field.
It was said that the suspension of the Doha Round of WTO negotiations might not imply an increase in protectionism over the short term, or a collapse of the multilateral

trade system. It might encourage new regional trade agreements favouring the more advanced countries and damaging the poorest ones in particular (F. Steinberg, 2007).

Following the deadlock of multilateralism talks and the beginning of a new wave of regionalism arrangements, different issues about free trade negotiations arise. The evolution and the stability of bilateral agreements between countries of different size and characteristics and most important, the efficiency of the resulting free trade scenario - for each group of countries – need to be studied, for instance, by means of the well known network formation theory. Some scholars (e.g., Bhagwati (1993) and Levy (1997)) have questioned whether bilateral arrangements would lead to broader liberalization. Moreover, also attention has been given to the welfare effects of regional free trade associations and customs unions (see for example Krugman (1991)). Other works concerned the incentives of countries to form regional free trade arrangements, and the strategic stability of particular trading configurations. For instance, Krishna (1998) with a political economy approach noticed that trade-diverting preferential arrangements are more likely to be supported politically. Preferential arrangements could critically change domestic incentives such that multilateral liberalization - initially politically feasible - could turn into infeasible due to a previous preferential arrangement. On the other hand, Ornelas (2005) has used an oligopolistic political-economy model and he noticed that free trade agreements tend to enhance support for further liberalization at the multilateral level. Among the applications of network theory to these issues the paper of Goyal and Joshi (2006) has investigated the formation of free trade agreements as a network formation game. They have shown that the process of bilateral trade agreements can lead to a global free trade configuration. According to Furusawa and Konishi (2002) when all countries are symmetric, the global free trade network is stable but if countries are asymmetric in the market size and/or in the size of industrial good sector, the global free trade network may not be reached.

In this chapter the evolution of free trade agreements process among countries and the efficiency of the resulting free trade scenario have been studied by means of the network formation theory. We analyze the efficiency of the outcome of a network formation game in which nodes are represented by countries and links by free trade agreements. The process through which links are formed differs from the one used in the first chapter. In this chapter we investigate the bilateral free trade agreement formation using a dynamic process. The stability concept, then, drawn from Watts (2001) is a dynamic concept as well: the stable state network.

As in the first chapter, we model the network formation game assuming the existence of just three countries. Firms' payoff functions in each country depend on equilibrium quantities and tariffs of the oligopolistic intra-industry trade model as the one in Krishna (1998). The payoff functions are represented by firm profits. Profits depend crucially on the number of firms competing in the country (and so belonging to the arrangements) and on the market size of the own and of the other countries. The total welfare is represented by a value function depending on the network architecture. We analyzed in which direction an increase in the number of links may lead the overall country incentives and the total welfare. In other words, we investigated whether allowing countries to build links pushes towards a further liberalization (global free trade) and to an increase in welfare (efficiency). In our model the value function is not component additive and the allocation rule consequently not component balanced. For this reason, externalities across and inside components are present and therefore we cannot use the concept of constrained efficiency. Moreover, the allocation rule is fixed and the value of each network configuration cannot be freely distributed among nodes, therefore, in order to evaluate the outcomes of the network formation process, we use the notion of Pareto efficiency.

We found out that whenever we assume that countries are symmetric both in market sizes and number of firms we always obtain that the stable network configuration is also the efficient one. On the other hand, if we assume a degree of asymmetry among countries we end up with a network configuration that is not the efficient one and that leads to a network architecture in which bigger countries are better off and small countries are worse off.
The chapter proceeds as follows. In the next section we introduce some important definitions on network games. In section 2.3 we develop the model, examining first the factors influencing countries' payoff functions. In section 2.4 we show the results on network stability and efficiency in the symmetric and asymmetric settings. In section 2.5 we conclude with some remarks.

2.2 Definitions

In this section we give a brief overview of the principal tools we need to proceed in the analysis.

2.2.1 Networks, value functions and allocation rules

A network g is a list of unordered pairs of players linked to each other. A network game is represented by a pair (N,v) where N represents the set of players and v is the *value function* that generates value for each network created linking players in N. The way this value is distributed among players in the network is defined by an *allocation rule*. Countries are represented by nodes of a graph and links indicate bilateral agreements between the countries. For instance, if $N=\{i,j,k\}$ then $g=\{ij\}$ is the network in which just nodes i and j are linked. Let $g+jk$ be the network derived by adding the link jk to network g; and $g-jk$ obtained by deleting it. Let $N(g)$ be the set of players who have at least one link in the network g. For every network g, $N(g)=\{i \mid \exists j \text{ s.t. } ij \in g\}$.
Network structure determines the overall utility or productivity of the society and of players. The study of network efficiency is based on the comparison of different *values* that different network architectures create. The means for evaluating the overall value generated by a specific network and the way in which this value is distributed among players are represented by the *value function* and the *allocation rule*. The value function derives directly from the characteristic function and the allocation rule from the imputation rule in cooperative game theory. In cooperative game theory those two concepts depend only by the set of player in the coalition. In the network approach these concepts depend on the global structure of the network rather than on a coalition. The possible valuations are identified through the use of the function $v: G \to \mathbb{R}$. In our model the normalization $v(\varnothing) \neq 0$ is not maintained. The value function can include costs and benefits to links. The network can be characterized by externalities within and across component of a network.

A value function is *component additive* if $v(g)=\sum_{g'\in C(g)} v(g')$[58] for all $g \in G$. With a component additive value function the value of a network is simply the sum of the value of its components. In this way the possibility of externalities across components is ruled out because this characteristic implies that the value of one component doesn't depend on the structure of the other components. A value function is *anonymous* if, for every permutation of the set of players π we have that $v(g^{\pi})=v(g)$ with $g^{\pi}=\{\pi(i)\pi(j) \mid ij \in g\}$. The network g^{π} is a network with the same structure as g but in which the set of nodes is relabelled according to π. *An allocation rule* is a function $Y: G \times v \rightarrow \mathbb{R}^N \;\; s.t. \sum_i y_i(g,v)=v(g)$ for all v and g. When a value function is component additive the allocation rule is often component balanced, Y can be arbitrary otherwise. An allocation rule is *component balanced* if for any component additive $v, g \in G, \text{ and } g' \in C(g) \quad \sum_{i \in N(g')} Y_i(g',v)=v(g')$. This concept implies that the value generated by any component is allocated to the players among that component. An allocation rule is a*nonymous* if for every permutation of the set of players $\pi, v \in V, \text{ and } g \in G \;:\; Y_{\pi(i)}(g^{\pi}, v^{\pi})=Y_i(g,v)$. That is, the allocation only changes according to the relabeling.

2.2.3 The Compatibility of Efficiency and Stability

The concept of efficiency has been defined in different ways, differing mainly on the applicability to different settings and economic situations. The basic concept is that of Pareto Efficiency. In this case, a network is Pareto efficient if there does not exist any network that leads to higher payoffs for all members of the society. Formally, a network g is *Pareto efficient* relative to v and Y if there does not exist any $g' \in G$ such that $Y_i(g',v) \geq Y_i(g,v)$ for all i with strict inequality for some i.

A second and stronger efficiency notion is based on the overall network payoff. It is assumed that value is fully transferable. A network g is *efficient* relative to v if $v(g) \geq v(g')$ for all $g' \in G$.

The first definition of efficiency applies for a given allocation rule (where transfers among players are not possible). The second concept applies in situations where any Y can be redistributed in arbitrary ways. The relationship between the two definitions may be understood in this way: A network g is efficient relative to v if it is Pareto efficient relative to v and Y for all Y. A third concept of efficiency is the *constrained efficiency*. This concept applies in situations where the value is not fully transferable but the allocation rule is component balanced and anonymous. For this reason it can be collocated between the two previous concepts.

A network g is *constrained efficient* relative to v if there does not exist any $g' \in G$ and a component balanced and anonymous Y such that $Y_i(g',v) \geq Y_i(g,v)$ for all i with strict inequality for some i.

[58] $C(g)$ represents the set of components of a network g.

In our model the value function is not component additive and the allocation rule consequently not component balanced. This means that we find externalities across and inside components and that we cannot use the concept of constrained efficiency. Indeed, we are going to use the notion of Pareto efficiency because the network value is also not fully transferable among nodes and we have a specific allocation rule.
As Jackson (2003) pointed out in his work on stability and efficiency, the relationship between the stability and efficiency of networks is *context dependent*. That is, they are not always compatible, but are compatible for certain classes of value functions and allocation rules. Individual incentives might not lead to overall efficiency because of the existence of externalities intra and inter networks. Sometimes individual payoff depends also on the position of each player in the network and not only on what value the network generates. For instance, in a Star network the hub node may have not incentives to move to another configuration but the overall efficiency may be lower than other network architectures. In this case the source of inefficiency comes from the bargaining power generated by the position in the network. In general, it has been noticed that,

> "There does not exist any component balanced and anonymous allocation rule (or even a component balanced rule satisfying equal treatment of equals) such that for every v there exists a constrained efficient network that is pairwise stable" (Jackson, 2003).

Whereas, if there is complete control over the allocation rule and not component balance, then to guarantee that all efficient networks are also pairwise stable it simply necessary to use an egalitarian allocation rule.

2.2.4. Dynamic network formation process and efficiency

We employ a dynamic framework in which networks are formed over time as the one used by Watts (2001) and Jackson and Watts (2002), in the context of the symmetric connections model. According to Watts' findings the formation process is path dependent and the process often converges to an *inefficient* network structure.
The concept of stability we are going to use is related to the dynamic process. In other words, a network is pairwise stable if and only if it has no improving paths emanating from it. An *improving path* is a sequence of network in which each network g_k is defeated by the subsequent network g_{k+1}. A network is pairwise stable then if and only if it is not defeated[59] by an (adjacent) network. An important difference with respect the previous definitions of stability we used is that in the latter, agents are allowed to simultaneously sever any of their existing links. That is, a network is stable if no player wants to sever a direct link and any two players both want to link each other and *simultaneously sever any other existing links*.

[59] A network g' is adjacent to a network g if $g' = g + ij$ or $g' = g - ij$ for some ij.
A network g' defeats another network g if either $g' = g - ij$ and $Y_i(g',v) > Y_i(g,v)$, or if $g' = g + ij$ with $Y_i(g',v) \geq Y_i(g,v), Y_j(g',v) \geq Y_j(g,v)$, and at least one inequality holding strictly.

Watts' dynamic process can be resumed in the following way. The sequence of networks begins with empty network because agents are initially unconnected to each other. Players meet over time and have the opportunity to form links. Time, T, is divided into periods and is modelled as a countable, infinite set, $T = \{1, 2, ..., t, ...\}$. In each period, a link ij is randomly identified to be updated with uniform probability. If the link ij already belongs to g_{t-1} then either player i or j can decide to sever the link. If $ij \notin g_{t-1}$, then players i and j can form link ij and simultaneously sever any of their other links if both players agree. Agents are myopic and thus decide to form or to sever links if doing so increases their current payoff. That is, a player decides whether or not to sever a link or to form a link (with corresponding severances) based on whether or not severing or forming a link will increase his period t payoff. If after some period t, no additional links are formed or broken, then the network formation process has reached a *stable state*. A stable state is any pairwise stable network that can be reached by an improving path from the empty network. The limit of this model consists in the fact that in some situations one can get stuck at the empty network because any single link results in a lower value even though the larger networks may be valuable. Moreover, if it could be possible to start at any network, then, any pairwise stable network could be reached by an improving path.

2.3 The Model

This section is organized in the following way. We describe the model structure and then we focus on payoff functions and the factors that affect them, such as the market size of countries and the number of firms in each country.

2.3.1 The Model and payoff functions structure

Assume there are three countries 1, 2 and 3. The payoff of each country depends on the network configuration. Let $g^j \in g(N)$ be the realized network architecture and g the set of all possible network configurations given the number of countries N. Since we start with the simple case of symmetric countries, we can limit our study to networks differing only by the number of links. Roughly speaking, we are not interested now in who is linked with whom.

The payoff of the player i belonging to the network g^j is represented by $\phi^j(g^j)$. The possible network configurations are four, the empty network g^e, the one-link network g^1, the star network g^S and the complete network g^C.

In each country i there are n_i symmetric firms. Countries' payoffs are given by the sum of the profits realized by each firm of the country selling in the own market and in foreign markets. The level of profits depends on the presence of tariffs. The absence of tariffs between two countries is represented graphically by a link between two nodes. That is, every link is a free trade agreement between the two nodes. If tariffs are present, on one hand, the quantity sold in each foreign country is lower and on the other hand the quantity sold in the domestic market is higher because there is very low competition coming from foreign firms (they don't have free access to the domestic market). Thus, tariffs represent a trade-off between market share in domestic and foreign countries. We have already shown that if countries are symmetric the abolition of economic barriers gives higher profits because the free access in foreign markets offsets the negative effect of increased competition in the domestic market. We are now interested in the efficiency of each network structure. That is, whether a realized configuration of free trade agreements, providing is absent any profitable deviation from it, leads to a high total value for the overall network society.

The payoff of country i belonging to network g is represented by:

(1) $$\phi^i(g) = n_i\left[\sum_{j=1}^{N}\pi_j^i\right]$$

(2) $$\phi^i(g) = n_i\left[\left(q_1^i\right)^2 + \left(q_2^i\right)^2 + \left(q_3^i\right)^2\right]$$

Where π_j^i indicates the profits of firm i in market j and 1, 2 and 3 the markets (countries), (Krishna 1998). Since we assume that firms competes *à la Cournot* we have:

(3) $$q_j^i = \left[\frac{A_j - c}{n+1} + \left(\frac{\sum_{k=1}^{N} n_k t_j^k}{n+1} - t_j^i\right)\right]$$

Eq. (3) represents the Nash-Cournot equilibrium quantity [60]of country i selling in market j, given the level of tariff, market size and the total number of firms $\left(n = \sum_i n_i\right)$.

2.3.2 Influence factors on symmetric and asymmetric payoffs

In this section we analyze the factors that affect payoffs in the symmetric and asymmetric case. The first network configuration $\left(g^e\right)$ represents the case of overall protectionism. Let $n = n_1 + n_2 + n_3$ be the total number of firms and $t_i^i = 0\ \forall i \in N$ with $t_j^i = t_j \underset{i \neq j}{\forall} i \notin N$ because we assume a uniform among countries. The market size of each country is defined as $A_j - c \equiv \alpha_j\ \ \forall j \in N$. Given the previous assumptions the payoff function for the empty network architecture turns into,

[60] For instance, the payoff of country 1 in the empty network will be:

$$\phi^1\left(g^e\right) = n_1\left[\left(q_1^1\left(g^e\right)\right)^2 + \left(q_2^1\left(g^e\right)\right)^2 + \left(q_3^1\left(g^e\right)\right)^2\right].$$

The payoffs for each country in all possible network architectures are illustrated in Appendix A.

(4) $$\phi^1(g^e)=n_1\left[\left(\frac{\alpha_1}{n+1}+\frac{n_2t_1+n_3t_1}{n+1}\right)^2+\left(\frac{\alpha_2}{n+1}+\frac{n_1t_2+n_3t_2}{n+1}-t_2^1\right)^2+\left(\frac{\alpha_3}{n+1}+\frac{n_1t_3+n_2t_3}{n+1}-t_3^1\right)^2\right]$$

Assume the market sizes of the three countries are the same but the number of firms is different. That is, $\alpha_1=\alpha_2=\alpha_3=\alpha$ with $t_1=t_2=t_3=t$ and $n_1>n_2>n_3$.

(4') $$\phi^1(g^e)=\frac{n_1}{(n+1)^2}\left[(\alpha+(n_2+n_3)t)^2+(\alpha-(n_2+1)t)^2+(\alpha-(n_3+1)t)^2\right]$$

From (4') differentiating with respect n_1 we obtain,

$$\frac{\partial\phi^1(g^e)}{\partial n_1}=\frac{(n_2+n_3+1-n_1)}{(n+1)^3}\left[(\alpha+(n_2+n_3)t)^2+(\alpha-(n_2+1)t)^2+(\alpha-(n_3+1)t)^2\right]$$

if $n_1<(n_2+n_3+1)$ *and* $\alpha>(n_2+1)t$ *then* $\frac{\partial\phi^1(g^e)}{\partial n_1}>0$

This result reflects the fact that under a protectionist setting, with a market size big enough but fixed, there exists a threshold value $\tilde{n}_1$ (number of domestic-firms) for which we have:

If $n_1=[0,\tilde{n}_1)$ *then* $\frac{\partial\phi^1(g^e)}{\partial n_1}>0$ every increase in the number of domestic firms will lead to a higher country's payoff;

If $n_1=[\tilde{n}_1,\infty)$ *then* $\frac{\partial\phi^1(g^e)}{\partial n_1}\leq 0$ any further increase in the number of domestic firms will reduce the payoff (*saturated market*).

Because countries are symmetric these conditions hold for the other two countries as well. This means that for the payoff being increasing in the number of domestic firm those other two conditions have to hold: $n_2<(n_1+n_3+1)$ and $n_3<(n_2+n_1+1)$.

The system of the three inequalities is always satisfied because it reduces to: $n<2n+3$. Thus, $n_1<\tilde{n}_1$, $n_2<\tilde{n}_2$, $n_3<\tilde{n}_3$ $\frac{\partial\phi^1(g^e)}{\partial n_1}>0$.

The number of firms of other countries also affects the country payoff. Define,

$$B\equiv n_1\left[(\alpha+(n_2+n_3)t)^2+(\alpha-(n_2+1)t)^2(\alpha-(n_3+1)t)^2\right];$$

$$B_{n_2}=(4n_1n_2+2n_1n_3+2)t^2;$$

$$B_{n_3}=(4n_1n_3+2n_1n_2+2)t^2;$$

$\frac{\partial\phi^1(g^e)}{\partial n_2}=\frac{B_{n_2}(n+1)-2[B]}{(n+1)^3}>0$ For values of α included in the interval:

$$\left(0<\alpha<\frac{4+2\sqrt{4+3[f(n_2)]}}{6}\right)\text{; Where } f'(n_2)>0\ .$$

$$\frac{\partial\phi^1(g^e)}{\partial n_2}=\frac{B_{n_2}(n+1)-2[B]}{(n+1)^3}<0\ \ for\ \ \alpha>\frac{4+2\sqrt{4+3[f(n_2)]}}{6}\ .$$

A similar result also occurs for country 3.

$$\frac{\partial \phi^1(g^e)}{\partial n_3} = \frac{B_{n_3}(n+1) - 2[B]}{(n+1)^3}$$

The intuition behind this result is the following. When the market size of the country is small enough (α belongs to the interval), an increase in the number of other countries' firms, makes the protectionist setting more valuable because it protects from potential harmful foreign competitors (since the country is one of relatively small size). On the other hand, when the market size is big enough, the increase on the number of other countries' firms makes the protectionist architecture no longer valuable.

Assume now that the market size of the three countries is different. In particular, assume that $\alpha_1 > \alpha_2 > \alpha_3$ and $t_1 = t_2 = t_3 = t$. Moreover, assume that $n_1 = n_2 = n_3 = n^*$ the number of firms in each country is equal to the same value $n*$.

The payoff for country 1 in the empty network architecture becomes:

(4'') $$\phi^1(g^e) = \frac{n^*}{(3n^*+1)^2}\left[(\alpha_1 + 2n^*t)^2 + (\alpha_2 - (n^*+1)t)^2 + (\alpha_3 - (n^*+1)t)^2\right]$$

From (4'') differentiating with respect α_1 we obtain,

$$\frac{\partial \phi^1(g^e)}{\partial \alpha_1} = \frac{n^*}{(3n^*+1)^2}\left[2\alpha_1 + 4n^*t\right] > 0$$

$$\frac{\partial \phi^1(g^e)}{\partial \alpha_2} = \frac{n^*}{(3n^*+1)^2}\left[2\alpha_2 - 2(n^*+1)t\right] < 0 \quad \text{if } n^* > \frac{(\alpha_2 - 1)}{t}$$

When the non-domestic market size increases, the protectionist payoff decreases (if the number of domestic firms is great enough). In other words, the protectionism strategy is less convenient because the domestic country has to renounce to have access to a market of bigger size (if the number of the domestic firms competing in the domestic market is high enough).

$$\frac{\partial \phi^1(g^e)}{\partial \alpha_3} = \frac{n^*}{(3n^*+1)^2}\left[2\alpha_3 - 2(n^*+1)t\right] < 0 \quad \text{if } n^* > \frac{(\alpha_3 - 1)}{t}$$

When the size of the smallest country increases, becoming closer to the other two, the negative effect on the domestic country's payoff (country 1 in this case) becomes stronger. That is, the greater is the distance among countries' sizes the lower is the value of the latter derivative with respect to the former, assuming that the two conditions on $n*$ are simultaneously satisfied. In other words, if $n* > (\alpha_2 - 1)/t$, the more the distance between α_2 and α_3 (and therefore between α_1 and α_3) the more the reduction in payoff function of country 1 (due to an increase in country 3 size) with respect an increase of country 2 size[61]. When there is significant market size distance among countries (strong asymmetry) the effect of an increase in the market size of the smallest one will lead to a stronger reduction in the biggest country size payoff (in the empty network). This result implies that strong asymmetry leads to higher protectionist payoffs.

[61] Given that $\alpha_1 > \alpha_2 > \alpha_3$, the previous result can be easily verified using $\frac{(\alpha_3 - 1)}{t} < \frac{(\alpha_2 - 1)}{t}$.

2.4 Results

In our model, drew on the dynamic framework of Watts (2001), every time a link is identified two nodes are activated. We can define the free trade formation game as a recurring game. In each period the stage game is played, that is, the free trade game. Every time a link is identified the set of players that can play changes. In a three player setting (starting from an empty network) players (when "activated") have the possibility to switch from one network architecture to the same sort of network structure, (for example the one-link architecture), just with a re-labelling operation. However, in our model – when countries are symmetric -we are interested only in the network structure and we don't take into account the node identities.

We consider two cases. First, a symmetric case in which each country has the same market size α and the same number of firms n. Second, an asymmetric case in which two countries have the same market size of α and one smaller country has market size of α_3. Using the study of the free trade agreements process and the evaluating network configurations through the Pareto efficiency concept we are able to analyse the tension between stability and efficiency in the free trade agreements formation game.

2.4.1 Symmetric countries

Assume that $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$, $t_1 = t_2 = t_3 = t$, and $n_1 = n_2 = n_3 = n$.

When countries are symmetric in both market size and number of firms the payoff functions are defined in the following way[62].

(Case 1)

$$\mathrm{a}_0 = \gamma\left[3\alpha^2 + 6nt^2 + 2t^2 - 4\alpha t\right];$$
$$\mathrm{b}_0 = \gamma\left[3\alpha^2 + 12n^2t^2 - 4\alpha nt + 8nt^2 - 4\alpha t + 2t^2\right];$$
$$\mathrm{b}_1 = \gamma\left[3\alpha^2 + 3n^2t^2 + 2\alpha nt + 2nt^2 - 2\alpha t + 2t^2\right];$$
$$\mathrm{c}_1 = \gamma\left[3\alpha^2 + 2n^2t^2 - 2\alpha nt + 2nt^2 - 2\alpha t + 2t^2\right];$$
$$\mathrm{c}_2 = \gamma\left[3\alpha^2 + 2n^2t^2 + 4\alpha nt\right];$$
$$\mathrm{d}_2 = \gamma\left[3\alpha^2\right];$$

- a_0 represents equations (7-8-9)[63], that is, the payoff of each player belonging to the empty network $\left(g^e\right)$ and therefore having no links (each player has no links);

- b_0 represents equation (12), that is, the payoff of each player belonging to a network architecture $\left(g^1\right)$ and having no links (when the other players have one each other);

[62] Where $\gamma = n/(3n+1)^2$.

[63] See appendix A.3 for the equations list.

- b_1 represents equation (10-11), that is, the payoff of each player facing a network architecture $\left(g^1\right)$ and having one link (when the non-partner player has no link);
- c_1 represents equations (13-15), that is, the payoff of each player belonging to a network architecture $\left(g^s\right)$ and having one link (be a "spoke");
- c_2 represents equation (14), that is, the payoff of each player facing a network architecture $\left(g^s\right)$ and having two links (be the "hub");
- d_2 represents equations (16-17-18), that is, the payoff of each player belonging to the complete network architecture $\left(g^C\right)$ and therefore having two links (each player is connected to each other).

The above relations between payoffs have been obtained studying a system of conditions[64] among outcomes of each country across every possible network position. From the analysis of the value function in each network configuration and from the payoff functions associated to it we obtained that,

1) If the size of countries is small and $\alpha < n/2$ (when countries are symmetric and small) the protectionism architecture (the empty network) gives always higher payoffs for each node. Therefore, the Pareto efficient network is the empty one.

2) If the size of countries is big and $\alpha > \left(10n^2+6n+1\right)/2(n+1)$ we have that the Pareto efficient network is the complete one.

Proposition 1. *If $\alpha < n/2$, then no links ever form. That is, when countries are symmetric and small the empty network is stable and Pareto efficient.*

Proof:

From the solution[65] of the inequalities system described in the Appendix B, we obtain that for $\alpha < n/2$ the payoffs order is the following:

$$a_0 > b_0 > b_1 > c_2 > c_1 > d_2$$

First, a network is a stable state (and a pairwise stable network) if it is not defeated by any (adjacent) network. The empty network has just one adjacent network, that is, the network ($g^1 = g^e + ij$). We know that, $y_i\left(g^e, v\right) = a_0 \textit{ and } y_i\left(g^1, v\right) = b_0 \textit{ or } b_1$ (depending on the position that the node i assumes in the new configuration). Because $b_1 < b_0 < a_0$ then $y_i\left(g^e, v\right) > y_i\left(g^1, v\right)\ \forall i \in N$. That is, The empty network is not defeated by any adjacent network. ■

[64] For an exhaustive explanation of the inequality system see Appendix B.
[65] Here we focus only on the two extreme cases:
$A)\ a_0 \le b_0 \le b_1 \le c_1 < c_2 \le d_2 \quad L)\ a_0 > b_0 > b_1 > c_2 > c_1 > d_2$.
In Appendix C have been described also the intermediate cases results.

Second, assume that the empty network is not Pareto efficient. Then there must exist a network $g^{'} \in G$ such that, $y_i\left(g^{'},v\right) \geq y_i\left(g^{e},v\right)$ *for every* i with strict inequality for some i.

Because $y_i\left(g^{e},v\right) = a_0$ *and* $a_0 > b_0 > b_1 > c_2 > c_1 > d_2$ then, $y_i\left(g^{'},v\right) < y_i\left(g^{e},v\right)$ *for every* $i \in N$ *and for any network* $g^{'} \in G$.■

So far we have shown that with small symmetric countries the protectionism architecture gives always higher payoffs. The stable network is the empty network. Besides that, if we avoid the possibility to simultaneously withdraw every previous agreements (e.g., for a long period), we can also conclude that starting from any network architecture the process might always be blocked because (given the payoffs order) there would not be any incentive to form new links at every starting configuration. Finally, the Pareto efficient network is the empty one.

Proposition 2. *If* $\alpha > \left(10n^2+6n+1\right)/2(n+1)$ *then every link forms and remains (no links are ever broken). That is, when countries are symmetric and big the complete network is stable and Pareto efficient.*

Proof:

From the solution of the inequalities system described in the Appendix B, we obtain that for $\alpha > \left(10n^2+6n+1\right)/2(n+1)$ the payoff order is the following:

$$a_0 \leq b_0 \leq b_1 \leq c_1 < c_2 < d_2$$

First, the complete network has one adjacent network, that is, the network ($g^S = g^C - ij$).

We know that,

$y_i\left(g^C,v\right) = d_2$ *and* $y_i\left(g^S,v\right) = c_1$ *or* c_2 (depending on the position that the node i assumes in the new configuration). Because $c_1 < c_2 < d_2$ then, $y_i\left(g^C,v\right) > y_i\left(g^S,v\right)\ \forall i \in N$. That is, The complete network is not defeated by any adjacent network.

Second, assume that the complete network is not Pareto efficient. Then there must exist a network $g^{'} \in G$ such that,

$y_i\left(g^{'},v\right) \geq y_i\left(g^C,v\right)$ *for every* i (with the inequality holding strictly for some i)

Because $y_i\left(g^C,v\right) = d_2$ *and* $a_0 \leq b_0 \leq b_1 \leq c_1 < c_2 < d_2$

Then,

$y_i\left(g^{'},v\right) < y_i\left(g^C,v\right)$ *for every* $i \in N$ *and for any network* $g^{'} \in G$.■

The efficient network is the complete one and it is also a pairwise stable network reached by an improving path emanating by the empty architecture. That is, there are always incentives to add new links. We can conclude that when countries are symmetric the efficient network is always the stable one. When countries are symmetric and their size big enough the complete network will be more valuable and it represents a stable state and the Pareto efficient network architecture. When countries' sizes are small enough we obtain the opposite result. The dynamic process may be blocked at the empty network architecture.

2.4.2 Asymmetric countries

In this section we analyse the case in which countries have different market sizes but the same number of firms. (Countries' payoffs in the asymmetric setting are displayed in Appendix A.4). We already noticed that,

$$\frac{\partial \phi^1(g^e)}{\partial \alpha_1} > 0 ; \frac{\partial \phi^1(g^e)}{\partial \alpha_2} < 0 \quad if \ n^* > \frac{(\alpha_2 - t)}{t} \text{ and } \frac{\partial \phi^1(g^e)}{\partial \alpha_3} < 0 \quad if \ n^* > \frac{(\alpha_3 - t)}{t}$$

When the non-domestic market size increases, the protectionist payoff decreases if the number of domestic firms is great enough. In other words, for country 1 it is not anymore convenient to renounce to the access of a foreign country if its market is bigger and if the number of the domestic firms competing in the domestic market is high.
This result is true for every position across the possible network structures.

$$\frac{\partial \phi^1(g^1)}{\partial \alpha_1} > 0 ; \frac{\partial \phi^1(g^1)}{\partial \alpha_2} > 0 \text{ and } \frac{\partial \phi^1(g^1)}{\partial \alpha_3} < 0 \ if \ n^* > \frac{\alpha_3 - t}{t}$$

We can observe that, for a given threshold value of n*, whenever the market size of the non-partner country increases, the country's payoff decreases. In general, an increase of the number of non-partner firms generates a gain in countries' payoffs; whereas an increase in the size generates a payoff reduction. It occurs because the country loses the access to a bigger market.
For the isolated node (in this case corresponding to the small country) in the one-link network we have,

$$\frac{\partial \phi^3(g^1)}{\partial \alpha_1} < 0 \quad if \ n^* > \frac{(\alpha_1 - t)}{2t} ; \frac{\partial \phi^3(g^1)}{\partial \alpha_2} < 0 \quad if \ n^* > \frac{(\alpha_2 - t)}{2t} \text{ and } \frac{\partial \phi^3(g^1)}{\partial \alpha_3} > 0$$

When the size of non-partner countries increases, as expected, the isolated node payoff decreases. When $\alpha_1 > \alpha_2 > \alpha_3$ $(or \ \alpha_1 = \alpha_2 > \alpha_3)$, country 3 gains to create a link with both county 1 and 2. On the other hand, because both country 1 and 2 sizes are bigger their payoffs increase and the incentives to link to country 3 reduce.
The effects on payoffs due to an increase in countries' market sizes considering the star network architecture and the spokes countries' payoffs are the followings:

$$\frac{\partial \phi^1(g^S)}{\partial \alpha_1} > 0 ; \frac{\partial \phi^1(g^S)}{\partial \alpha_2} > 0 \text{ and } \frac{\partial \phi^1(g^S)}{\partial \alpha_3} < 0 \ if \ n^* > \frac{(\alpha_3 - t)}{2t}$$

For the hub country,

$$\frac{\partial \phi^2(g^S)}{\partial \alpha_1} > 0 ; \frac{\partial \phi^2(g^S)}{\partial \alpha_2} > 0 \text{ and } \frac{\partial \phi^2(g^S)}{\partial \alpha_3} > 0$$

In the star configuration, spoke countries will see a reduction in their payoffs if the non-partner country size increases. If we assume that one of the two spoke countries has a smaller market size, in the star network architecture, the payoff of the bigger spoke country will be higher and the hub country's payoff lower. We expect to see (depending on the degree of payoff reduction) the hub node severing one of the two links (star configuration instability) or at least a reduction in the incentive towards the complete network configuration (due to the increase in the bigger spoke country's payoff).

Finally, if we examine payoffs in the complete network configuration we have,

$$\frac{\partial \phi^1(g^C)}{\partial \alpha_1} > 0 ; \frac{\partial \phi^1(g^C)}{\partial \alpha_2} > 0 \text{ and } \frac{\partial \phi^1(g^C)}{\partial \alpha_3} > 0$$

In the global free trade scenario, an increase in the other countries size will always increase the payoff. This result implies that when $\alpha_1 > \alpha_2 > \alpha_3$ $(or\ \alpha_1 = \alpha_2 > \alpha_3)$ the small country will be better off in the asymmetric solution with respect to a symmetric one because is linked to countries with bigger market sizes.

2.4.2.1 Efficiency and Asymmetry

Assume that $n^* = \alpha_1 = \alpha_2 = \alpha$ and $\alpha > \alpha_3$. That is, country 3 has the smallest market size and the number of firms in each country is greater than the threshold value previously described. Countries payoffs in the asymmetric case are illustrated in the following way,

(Case 2)

- $\mathrm{a} = \lambda\left[9\alpha^2 + 1 + (\alpha_3 - \alpha - 1)^2\right]$, $\phi^1(g^e) = \phi^2(g^e)$ where $g^e = \{\varnothing\}$;
- $\mathrm{a}_3 = \lambda\left[\alpha_3^2 + 4\alpha^2 + 4\alpha_3\alpha + 2\right]$, $\phi^3(g^e)$ where $g^e = \{\varnothing\}$;
- $\mathrm{b} = \lambda\left[8\alpha^2 + (\alpha_3 - \alpha - 1)^2\right]$, $\phi^1(g^1) = \phi^2(g^1)$ where $g^1 = \{12\}$;
- $\mathrm{b}_3 = \lambda\left[\alpha_3^2 + 4\alpha^2 + 4\alpha_3\alpha + 2(\alpha^2 + 1 + 2\alpha)\right]$, $\phi^3(g^1)$ where $g^1 = \{12\}$;
- $\mathrm{c}_1 = \lambda\left[5\alpha^2 + (\alpha_3 - 2\alpha - 1)^2\right]$, $\phi^1(g^s)$ where $g^S = \{12, 23\}$;
- $\mathrm{c}_2 = \lambda\left[5\alpha^2 + \alpha_3^2 + \alpha^2 + 2\alpha_3\alpha\right]$, $\phi^2(g^s)$ where $g^S = \{12, 23\}$;
- $\mathrm{c}_3 = \lambda\left[\alpha^2 + \alpha_3^2 + \alpha^2 + 2\alpha_3\alpha + \alpha^2 + 1 + 2\alpha\right]$, $\phi^3(g^s)$ where $g^S = \{12, 23\}$;
- $\mathrm{d} = \lambda\left[2\alpha^2 + \alpha_3^2\right]; \phi^1(g^C) = \phi^2(g^C) = \phi^3(g^C)$ where $g^C = \{12, 23, 13\}$;
- $\lambda = \alpha/(3\alpha + 1)^2$.

We first provide some remarks on payoffs for each network architecture in the asymmetric case. In the empty network, the small country's payoff is always lower than the payoff of the bigger countries $(a > a_3)$. Moreover, bigger countries' payoffs in the empty network (eq. (7') and (8') in the Appendix A.4) are always greater than the one-link ones ((10') and (11')) implying that there are no incentives to switch to the one-link configuration in which the two bigger countries are linked. On the other hand, the small country has always incentives to switch from the empty network to the one-link network.

So far we have assumed that the only link formed in the one-link network configuration is between the two bigger countries. Consider the one-link configuration formed by two asymmetric countries linked together. The payoff functions in the one-link configuration turn in to:

10") $$\mathrm{b}_1 = \phi^1(g^1) = \frac{\alpha}{(3\alpha + 1)^2}\left[\alpha^2 + (\alpha_3 - \alpha - 1)^2\right]$$

11") $$\mathrm{b}_2 = \phi^2(g^1) = \frac{\alpha}{(3\alpha + 1)^2}\left[9\alpha^2 + (\alpha_3 - 2\alpha - 1)^2 + (-\alpha - 1)^2\right]$$

12") $$\mathrm{b}_3^{'} = \phi^3(g^1) = \frac{\alpha}{(3\alpha + 1)^2}\left[4\alpha^2 + (\alpha_3 + \alpha)^2 + (-\alpha - 1)^2\right]$$

The overall result is unchanged. In the empty network configuration, country 1, (zero-degree in the empty network and one-degree in the one-link configuration), will still have no incentives to form a link with country 3. On the other hand, country 3 (switching from zero-degree to one-degree) and 2 (zero-degree in both cases) will be better off in the one link architecture $g^1 = \{13\}$.

Assume also that in the star configuration the hub node is represented by the small country. Countries' payoffs turn in to:

13") $$c_1^{'} = c_2^{'} = \phi^1\left(g^{S^{'}}\right) = \frac{\alpha}{(3\alpha+1)^2}\left[5\alpha^2 + \alpha_3^2 + 2\alpha + 1\right]$$

15") $$c_3^{'} = \phi^3\left(g^{S^{'}}\right) = \frac{\alpha}{(3\alpha+1)^2}\left[8\alpha^2 + \alpha_3^2\right]$$

In this case, country 1, in spoke position (one-degree in the star network), would prefer the network architecture $g^1 = \{12\}$ instead of $g^S = \{13,23\}$ (i.e., $\left(b > c_1^{'}\right)$). The small country in hub position is better off in the star architecture if the asymmetry with respect to countries 1 and 2 is strong enough. This result derives from the fact that $\left(c_3^{'} > b_3^{'}\right)$ if $\alpha > 2\alpha_3 + 2$. However because $c_2^{'} < b_2$, country 2 (zero degree in the one-link network) will never link to the small country and the star configuration with the small country in hub position will never form.

So far the results are the followings,

- Bigger countries in the empty network are always better off than in the one link one, because $(a > a_3)$, and $(a > b)$ where payoff b refers to the network $g^1 = \{12\}$;
- Whenever the one-link network is formed there are no incentives for country 1 to switch from $g^1 = \{12\}$ to the star configuration $g^S = \{12,23\}$ (to a spoke position);
- Country 2 will have no incentives as well if $\alpha_3 < \alpha/2$, that is , if country 3 is very small;
- Country 3 will be worse off in the star configuration than in the one-link configuration because the payoffs order is $(b > c_1);(b > c_2 \text{ if } a_3 \le a/2);(b_3 > c_3)$ where c_1, c_2, c_3 refer to $g^S = \{12,23\}$;
- There are no incentives to switch from $g^S = \{12,23\}$ to $g^C = \{12,23,13\}$ because $(c_1 > d);(c_2 > d);(c_3 > d)$.

The overall results don't change if we allow asymmetric countries to form links between each other:

- Country 1 will be better off in the empty network with respect to the one-link configuration because $(a > b_1);(a < b_2);(a_3 < b_3)$ where $b_1, b_2, b_3^{'}$ refer to the network $g^1 = \{13\}$;
- Whenever the one-link network is formed between two asymmetric countries there are no incentives for country 2 to switch from $g^1 = \{13\}$ to the star configuration $g^S = \{13,23\}$ (to a spoke position);
- Finally, as in the previous case, there are no incentives to switch from $g^S = \{13,23\}$ to the complete configuration because the payoffs structure is $\left(c_1^{'} > d\right);\left(c_2^{'} > d\right);\left(c_3^{'} > d\right)$.

Proposition 3 *if* $\alpha > 3\alpha_3$ *, then no links ever form. When countries are asymmetric the empty network is stable but not Pareto efficient.*

Proof:

First, from the analysis of the payoffs resulting in all network architectures, the payoffs order is: $d < c_3 < a_3 < c_2 < b_3 < c_1 < b < a$ if $\alpha > 3\alpha_3$

The empty network has one adjacent network, that is, the network ($g^1 = g^e + ij$). We know that, if $i = \{1,2\}$ and the one link configuration is formed by two big countries we have,

$y_i(g^e, v) = a$ *and* $y_i(g^1, v) = b$. Because $b < a$ then $y_i(g^e, v) > y_i(g^1, v)$ $\forall i = \{1,2\}$.

Whenever $i = \{3\}$ and the one link configuration is formed by two asymmetric countries we have, $y_i(g^e, v) = a_3$ *and* $y_i(g^1, v) = b_3^{'}$.

Because $(a > b_1), (a < b_2), (a_3 < b_3^{'})$ - where $b_1, b_2, b_3^{'}$ refer to the network $g^1 = \{13\}$ - then, even if $y_i(g^e, v) < y_i(g^1, v)$ we have that $y_j(g^e, v) > y_j(g^1, v)$ $\forall j = \{1,2\}$ and then no links between asymmetric countries will be formed. This means that the empty network is not defeated by any adjacent network in both cases. ■

Second, assume that the empty network is Pareto efficient. Then there must not exist a network $g^{'} \in G$ such that,

$y_i(g^{'}, v) \geq y_i(g^e, v)$ *for every* i (with strict inequality for some i)

Because $y_i(g^e, v) = a$ for big countries and a_3 for the small country and $d < c_3 < a_3 < c_2 < b_3 < c_1 < b < a$ if $\alpha > 3\alpha_3$.

Then,

$y_i(g^{'}, v) < y_i(g^e, v)$ *for* $i = \{1,2\}$ but not for every i and g, because for $y_i(g^{'}, v) = b_3$ (g^1) we have $y_i(g^{'}, v) > y_i(g^e, v)$.

It can be shown that this is true for every network configuration because the bigger countries' payoffs (belonging to any network architecture except the empty one) will be always lower than the one in the empty configuration. ■

In an asymmetric setting in which there are two large countries (of the same market size) and a small one, if the latter is small enough there are no incentives to form free trade agreements and the stable network architecture is not Pareto efficient. Furthermore, starting from any network architecture the process is always blocked.

2.5 Concluding remarks

Over the course of the negotiations at the Doha round, the influence of groups that want to protect themselves from foreign competition (mainly farmers in rich countries and

manufacturers in emerging ones) seems to have been greater than that of those who sought more integration. This made it impossible to strike a deal acceptable for the main countries of the WTO (F. Steinberg, 2007).
In this chapter we have analyzed the efficiency and stability of the bilateral free trade agreement formation in a dynamic network formation game setting. Making use of a dynamic formation process and the Pareto efficiency concept we have described the relationship between stability and efficiency in a symmetric and asymmetric countries environment. We considered two cases. First, a symmetric case in which each country has the same market size α and the same number of firms n. Second, an asymmetric case in which two countries have bigger market sizes α and one smaller market size α_3.
From the analysis of the value functions of each network configuration and from the associated payoff functions we obtained the following results on Pareto efficiency:

1) If the size of countries is small and $\alpha < n/2$ (when countries are symmetric and small) the protectionism architecture (the empty network) gives always higher payoffs for each node. Therefore, the Pareto efficient network is the empty one.
2) If $\alpha > \left(10n^2+6n+1\right)/2(n+1)$ we have that the Pareto efficient network is the complete one.
3) Assuming that $n^* = \alpha_1 = \alpha_2 = \alpha$ and $\alpha > \alpha_3$ (countries are asymmetric) we found out that the empty network is not Pareto efficient.

From the study of the dynamic process of free trade agreement formation together with the efficiency analysis we found that,

Proposition 1. *If $\alpha < n/2$, then no links ever form. That is, when countries are symmetric and small the empty network is stable and Pareto efficient.*
Proposition 2. *If $\alpha > \left(10n^2+6n+1\right)/2(n+1)$ then every link forms and remains (no links are ever broken). That is, when countries are symmetric and big the complete network is stable and Pareto efficient.*
Proposition 3. *if $\alpha > 3\alpha_3$, then no links ever form. With symmetric countries the empty network is stable but not Pareto efficient.*

Thus, whenever we assume that countries are symmetric both in market sizes and number of firms we always obtain that the stable network configuration is also the Pareto efficient one. On the other hand, if we assume a degree of asymmetry among countries we end up with a network configuration that is not the efficient one and that leads to a network architecture in which bigger countries are better off and small countries are worse off.

2.6 Appendix A

2.6.1 Appendix (A.1): Payoff functions structure

The payoffs associated to the empty network for each country are the followings,

$$(4)\quad \phi^1\left(g^e\right)=n_1\left[\left(\frac{A_1-c}{n+1}+\frac{n_1t_1^1+n_2t_1^2+n_3t_1^3}{n+1}-t_1^1\right)^2+\left(\frac{A_2-c}{n+1}+\frac{n_1t_2^1+n_2t_2^2+n_3t_2^3}{n+1}-t_2^1\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1+n_2t_3^2+n_3t_3^3}{n+1}-t_3^1\right)^2\right]$$

$$(5)\quad \phi^2\left(g^e\right)=n_2\left[\left(\frac{A_2-c}{n+1}+\frac{n_1t_2^1+n_2t_2^2+n_3t_2^3}{n+1}-t_2^2\right)^2+\left(\frac{A_1-c}{n+1}+\frac{n_1t_1^1+n_2t_1^2+n_3t_1^3}{n+1}-t_1^2\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1+n_2t_3^2+n_3t_3^3}{n+1}-t_3^2\right)^2\right]$$

$$(6)\quad \phi^3\left(g^e\right)=n_3\left[\left(\frac{A_1-c}{n+1}+\frac{n_1t_1^1+n_2t_1^2+n_3t_1^3}{n+1}-t_1^3\right)^2+\left(\frac{A_2-c}{n+1}+\frac{n_1t_2^1+n_2t_2^2+n_3t_2^3}{n+1}-t_2^3\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1+n_2t_3^2+n_3t_3^3}{n+1}-t_3^3\right)^2\right]$$

The payoffs in the one-link network for each country are the followings,

$$(7)\quad \phi^1\left(g^1\right)=n_1\left[\left(\frac{A_1-c}{n+1}+\frac{n_3t_1^3}{n+1}\right)^2+\left(\frac{A_2-c}{n+1}+\frac{n_3t_2^3}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1+n_2t_3^2}{n+1}-t_3^1\right)^2\right]$$

$$(8)\quad \phi^2\left(g^1\right)=n_2\left[\left(\frac{A_2-c}{n+1}+\frac{n_3t_2^3}{n+1}\right)^2+\left(\frac{A_1-c}{n+1}+\frac{n_3t_1^3}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1+n_2t_3^2}{n+1}-t_3^2\right)^2\right]$$

$$(9)\quad \phi^3\left(g^1\right)=n_3\left[\left(\frac{A_1-c}{n+1}+\frac{n_3t_1^3}{n+1}-t_1^3\right)^2+\left(\frac{A_2-c}{n+1}+\frac{n_3t_2^3}{n+1}-t_2^3\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1+n_2t_3^2}{n+1}\right)^2\right]$$

The payoffs in the star network:

$$(10)\quad \phi^1\left(g^s\right)=n_1\left[\left(\frac{A_1-c}{n+1}+\frac{n_3t_1^3}{n+1}\right)^2+\left(\frac{A_2-c}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1}{n+1}-t_3^1\right)^2\right]$$

$$(11)\quad \phi^2\left(g^s\right)=n_2\left[\left(\frac{A_2-c}{n+1}\right)^2+\left(\frac{A_1-c}{n+1}+\frac{n_3t_1^3}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1}{n+1}\right)^2\right]$$

$$(12)\quad \phi^3\left(g^s\right)=n_3\left[\left(\frac{A_1-c}{n+1}+\frac{n_3t_1^3}{n+1}-t_1^3\right)^2+\left(\frac{A_2-c}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}+\frac{n_1t_3^1}{n+1}\right)^2\right]$$

The payoffs in the complete network are:

$$(13)\quad \phi^1\left(g^c\right)=n_1\left[\left(\frac{A_1-c}{n+1}\right)^2+\left(\frac{A_2-c}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}\right)^2\right]$$

$$(14)\quad \phi^2\left(g^c\right)=n_2\left[\left(\frac{A_2-c}{n+1}\right)^2+\left(\frac{A_1-c}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}\right)^2\right]$$

(15) $$\phi^3\left(g^c\right)=n_3\left[\left(\frac{A_1-c}{n+1}\right)^2+\left(\frac{A_2-c}{n+1}\right)^2+\left(\frac{A_3-c}{n+1}\right)^2\right]$$

2.6.2 Appendix (A.2): Symmetric payoff functions

The payoff functions for the empty network architecture assuming,

- $n = n_1 + n_2 + n_3$
- $t_i^i = 0 \ \forall i \in N$
- $t_j^i = t_j \underset{i \neq j}{\forall} i \notin N$ (uniform tariff assumption)
- $A_j - c \equiv \alpha_j \forall j \in N$

(5') $$\phi^2\left(g^e\right)=n_2\left[\left(\frac{\alpha_2}{n+1}+\frac{n_1t_2+n_3t_2}{n+1}\right)^2+\left(\frac{\alpha_1}{n+1}+\frac{n_2t_1+n_3t_1}{n+1}-t_1^2\right)^2+\left(\frac{\alpha_3}{n+1}+\frac{n_1t_3+n_2t_3}{n+1}-t_3^2\right)^2\right]$$

(6') $$\phi^3\left(g^e\right)=n_3\left[\left(\frac{\alpha_1}{n+1}+\frac{n_2t_1+n_3t_1}{n+1}-t_1^3\right)^2+\left(\frac{\alpha_2}{n+1}+\frac{n_1t_2+n_3t_2}{n+1}-t_2^3\right)^2+\left(\frac{\alpha_3}{n+1}+\frac{n_1t_3+n_2t_3}{n+1}\right)^2\right]$$

The payoff functions with this set of assumptions:

- $\alpha_1 = \alpha_2 = \alpha_3 = \alpha$
- $t_1 = t_2 = t_3 = t$
- $n_1 > n_2 > n_3$

(4'') $$\phi^1\left(g^e\right)=\frac{n_1}{(n+1)^2}\left[\left(\alpha+(n_2+n_3)t\right)^2+\left(\alpha-(n_2+1)t\right)^2+\left(\alpha-(n_3+1)t\right)^2\right]$$

(5'') $$\phi^2\left(g^e\right)=\frac{n_2}{(n+1)^2}\left[\left(\alpha+(n_1+n_3)t\right)^2+\left(\alpha-(n_1+1)t\right)^2+\left(\alpha-(n_3+1)t\right)^2\right]$$

(6'') $$\phi^3\left(g^e\right)=\frac{n_3}{(n+1)^2}\left[\left(\alpha+(n_2+n_1)t\right)^2+\left(\alpha-(n_1+1)t\right)^2+\left(\alpha-(n_2+1)t\right)^2\right]$$

2.6.3 Appendix (A.3): Asymmetric payoff functions

Given the following assumptions on market size, tariffs and number of firms,

- $\alpha_1 \neq \alpha_2 \neq \alpha_3$
- $t_1 = t_2 = t_3 = t$
- $n_1 = n_2 = n_3 = n^*$

The Payoff functions in each network configurations are the following:

<u>Empty network configuration</u>

7) $\phi^1(g^e)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_1+2n^*t)^2+(\alpha_2-(n^*+1)t)^2+(\alpha_3-(n^*+1)t)^2\right]$

8) $\phi^2(g^e)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_2+2n^*t)^2+(\alpha_1-(n^*+1)t)^2+(\alpha_3-(n^*+1)t)^2\right]$

9) $\phi^3(g^e)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_3+2n^*t)^2+(\alpha_2-(n^*+1)t)^2+(\alpha_1-(n^*+1)t)^2\right]$

<u>One link configuration</u>

10) $\phi^1(g^1)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_1+n^*t)^2+(\alpha_2+n^*t)^2+(\alpha_3-(n^*+1)t)^2\right]$

11) $\phi^2(g^1)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_2+n^*t)^2+(\alpha_1+n^*t)^2+(\alpha_3-(n^*+1)t)^2\right]$

12) $\phi^3(g^1)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_3+2n^*t)^2+(\alpha_2-(2n^*+1)t)^2+(\alpha_1-(2n^*+1)t)^2\right]$

<u>Star configuration</u>

13) $\phi^1(g^S)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_1+n^*t)^2+(\alpha_2)^2+(\alpha_3-(2n^*+1)t)^2\right]$

14) $\phi^2(g^S)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_2)^2+(\alpha_1+n^*t)^2+(\alpha_3+n^*t)^2\right]$

15) $\phi^3(g^S)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_3+n^*t)^2+(\alpha_2)^2+(\alpha_1-(2n^*+1)t)^2\right]$

<u>Complete configuration</u>

16) $\phi^1(g^C)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_1)^2+(\alpha_2)^2+(\alpha_3)^2\right]$

17) $\phi^2(g^C)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_2)^2+(\alpha_1)^2+(\alpha_3)^2\right]$

18) $\phi^3(g^C)=\dfrac{n^*}{(3n^*+1)^2}\left[(\alpha_3)^2+(\alpha_2)^2+(\alpha_1)^2\right]$

19) $\dfrac{\partial\phi^1(g^e)}{\partial\alpha_1}=\dfrac{n^*}{(3n^*+1)^2}\left[2\alpha_1+4n^*t\right]>0$

20) $\dfrac{\partial\phi^1(g^e)}{\partial\alpha_2}=\dfrac{n^*}{(3n^*+1)^2}\left[2\alpha_2-2(n^*+1)t\right]<0 \quad if\ n^*>\dfrac{(\alpha_2-t)}{t}$

21) $\dfrac{\partial\phi^1(g^e)}{\partial\alpha_3}=\dfrac{n^*}{(3n^*+1)^2}\left[2\alpha_3-2(n^*+1)t\right]<0 \quad if\ n^*>\dfrac{(\alpha_3-t)}{t}$

22) $\dfrac{\partial\phi^1(g^1)}{\partial\alpha_1}=\dfrac{n^*}{(3n^*+1)^2}\left[2(\alpha_1+n^*t)\right]>0$

23) $\frac{\partial \phi^1(g^1)}{\partial \alpha_2} = \frac{n^*}{(3n^*+1)^2}\left[2(\alpha_2 + n^* t)\right] > 0$

24) $\frac{\partial \phi^1(g^1)}{\partial \alpha_3} = \frac{n^*}{(3n^*+1)^2}\left[2(\alpha_3 - (n^*+1)t)\right] < 0 \text{ if } n^* > \frac{\alpha_3 - t}{t}$

25) $\frac{\partial \phi^3(g^1)}{\partial \alpha_1} = \frac{n^*}{(3n^*+1)^2}\left[2\alpha_1 - 2(2n^*+1)t\right] < 0 \quad \text{if } n^* > \frac{(\alpha_1 - t)}{2t}$

26) $\frac{\partial \phi^3(g^1)}{\partial \alpha_2} = \frac{n^*}{(3n^*+1)^2}\left[2\alpha_2 - 2(2n^*+1)t\right] < 0 \quad \text{if } n^* > \frac{(\alpha_2 - t)}{2t}$

27) $\frac{\partial \phi^3(g^1)}{\partial \alpha_3} = \frac{n^*}{(3n^*+1)^2}\left[2\alpha_3 + 4n^* t\right] > 0$

28) $\frac{\partial \phi^1(g^S)}{\partial \alpha_1} = \frac{n^*}{(3n^*+1)^2}\left[2(\alpha_1 + n^* t)\right] > 0$

29) $\frac{\partial \phi^1(g^S)}{\partial \alpha_2} = \frac{n^*}{(3n^*+1)^2}[2\alpha_2] > 0$

30) $\frac{\partial \phi^1(g^S)}{\partial \alpha_3} = \frac{n^*}{(3n^*+1)^2}\left[2\alpha_3 - 2(2n^*+1)t\right] < 0 \text{ if } n^* > \frac{(\alpha_3 - t)}{2t}$

31) $\frac{\partial \phi^2(g^S)}{\partial \alpha_1} = \frac{n^*}{(3n^*+1)^2}\left[2(\alpha_1 + n^* t)\right] > 0$

32) $\frac{\partial \phi^2(g^S)}{\partial \alpha_2} = \frac{n^*}{(3n^*+1)^2}[2\alpha_2] > 0$

33) $\frac{\partial \phi^2(g^S)}{\partial \alpha_3} = \frac{n^*}{(3n^*+1)^2}\left[2(\alpha_3 + n^* t)\right] > 0$

34) $\frac{\partial \phi^1(g^C)}{\partial \alpha_1} = \frac{n^*}{(3n^*+1)^2}[2\alpha_1] > 0$

35) $\frac{\partial \phi^1(g^C)}{\partial \alpha_2} = \frac{n^*}{(3n^*+1)^2}[2\alpha_2] > 0$

36) $\frac{\partial \phi^1(g^C)}{\partial \alpha_3} = \frac{n^*}{(3n^*+1)^2}[2\alpha_3] > 0$

With the following assumptions,

- $n^* = \alpha_2 = \alpha_1$
- $\alpha_1 = \alpha_2 > \alpha_3$

The payoff functions in each network configuration are:

<u>Empty configuration</u>

7') $\phi^1\left(g^e\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha+2\alpha t)^2+\left(\alpha-(\alpha+1)t\right)^2+\left(\alpha_3-(\alpha+1)t\right)^2\right]$

8') $\phi^2\left(g^e\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha+2\alpha t)^2+\left(\alpha-(\alpha+1)t\right)^2+\left(\alpha_3-(\alpha+1)t\right)^2\right]$

9') $\phi^3\left(g^e\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha_3+2\alpha t)^2+\left(\alpha-(\alpha+1)t\right)^2+\left(\alpha-(\alpha+1)t\right)^2\right]$

<u>One link configuration</u>

10') $\phi^1\left(g^1\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha+\alpha t)^2+(\alpha+\alpha t)^2+\left(\alpha_3-(\alpha+1)t\right)^2\right]$

11') $\phi^2\left(g^1\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha+\alpha t)^2+(\alpha+\alpha t)^2+\left(\alpha_3-(\alpha+1)t\right)^2\right]$

12') $\phi^3\left(g^1\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha_3+2\alpha t)^2+\left(\alpha-(2\alpha+1)t\right)^2+\left(\alpha-(2\alpha+1)t\right)^2\right]$

<u>Star configuration</u>

13') $\phi^1\left(g^S\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha+\alpha t)^2+(\alpha)^2+\left(\alpha_3-(2\alpha+1)t\right)^2\right]$

14') $\phi^2\left(g^S\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha)^2+(\alpha+\alpha t)^2+(\alpha_3+\alpha t)^2\right]$

15') $\phi^3\left(g^S\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha_3+\alpha t)^2+(\alpha)^2+\left(\alpha-(2\alpha+1)t\right)^2\right]$

<u>Complete configuration</u>

16') $\phi^1\left(g^C\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha)^2+(\alpha)^2+(\alpha_3)^2\right]$

17') $\phi^2\left(g^C\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha)^2+(\alpha)^2+(\alpha_3)^2\right]$

18') $\phi^3\left(g^C\right)=\frac{\alpha}{(3\alpha+1)^2}\left[(\alpha)^2+(\alpha)^2+(\alpha_3)^2\right]$

2.7 Appendix B

2.7.1 Appendix (B.1): The parametric system of inequalities

The relations between the following payoffs, $a_0 > c_1$ and $c_1 < c_2$, hold for any value of n. The other payoffs relations depend on some value conditions on the number of firms with respect to country sizes (or vice versa). In other words,

1. $a_0 > d_2 \quad \alpha < \frac{3n^2+2n+1}{2}; \; n > \frac{-2+\sqrt{24\alpha-8}}{6};$
2. $a_0 > b_1 \quad \alpha < \frac{3n^2+2n+1}{2(n+1)}; \; n > \frac{2(\alpha-1)+\sqrt{4\alpha^2+20\alpha-8}}{6};$
3. $a_0 > c_2 \quad \alpha < \frac{2n^2+2n+1}{2(n+1)}; \; n > \frac{2(\alpha-1)+\sqrt{4\alpha^2-24\alpha-8}}{4};$
4. $a_0 > b_0 \quad \alpha < \frac{3n}{2}+1; \; n > \frac{2\alpha-2}{3}$
5. $c_1 > d_2 \quad \alpha < \frac{2n^2+2n+1}{2(n+1)}; \; n > \frac{2(\alpha-1)+\sqrt{4\alpha^2-24\alpha-8}}{4};$
6. $b_0 > c_1 \quad \alpha < \frac{10n^2+6n+1}{2(n+1)}; \; n > \frac{2\alpha-6+\sqrt{4\alpha^2+56\alpha-4}}{20};$
7. $b_0 > c_2 \quad \alpha < \frac{5n^2+4n+1}{2(2n+1)}; \; n > \frac{4(\alpha-1)+\sqrt{16\alpha^2+8\alpha-4}}{10};$
8. $b_0 > d_2 \quad \alpha < \frac{6n^2+2n+1}{2(n+1)}; \; n > \frac{2(\alpha-1)+\sqrt{4\alpha^2+40\alpha-20}}{12};$
9. $b_1 > c_1 \quad \alpha < \frac{n^2}{4}; \; n > 2\sqrt{\alpha}$
10. $b_1 > c_2 \quad \alpha < \frac{n^2+2n+1}{2(n+1)}; \; n > 2\alpha-1$
11. $c_2 > d_2 \quad \alpha < \frac{n}{2}; \; n > 2\alpha$
12. $b_1 > d_2 \quad \alpha < \frac{3n^2+2n+1}{2(1-n)}; \; n > \frac{-2(\alpha+1)+\sqrt{4\alpha^2+32\alpha-8}}{6};$
13. $b_0 > b_1 \quad n > \frac{6(\alpha-1)+\sqrt{36\alpha^2+144\alpha}}{18}.$

Table 2.1 Complete inequalities system results (relationships between all the cross network payoffs)

(12)	(1)	(6)	(8)	(4)	(9)	(13)	(2)	(3)=	=(5)	(7)	(10)	(11)	
b1<d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2	b1>d2
ao<d2	ao<d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2	ao>d2
bo<c1	bo<c1	bo<c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1
bo<d2	bo<d2	bo<d2	bo<d2	bo>d2	bo>d2	bo>d2	bo>d2	bo>d2	bo>d2	bo>d2	bo>d2	bo>d2	bo>d2
ao<bo	ao<bo	ao<bo	ao<bo	ao<bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo
b1<c1	b1<c1	b1<c1	b1<c1	b1<c1	b1<c1	b1>c1	b1>c1	b1>c1	b1>c1	b1>c1	b1>c1	b1>c1	b1>c1
bo<b1	bo<b1	bo<b1	bo<b1	bo<b1	bo<b1	bo<b1	bo>b1	bo>b1	bo>b1	bo>b1	bo>b1	bo>b1	bo>b1
ao<b1	ao<b1	ao<b1	ao<b1	ao<b1	ao<b1	ao<b1	ao<b1	ao>b1	ao>b1	ao>b1	ao>b1	ao>b1	ao>b1
ao<c2	ao<c2	ao<c2	ao<c2	ao<c2	ao<c2	ao<c2	ao<c2	ao<c2	ao>c2	ao>c2	ao>c2	ao>c2	ao>c2
c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1>d2	c1>d2	c1>d2	c1>d2
bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo>c2	bo>c2	bo>c2
b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1>c2	b1>c2
c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2>d2
ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1	ao>c1
c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2

The table above represents the system of conditions among country payoffs across every possible network position. The relations between payoffs described by the red line are always verified. For all the others, the dotted line shows where the opposite is verified and the dark line shows where the condition (expressed by an equation number) is verified. We have a system of thirteen inequalities.
Table 2.1 shows the results also for cross-network payoffs. From now on, we will take in to account only the relations among adjacent-network payoffs (Table 2.2).

Tab.2.2 Reduced inequalities system results

(6)	(4)	(9)	(13)	(2)	(5)	(7)	(10)	(11)	
bo<c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1	bo>c1
ao<bo	ao<bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo	ao>bo
b1<c1	b1<c1	b1<c1	b1>c1	b1>c1	b1>c1	b1>c1	b1>c1	b1>c1	b1>c1
bo<b1	bo<b1	bo<b1	bo<b1	bo>b1	bo>b1	bo>b1	bo>b1	bo>b1	bo>b1
ao<b1	ao<b1	ao<b1	ao<b1	ao<b1	ao>b1	ao>b1	ao>b1	ao>b1	ao>b1
c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1<d2	c1>d2	c1>d2	c1>d2	c1>d2
bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo<c2	bo>c2	bo>c2	bo>c2
b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1<c2	b1>c2	b1>c2
c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2<d2	c2>d2
A	B	C	D	E	F	G	H	I	L
c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2	c1<c2

The result is represented by a nine inequalities system (Tab.2.2). We obtained nine possible configuration results. For each of them we have a condition on the number of firms and/or the size of the country market (because the parametric inequalities depend on the market size and the number of firms). The results system (L), for example, represents the case in which countries are small.

2.8 Appendix C

2.8.1 Appendix (C): Further results

Depending on the size of the countries we can have different solutions:

B) $a_0 \le b_1 = c_1 < b_0 \le c_2 \le d_2$

- Protectionism is still the worst configuration;

- There are no adverse effects losing the exclusive access to the partner country $(b_1 = c_1)$;
- The zero-degree vertex (the isolated one in g^1) has a bigger payoff than a one-link situation and having zero-degree in g^e;
- The hub position is better than the spoke and of every zero-degree position;
- Global free trade gives the best payoff configuration.

In this case g^1 is a stable emanating from two improving paths, one starting from g^e and the other from g^S. The network configuration g^C is a stable state emanating from the improving path starting from g^S and the *Pareto efficient* network configuration.

D) $c_1 < b_0 \le b_1 = a_0 \le c_2 \le d_2$

- The "spoke" position is the worst;
- Countries are indifferent between the protectionism situation and opening domestic market to one country when the rest of the world doesn't belong to the agreement;
- The isolated vertex in g^1 is worse off than to be a zero-degree vertex in g^e.This is due to the intra and inter network externalities deriving from the relationship between the isolation in g^e (where every country don't have market access to the other countries) and the isolation in g^1.
- To be isolated in g^1 is still better than to bear the (market) accession diversion to another country by the own partner as in the spoke position in g^S;
- The hub position gives a lower or equal payoff than the global free trade position. That is, (n-1)-degree vertex's payoff when the other countries are (n-1)-degree vertices is greater than (n-1)-degree vertex's payoff when the others are (n-2)-degree vertices. In some way, the homogenous behaviour gives higher payoffs;
- In this case g^C is the Pareto efficient network.

F) $c_1 < b_1 < b_0 \le c_2 \le d_2 < a_0$

- The spoke position is the worst;
- Because the protectionism outcome is the best one, we don't have any incentive to move from the empty network to the one-link network;
- An N-1 degree vertex is always better than any other situations except for the 0-degree vertex cases (both g^e and g^1 for the isolated node);
- In this case g^e and g^C are two stable states but only g^e is the Pareto efficient network.

3

Market Structure and Strategic Behaviour

3.1 Introduction

In last years, the structure of agriculture and food markets appears to be changing in fundamental ways. Usually the grocery market was composed by a very large number of buyers and/or sellers operating as small price takers, whereas, large buyers and sellers, each with significant power, drive the current market. It has been observed that a great number of mergers involving major retailers across Europe and cross-border mergers become increasingly common. According to studies on competition in food retail distribution sector developed for the European Commission the consolidation in food retailing emerged clearly as general feature across most EU countries. However, even if this tendency is sometimes associated with benefits deriving from improved efficiency and service, the increasing concentration brings concerns about the exercise of market power by retailers (buyers and/or sellers) and its consequence on welfare.
For instance, in Dobson (1999) it has been argued that, when efficiency benefits are not able to off-set the negative effect of concentration, seller power by retailers could result in higher prices for consumers and perhaps reduced choice. In general, the economic welfare effects arising from the exploitation of buyer power are ambiguous. In fact, suppliers will generally suffer if the prices obtained for their goods are reduced while

consumers might gain if lower upstream market prices result in retailers setting lower final consumers' prices. Although the net effect is not clear, there seems to be a general belief which maintains that retail concentration is bad for social welfare. To some extent, in economic theory this belief finds support. According to Dobson (1997), under particular conditions, an increase in retail concentration may result in a net social welfare loss. On the other hand, Dobson and Waterson (1998) show that even though retail concentration reduces competition at all stages of the marketing chain, it can generate productive efficiency benefits that enhance consumer welfare.

Moreover, recent literature emphasises the role of market structure in determining the degree of price transmission along the marketing chain

Some studies, as Weldegebriel (2004), model vertical price transmission allowing for both oligopoly (seller) power in the retail sector and oligopsony (buyer) power in the supply sector. They suggest that the exercise of market power by retailers does not totally explain why suppliers' price changes are not fully reflected as retail price changes. Indeed, they found that, apart from the special cases where the retail demand and supply functions are linear, market power's impact on the degree of price transmission is often ambiguous. Due to market concentration concern and the increase of market power by retailers in both vertical and horizontal dimension, in recent years, public institutions in the EU have performed studies focused on the potential for asymmetric price transmission in food-markets. The main concern is that consumers may not benefit as much as expected from liberal agricultural policy reforms if suppliers and retailers do not fully transform it into the proposed price reductions. According to the survey of Meyer at al (2004) on the asymmetric price transmission: "Asymmetry with respect to the speed of price transmission leads to a temporary transfer of welfare – in this case from buyers of the output good to sellers – whereas, asymmetry with respect to the magnitude of price transmission leads to a permanent transfer of welfare the size of which depends only on the price changes and transaction volumes involved". The exercise of market power can be a source of asymmetric price transmission with respect to magnitude and could be used to impose oligopoly or monopoly pricing. In the suppliers retailers' relationship the price transmission mechanism is classified as vertical and when it is affected by asymmetry it focuses specifically on transmission between different stages of market chain.

In Peltzman (2000) the asymmetric price transmission is defined as either positive or negative. If the output price reacts more fully or rapidly to an increase in the input price than to a decrease, the asymmetry is termed 'positive', whereas the 'negative' asymmetry denotes a situation in which the output price reacts more fully or rapidly to a decrease in the input price than to an increase. This distinction focuses on the different direction of welfare transfers due to asymmetric price transmission. Interesting study on the determination of positive or negative asymmetric price transmission is, for instance, the work of Ward (1982), where he suggests that market power can lead to negative asymmetric price transmission if oligopoly firms are reluctant to risk losing market share by increasing output prices. Bailey & Brorsen (1989) show that if a firm believes that no competitor will match a price increase but all will match a price cut, negative asymmetry will result. Otherwise if the firm conjectures that all firms will match an increase but none will match a price cut, positive asymmetry will result. The general result is ambiguous again.

In summary, many authors have suggested that market power can lead to a not symmetric price transmission. However most scholars predict that in the common oligopoly context, both positive and negative price transmission are likely to emerge, depending on market structure and strategic behaviour. *For instance when the wholesale price raises, retailers try to save their normal profit margin, while they try to* *widen it, at least temporarily, when the input price falls.* Because of consumers search costs, eventually profits go down and prices tend to the competitive levels. However, another situation may happen. When wholesale prices grow, each firm increase in short time its selling price as signal to competitors to agree to a tacit agreement; in the opposite case in order to save the tacit agreement it slowly decreases the selling price. As Peltzman (2000) pointed out the asymmetric price transmission is the rule, rather than the exception. More research is needed to shed light on the increasingly complicated relationships among behaviour of agents along the supply chain and the resulting prices and quantities.

The aim of this final chapter is to model retailers' strategic behaviour considering the vertical relation between retailers and suppliers in the food industry whereby retailers exercise seller power in their relation with consumers and buyer power in their relation with producers. In order to describe the vertical structure of the supplier-retailer relation we distinguish between upstream firms (producers or suppliers) and the downstream firms (processors, manufacturers or retailers).

Due to the not competitive nature of the downstream level of the market we can observe that the reduction for the purchasing prices received by the suppliers is often not proportional[66] to the reduction of the final prices set by retailers. In other words, a countervailing power[67] may lead to social welfare benefits but a downstream market power may be expected to have an adverse effect. This evidence suggests that when both forms of power are simultaneously present, for consumers to gain (obtaining lower prices), the increase in final prices (due to the selling power of buyers) needs to be more than offset by lower suppliers' prices (resulting from buyer power).

Suppliers' profits also suffer in this situation and producers can react trading with only one of the retailers, refusing to supply the rest (when a highly concentrated downstream market and intense competition is present). The result is that retailers derive monopoly profits and consumers bear higher prices and less retail products variety.

The strategic interaction among retailers embedded in an upstream and a downstream market has been studied using game theory and network theory analysis. Game theory and Network theory capture strategic interactions among players aware that their payoffs depend on other players' decisions. Our main purpose is to show how the strategic interaction among agents shape the power structure of the market (described by means of a graph structure) allowing for equilibrium architectures in which retailers have both market power and buyer power (countervailing power). Furthermore, strategic market interaction in turn leads to a bad price transmission causing the worst outcome for consumers.

The chapter is organized as follows. Next section describes the Upstream-Downstream Model (U-D). Results are described in section 3.3. In section 3.4 some concluding remarks are given.

3.2 The Model

3.2.1 The Upstream and Downstream markets Interaction

[66] We can define this situation as a *bad price transmission mechanism* or as previously quoted *"a positive asymmetric price transmission"*.

[67] It has been coined by Galbraith (1952) to describe the ability of large buyers in concentrated downstream markets to extract price concessions from suppliers.

Competition is usually regarded as a desirable feature of an economic system. There are different ways to define and measure it. The neoclassical approach focuses on market structure, the number of members in a particular sector and their market shares. This approach considers market concentration as an important measure of competition. There is a presumption that the more concentrated is the market, the less competitive it is. Both theory and empirical evidence suggest that other things being equal, markets with a higher Herfindahl-Hirschman index generate higher prices and bigger profits for market firms. The mechanism linking prices to the number of firms depends on a complicated mix of factors as the type of competition and the potential for collusion between sellers. Thus, market structures depend on more than just the market behaviour of retailers, such as, for instance, the economies of scale, the institutional factors and the buying power of the retailers. All of these variables may affect competition by preventing, restricting and distorting competition. Economists generally distinguish three main methods for organizing the transfer of commodities from the upstream stage to the next stages of food production: a) spot markets (prices set at the time of sale), b) vertical integration (upstream and downstream under a single ownership), and c) contracts (formal agreements). Spot markets represent the dominant traditional method for organizing the transfer even though the observed increased concentration downstream has had as a direct implication a fundamental change in terms of trade transactions. Nowadays, more upstream-downstream transactions are organized through agricultural specific contracts. Furthermore, the gradual removal of trade barriers has accelerated the formation of such vertical contracts. An alternative to shorter or longer term contracting is for the firms to follow vertical integration strategies.

Sometimes, it has been suggested that a larger number of stages in the vertical chain (i.e., a larger number of intermediaries) leads to higher final product prices (Vettas N. 2007). The structure of the vertical chain shapes the nature of the strategic interaction among agents. Therefore, it helps to understand the outcomes of the interactions in term of prices and quantities. Assume that there are only two stages with an upstream firm as producer (e.g., a supplier S) and a downstream retailer (R). The upstream firm's product is sold to firm downstream firm, which in turn sells to the final consumers. If we add to the basic vertical externality a horizontal externality like the (inter-brand) oligopolistic competition, it may correspond to a situation in which we have one supplier that deals with two (or more) retailers. In such cases, market power exists not only in the vertical sense but also in the horizontal one. *We need then to take into account not only strategic interaction between producers and retailers but also the same interaction among retailers*. The situation could be very different when there is both inter-brand and intra-brand competition at the same time. Imagine, for simplicity, there are two upstream firms, of which one is trading exclusively with *n* downstream firms and the other with *m* downstream firms. In this case, the each vertical chain pushes its own retailers to have more aggressive competition against rivals. Then, the equilibrium prices of suppliers may depend on the number of retailers each upstream firm is associated with.

The opposite scenario is given by two upstream firms facing a single downstream firm (D firm). In such a case, the D firm uses its bargaining power to obtain high profits. A more complex situation could be given by two large upstream and two large downstream firms. It is not straightforward to determine which kind of prices for the final consumers are set.

The answers will depend crucially on what kind and degree of bargaining powers each party has and if there exist outside options (Vettas, 2007).

Retailing buyer power consist in the ability to obtain from suppliers more favourable terms than those available to other retailers or that would otherwise be expected under normal competitive conditions. Apart from discounts on transactions from suppliers, buyer power can be expressed by means of contractual obligations on suppliers. Contractual obligations take different forms such as listing charges[68] , slotting allowances[69], retroactive discounts on goods already sold, buyer forced application of most favoured nation clauses[70], unjustified high contribution to retailer promotional expenses, and insistence on exclusive supply (Dobson 1998).

Retailers strategic choices involve multiple decisions act to utilize firms' resources at best and to exploit the external opportunities. Moreover, choices may conflict or overlap each others. We can describe common retailers' strategies using four directions. First, the so called internal strategies deal with reform and change at the intra-organizational level. One important internal strategy proliferating in the last years is related to the product differentiation strategies, *the development of in-house brands and labels*. Second, the horizontal strategies deal with retail expansion (domestic and international). Mergers and acquisitions are an example of retail expansion. Significantly, retail concentration can be a possible result deriving from this strategy. The most powerful horizontal strategy is represented by *retailer alliances*. Retail alliances may take different forms. An alliance at the purchasing level among independent retailers can result in key discount prices from suppliers. These kinds of alliances are officially known as *buying groups*. It is important to stress to better understand the further analysis that buyer power may exist in isolation (i.e., when the selling power of retailers is limited by intense competition) where retailing is highly fragmented on the selling side but coordinated (through *buying groups*) on the buying side. But often it might be that the buyer power of retailers is linked with their selling power, where one power reinforces the other. When it exists in isolation the power that retailers have against suppliers need not to depend on the number of retailers in the market nor on their concentration but on their *alliances.*

Vertical strategies extend the retailers' decision territory also to the supplier one. Vertical strategies can be of three types. The first leads to a vertical integration, that is, retailers' acquisition of suppliers. This vertical strategy can often give support to the creation of own-label product (which can strengthen the effect of the horizontal one). The second vertical strategy involves the direct change in the buyer-seller relationship is available in presence of strong retailer's bargaining power. A power shift toward retailers depends on the relative size and concentration of retailers with respect the suppliers and the threat of vertical integration. Within the third strategy retailers can sign exclusive agreements with suppliers.

Finally, *migration strategies* represent extra-territorial and evolutionary strategies (outlet size, location type, outlet type) (Mudambi, 1994).

3.2.2 Assumptions

Assume there are only two stages in the market chain with a set $S=\{S_1,S_2,...,S_{n^s}\}$ of suppliers (S) and a set $R=\{R_1,R_2,...,R_{n^r}\}$ of retailers (R). Let the cardinality of the two previous sets $|S|=N^s$ and $|R|=N^r$ be equivalent to N. Firms' S product is sold to firms R, which in turn sell to final consumers. Generally, suppliers may act as price-takers or price-makers. Retailers in turn may act as downstream and upstream price-takers or they can exert a combination of upstream and downstream powers. *If retailers have market power and suppliers are price-takers then the product price determined between*

[68] Buyers require payment of a fee before goods are purchased from the listed supplier. Dobson (1998).
[69] Fees are charged for store shelf-space allocation Dobson (1998).
[70] Contractual obligations for the supplier not to sell to another retailer at a lower price. Dobson (1998).

suppliers and retailers should be at the lowest level that retailers can pay to suppliers based on the current degree of horizontal competition among retailers. Alternatively, if suppliers have market power and retailers are price takers, then the level of market price suppliers can obtain from retailers should be the highest one given the degree of horizontal competition among suppliers. In reality, since both retailers and suppliers likely have some degree of market power, the actual price lies somewhere between the highest and the lowest level. Thus, it becomes difficult to produce a unique solution in modelling such markets. Consequently, most studies have assumed that one side of the market is a price-taker while the other has completely dominant vertical power.
We assume that suppliers are price takers whereas retailers, depending on strategic interactions, may exercise power either as buyer power on firms S or as market power only on consumers or on both groups of agents. We assume that:

1. Producers are price-takers with an inverse supply function as $P = c + dQ$;
2. Retailers have both countervailing power and selling power*;*
3. Consumers are price-takers with an inverse demand function as $P = a - bQ$.

The output market is not perfectly competitive. Because retailers have market power both on the intermediate and final product market suppliers' and retailers' price are function of the quantity. In other words, retailer's profits function is given by:

(1) $$\pi = \left(p_c(Q) - p_s(Q) - cmg \right).Q$$

Where p_c represents the price of the good sold to consumers (retailers' price) and p_s represents the price of the wholesale product sold to retailers.
Market consolidation on the buyers' side may offer socially beneficial countervailing power (against the *original* power of suppliers) but may have also a detrimental effect due to increased buyer concentration on the downstream level, leading to a *successive* increase in power further reducing social welfare (Office of Fair Trading. Research paper n16, 1998, Uk). *According to this view market structures assume a relevant role. Different combinations of downstream and upstream structures influence the exercise of power in the strategic interaction and in turn strategic interaction may shape market structure.*
Market structures related to retailers' power exerted only on suppliers in the upstream market are the monopsony[71] structure, the oligopsony (in which the greater the concentration of buyers, the greater the distortion in factor price) and buyer cartels (buyer coordination reducing suppliers' prices by restricting the overall demand reduces social welfare as in the monopsony case). Where demand side has power in two directions, upstream and downstream (welfare loss from exercising buyer power is made worse by the presence of seller power), we have the monempory or oligempory case (both factor producers and final consumers is likely to be adversely affected).

3.2.3 The U-D Game

Assuming that downstream firms may exert price setting (or bargaining) power against both the final consumers and also the upstream firms adds to the basic vertical externality of market chain also the horizontal one.

[71] In the *monopoly case*, the seller dominates and sets price and the buyer establishes the quantity. In the *monopsony case*, the buyer dominates and sets price while the seller chooses the output level.

Due to the vertical and horizontal nature of the strategic interaction, we can describe the Upstream-Downstream (U-D) Game by means of a graph $G=(R\cup S,E)$ with $R\cap S=\varnothing$ and $E\cap(S\times S)=\varnothing$, where R and S are respectively the set of retailers and the set of suppliers (disjoint union of nodes) and E represents the set of links between couple of nodes (assuming there are no links among suppliers). In the U-D interaction model, links' structure shapes market structures and for this reason, the latter can be represented as different network (or graph) configurations (i.e., upstream-downstream combinations). Retailers form pair-wise links with other retailers shaping a network (intra-network relationships). Market structures are described by different links configurations. Agents' payoffs depend on network architectures that in turn derive from the profile of equilibrium strategies selected by retailers. The intra-network links (or collaboration links) represent a kind of *horizontal-strategy* that enhances the profit in different ways: *purchasing-led retailer alliances* increasing buyer power towards suppliers, tacit collusion, acquisitions, mergers, etc. A collaboration link is cost reducing [72] and profits increasing[73] and may be interpreted as "tacit, profits-increasing agreements". An alliance at the purchasing level among independent retailers can result in key discount prices from suppliers. These kinds of alliances are known officially as *buying groups*.

Retailers form pair-wise links with suppliers (inter-networks relationships) as well. The inter-networks links represent a kind of "*vertical-strategy*". Vertical strategies move the retailer from its territory to its supplier's one. The principal types of vertical strategies are two: integration and changes in buyer-supplier relationships. The retailer can acquire a supplier of a broad range of products or one key product facilitating, for example, the development of own-brand products. The change in the relationship is directly related to the size and concentration of buyers relatively to suppliers, the quality of information buyers have about the suppliers and the extent, or credible threat of vertical integration. A retailer can exercise big power if it is large enough and the supplier is vulnerable. Finally, an alternative to the acquisition is the "*exclusive agreement*" with the supplier.

In the U-D Game market structures represent both the set of rules (i.e. the network) according to which players take their decisions and the degree of power players are endowed with. Firms are symmetric with zero fixed costs and identical constant return to scale functions. The payoff functions are represented by firms' profits depending on the network structure and on equilibrium quantities and prices as well. The shift from one configuration to another affects payoff functions. We define two different kind of buyer power: the *single-buying power* and the *buying group power*. The first form of power refers to the number of buyers linked to the same supplier; the second derives directly from the alliances with other retailers. In particular, the less the number of buyers linked to single supplier the greater the single-buying power over it. In other words, when the two sub-networks are connected to each other with the minimum number of links retailers exert the maximum single-buying power over suppliers. The buying group power refers to the extent of alliances with other retailers when a group of them is linked to the same supplier. When the number of collaboration links is greater, the power of retailers over consumers and over "common" suppliers increases. For instance, every collaboration link affects the degree of the oligopoly, potentially leading to the tacit collusion behaviour by retailers, and in turn, to a monopoly outcome for consumers.

---

[72] It is easy to see that if retailers have market power at the downstream level the final price set by producers at upstream level will be affected. Retailers' market power at downstream level affects (reduces) suppliers' prices at upstream level.

[73] Market power at the downstream level affects final prices for the consumers.

The U-D game is represented by a three stages game. In the first stage, retailers choose the number of supplier/s are going to provide them wholesale products (we assume that each producers sells the same good) creating a vertical link with them. The set of *vertical strategies* available for retailers in the first stage of the game is represented by $L(I) = 2^S \quad \forall i \in R$, where $S = \{s_1, s_2, ..., s_i, ..., s_{n_s}\}$ is the set of suppliers. The set of linking strategies for each retailer in the first stage is simply defined as $L_i(I) = \left\{ l_i(I) : \mu_i = \sum_{j=1}^{N^s} \mu_{ij} \left| \mu_{ij} = \{0,1\} \forall j \in S \right. \right\}$ where μ_i represents the number of links retailer i establishes with each supplier and μ_{ij} is equal one when retailer i establishes a link with supplier j. The first stage of the game is represented by a simultaneous move game among retailers that independently determine the number of links to establish with suppliers $G(\hat{\Gamma})_I = \left\{ R, \{L_i^I\}_{i=1}^N, \{\xi(\theta)\} \right\}$, where R is the set of players, L_i^I the strategy space for each retailer in the first stage and $\xi(\theta)$ is the outcome of the simultaneous subgame that depends on the parameter θ.

At the beginning of the second stage each retailer i observe the result of the previous stage and chooses a strategy belonging to the *horizontal strategy* space $L(II) = 2^{R \backslash \{i\}} \ \forall i \in R$, where $R \backslash \{i\} = \{R_1, R_2, ..., R_{i+1}, ..., R_{n_r}\}$ is the set of retailers not including retailer *i*. The linking strategy for each retailer in the second stage of the game is defined as $L_i(II) = \left\{ l_i(II) : \eta_i = \sum_{j=1}^{N^r - 1} \eta_{ij} \left| \eta_{ij} = \{0,1\} \forall j \in R \backslash \{i\} \right. \right\}$ where η_i represents the number of collaboration links retailer i establishes with other retailers and η_{ij} is equal one when retailer i establishes a link with retailer j. The second stage of the game is represented by a simultaneous move game among retailers that independently determine the number of links to establish with other retailers. That is, $G(\hat{\Gamma})_{II} = \left\{ R, \{L_i^{II}\}_{i=1}^N, \{\theta^*; \xi^*\} \right\}$, where R is the set of players, L_i^{II} the strategy space for each retailer in the second stage of the game. We have two outcomes. The first $\theta^* : L_1^{II} \times L_2^{II} \times ... \times L_N^{II} \to [0,1]$ is obtained from the interaction among retailers in the subgame and the second is the realized value of $\xi^*(\theta^*) : L_1^I \times L_2^I \times ... \times L_N^I \to [0,1]$ given collaboration links choices. Thus, each retailer decides whether to create a collaboration link with others retailers in order to exploit market power in the downstream market and/or buying power in the upstream market.

Retailers observe the outcome generated by the two previous subgames that shape the network structure in which they are going to play the third stage of the game.

In the last stage retailers play a quantity game and maximize their profits given the upstream and downstream market structure resulted from retailers strategic interactions in the previous stages of the game. The strategy space for retailers in stage three is $L_i^{III} = \left\{ l_i(III) : \mathbb{R}_+ \to \mathbb{R}_+, q_i = l_i(q_{-i}, \theta^*, \xi^*) \right\}$ where the strategic variable is the quantity.

The strategic form representation of the final subgame is $G(\hat{\Gamma})_{III} = \left\{ R, \{L_i^{III}\}_{i=1}^N, \{\pi_i\}_{i=1}^N \right\}$ where the payoff functions are determined by the simultaneous quantity choices, that is, $\pi_i(q^*, \theta^*, \xi^*) : L_1^{III} \times L_2^{III} \times ... \times L_N^{III} \to \mathbb{R}$.

The equilibrium outcome is characterized by a network architecture and a payoffs structure associated to it (deriving by strategic interaction among nodes). A semi-bipartite graph represents, in our model, the outcome, in terms of links architecture, of the equilibrium strategies in the first two stages. The payoffs structure corresponds to the *monempory* payoff for every retailer and in turn consumer prices are at their maximum level.

The first and the second stages outcomes - θ and ξ - are two conjectural parameters: $\theta = [0,1]$ and $\xi = [0,1]$. In the U-D game retailers profits depend on quantity and also on the two conjectural parameters ξ, θ representing respectively the degree of *buyer power* and *market power* of retailers. The combination of these two parameters shapes the structure in which retailers are going to take strategic decisions. For instance, the monempory market structure corresponds to a value of θ and ξ equal one.

Thus, retailers create horizontal and vertical links in order to influence the conjectural parameters that in turn shape the graph configuration.

The parameter ξ depends on the number of links retailers choose to build with suppliers and on parameter θ (eq.2):

$$\xi = \sum_{1}^{N^s} \frac{\left(N^r - \left|S^r(j)\right|\right)}{N^s\left(N^r - 1\right)} \left[\det(P) - \left(\det(P) - 1\right) d(\mu)\right] + \theta \sum_{1}^{N^s} \frac{\left(\left|S^r(j)\right| - 1\right)}{N^s\left(N^r - 1\right)} + \left[\theta\left(1 - \det(P)\right)\right]\left[1 - d(\mu)\right]$$

Where the term $\left|S^r(j)\right|$ represents the cardinality of the set of retailers the supplier *j* is connected with and N^r, N^s represent respectively the number of retailers and the number of suppliers (we assume for simplicity they are equivalent).

Equation (2) can be rearranged in order to take in to account only retailers decisions because we assume suppliers always accept to create link with retailers.

(2')

$$\xi = \sum_{j=1}^{N^s} \frac{\left(N^r - \sum_{i=1}^{N^r} \mu_{ij}\right)}{N^s\left(N^r - 1\right)} \left[\det(P) - \left(\det(P) - 1\right) d(\mu)\right] + \theta \sum_{j=1}^{N^s} \frac{\left(\sum_{i=1}^{N^r} \mu_{ij} - 1\right)}{N^s\left(N^r - 1\right)} + \left[\theta\left(1 - \det(P)\right)\right]\left[1 - d(\mu)\right]$$

The parameter $\sum_{i=1}^{N^r} \mu_{ij}$ represents the sum of links that each retailer *i* *for* $i = 1, 2, ..., N^r$ has built with supplier *j*. The term $\mu_{ij} = \{0,1\}$ is equal one when retailer *i* chooses to build a vertical link with supplier *j* and it is equal zero otherwise.

The parameter ξ depends also on the market power because an increase in collaboration links affects also the buyer power of retailers. The exercise of buyer power by retailers can take two forms. It can be exerted as *single buying power* (or "pure buyer power") when for each supplier there is only one retailer buying from him, or it can be exploited through the *buying group power*. The second form of buyer power is directly related to the downstream market structure. In other words, the greater the number of links connecting retailers the more they can exploit the buying group power in the upstream market when they have vertical links with the same set of suppliers.

Equation (2') takes into account both buyer powers. Finally, in order to discriminate between single buying power or a potential buyer group power depending on the distribution of links in the downstream market, we have to consider the term $\det(P)$ and $d(\mu)$. The former represents the determinant of an $N^r \times N^s$ matrix of vertical links. It corresponds neither to the adjacent matrix nor to the incidence matrix of the bipartite graph formed by retailers and suppliers. Each element of matrix P correspond to $\mu_{ij} = \{0,1\}$ so that in each row there will be an entry of value 1 in correspondence of column j if retailer i has a link with supplier j. That is,

$$P = \begin{pmatrix} \mu_{11} & \mu_{12}\cdots & \cdots\mu_{1j}\cdots & \cdots\mu_{1N^s} \\ \mu_{21} & \mu_{22}\cdots & \cdots\mu_{2j}\cdots & \cdots\mu_{2N^s} \\ \cdots & \cdots & \cdots & \cdots \\ \mu_{31} & \mu_{32}\cdots & \cdots\mu_{ij}\cdots & \cdots\mu_{1N^s} \\ \cdots & \cdots & \cdots & \cdots \\ \mu_{31} & \mu_{32}\cdots & \cdots\mu_{3j}\cdots & \cdots\mu_{N^r N^s} \end{pmatrix}$$

Define $\mu_i = \sum_{j=1}^{N^s} \mu_{ij}$ as the number of links retailer i has with suppliers. When $\mu_i = 1\ \forall i \in R$ (where the term μ_i is simply the sum of the entries in row i of matrix P) the determinant of the "power matrix" P is either 0 or 1, depending on the distribution of the entries 1 for each column. When there is single buying power the power matrix is a unit matrix and its determinant is always equal 1. When P is not the identity matrix (and $\mu_i = 1\ \forall i \in R$), its columns will be not linearly independent and then its determinant will be always equal 0. The variable $d(\mu)$ in equation 2' is a dicotomic variable:

$$d(\mu) = \begin{cases} 0 & if\, \mu_i = 1 \forall i \\ 1 & otherwise \end{cases}$$

It discriminates between the single buying power and the potential for buying group power when each retailer chooses one supplier. The relationship between ξ *and* θ can be explained in the following way:

- When $\mu_i = 1\ \forall i \in R$ and each retailer is connected to a *different* supplier, $\det(P) = 1$ and $d(\mu) = 0$ the resulting value of parameter $\xi = 1$ for every value of θ.
- When $\mu_i = 1\ \forall i \in R$ and each retailer is connected to the *same* supplier, $\det(P) = 0$ and $d(\mu) = 0$ the resulting value of parameter $\xi = \theta$ and depends completely by the buying group power deriving from the interaction in the downstream level.
- In all the other cases because $d(\mu) = 1$ equation (2') becomes :

(2'') $$\xi = \sum_{j=1}^{N^s} \frac{\left(N^r - \sum_{i=1}^{N^r} \mu_{ij} \right)}{N^s \left(N^r - 1 \right)} + \theta \sum_{j=1}^{N^s} \frac{\left(\sum_{i=1}^{N^r} \mu_{ij} - 1 \right)}{N^s \left(N^r - 1 \right)}$$

- In this case, for $\theta = 1$, the buyer group power in the downstream market affects the level of power in the upstream resulting in $\xi = 1$ as well.
- For the value of parameter $\theta = 0$, parameter ξ corresponds only to the first term of equation (2'') and is equal one only when there is single buying group power.
- For values of $0 < \theta < 1$, the value of ξ (without single buyer power) will be:

$$\sum_{j=1}^{N^s} \frac{\left(N^r - \sum_{i=1}^{N^r} \mu_{ij} \right)}{N^s \left(N^r - 1 \right)} < \xi < 1$$

The parameter θ depends only on the number of links retailers build one each other (eq.3 and 3').

(3) $$\theta = \sum_{i=1}^{N^r} \frac{\left| R^r(i) \right|}{N^r \left(N^r - 1 \right)}$$

(3') $$\theta = \sum_{1}^{N^r} \frac{\eta_i}{N^r \left(N^r - 1 \right)}$$

The set $R^r(i) = \left\{ j \in R^r \middle| ij \in g, j \neq i \right\}$ is the set of retailers with whom retailer i is connected with. The parameter η_i represents the number of players retailer i is connected with (i.e., the cardinality of the set $R^r(i)$).

3.3 Results

3.3.1 Definitions

In the U-D game, each stage is represented by a simultaneous-move game among retailers. Each retailer, at the end of every stage is full aware of the strategies chosen by the other retailers, so each stage represents a proper subgame of the U-D game. The natural approach to analyse such a multistage game is to focus on its subgame-perfect equilibria.

Definition 1. *A profile $\gamma^* \in \Psi$ is a subgame-perfect equilibrium of Γ if, for every proper subgame $\hat{\Gamma} \subseteq \Gamma$, $\gamma^* \big|_{\hat{\Gamma}}$ is a Nash equilibrium of $\hat{\Gamma}$.* (Selten, 1965)

In the first two stages, retailers' interaction - in the linking games - defines a network equilibrium structure that, together with the equilibrium quantity in the final stage, determines players' payoff and the final price for consumers. Given the particular graph representation of the U-D game the equilibrium configurations may have different characteristics. Two classes of network configurations need to be defined.

Definition 2. *A Bipartite Graph $G = (V, E)$ is a graph in which the vertex set V can be divided into two disjoint subsets X and Y such that every edge $e \in E$ has one end in X and the other in Y.* Wilson (1996)

This kind of structure, in the U-D model corresponds to a situation in which retailers (vertices *Y*) have built links only with suppliers (vertices *X*) and there are no collaboration links. Depending on the number of links among the two sub-sets of vertices – if it is minimum or maximum (e.g., in case of *perfect or maximum matching*)

- we can derive the extent of single-buying power and consequently deduce the resulting final price for consumers in the perfect competitive third stage of the game (i.e., no horizontal links are formed).

Definition 3. *A Semi-Bipartite Graph*[74] $G=(V,E)$ *is a graph in which the vertex set V can be divided into two disjoint subsets X and Y and also the edge set E is divided into two disjoint subset* E_{xx} *and* E_{xy} *. The graph consists in two parts. In the first part which is called "centre" all the x are connected to each other and* $e \in E_{xx}$*. The nodes of the second part, the y (peripheral nodes) are not connected to each other, but there are some links between them and the nodes of the centre (central nodes) such that every edge* $e \in E_{xy}$ *has one end in X and the other in Y. A semi-bipartite network model consists of a semi-bipartite graph* $G=(R,S,E)$ *where nodes on one side of the bipartition represent retailers (R), and nodes on the other side of the bipartition represent sellers (S), and there no edges among Suppliers.*

The Semi-Bipartite configuration corresponds to a particular outcome of the game. Retailers in the second stage build a complete network of collaboration links, whereas the number of links between retailers and suppliers is not defined. It depends on whether retailers exploit the advantages of a *perfect matching* (i.e., each retailer is connected to only one different supplier (of one-degree)).

Definition 4. *A matching of a graph G is a sub-graph of G such that every edge shares no vertex with any other edge.*

That is, each vertex in matching M has degree one. The size of a matching is the number of edges in that matching. A matching is *maximum* when it has the largest possible size. The matching number of a graph is the size of a maximum matching of the graph. A complete matching of a graph G is a matching that contains all of G's vertices. *The matching number of a bipartite graph* G is equal to $|U|-D_U(G)$, where U is the set of the upper vertices. Likewise the matching number is also equal to $|D|-D_D(G)$, where D is the set of the down vertices. $D_U(G)$ and $D_D(G)$ are defined in the following way:

$$D(S)=\begin{cases}|S|-k & \textit{if positive}\\ 0 & \textit{otherwise}\end{cases}$$

Where $S \subseteq U$ and *k* is the number of downstream vertices the set S is connected to. That is, $D(S)$ is the number of vertices missing for having a perfect match (the number of vertices we need to have a perfect match). Obviously, if the size of the set of upstream and downstream nodes is different $D_U(G)$ and $D_D(G)$ will be never zero.

The conditions for having a complete matching in a bipartite graph are the followings.

(i) *The number of nodes in the two disjoint set has to be equal;*

[74] It has been shown that such a network has a high clustering coefficient and a short average path length. These two characteristics of the network are independent of the size of the network directly, but it depends on the fraction of central nodes. The average path length has an upper bound and the clustering coefficient has a lower bound regardless of the size of the network. (Characteristics of 'semi-bipartite' networks Asieh Karami et al 2007 Phys. Scr. 75 316-319).

(ii) $D_U(G) = D_D(G) = 0$.

We expect to observe a bad price transmission from suppliers to consumers. Depending on the network of relationships built in the first stages of the U-D game we may obtain different equilibrium quantities and therefore different consumer prices.

Definition 5. *The highest price retailers can extract from consumers is the result of different kinds of semi-bipartite network architectures* in which the *centre* is complete $\left(\eta_i = \left(N^r - 1\right) \forall i \in R\right)$ and it is connected to *peripheral nodes* such that either $0 < \mu_i \le N^s$ or $\mu_i = 1 \forall i \in R$ $\left(\text{i.e.,} \left|R^s_{(i)}\right| = 1\right)$[75].

We have to distinguish between two cases. The first case is represented by a network structure in which the source of power derives only from the downstream market, that is, the buyer group power, and the second, when the upstream market structure is one of single buying power. The set of "highest prices" semi-bipartite configurations is characterized by a complete centre, then, this means that the buying group power is at work.
Buying group power in a semi-bipartite network configuration is described in definition 6.

Definition 6. *There does exist buying group power if for every vertical link structure belonging to the subset of suppliers' vertices* $S^r = \left\{S^r_{(1)}, ..., S^r_{(j)}, ..., S^r_{(N^r)}\right\}$ *where* $S^r_{(j)} = \left\{i \in R \mid ij \in g\right\}$ *is the set of retailers supplier j is connected with; and for every horizontal links structure* $R^r = \left\{R^r_{(1)}, ..., R^r_{(j)}, ..., R^r_{(N^r)}\right\}$ *where* $R^r_{(i)} = \left\{j \in R \middle| ij \in g, j \ne i\right\}$ *is the set of retailers retailer i is connected with; each element of* S^r *has a corresponding element in* R^r *such that each* $R^r_{(i)} \supseteq S^r_{(j)} \setminus \{i\}$[76].

In other words, to have buyer group power, for every pair of players connected with the same supplier in the upstream network, a link between them have to be formed in the downstream market.
The second type of configuration corresponds to a semi-bipartite network with perfect matching and focuses on single-buying power. Single-buying power in a semi-bipartite network configuration can be defined as follows.

Definition 7. *There does exist single- buying power if for every vertical links structure belonging to the subset of suppliers' vertices* $S^r = \left\{S^r_{(1)}, ..., S^r_{(j)}, ..., S^r_{(N^s)}\right\}$ *where* $S^r_{(j)} = \left\{i \in R \mid ij \in g\right\}$ *is the set of retailers supplier j is connected with; and for every vertical links structure belonging to the subset of retailers' vertices* $R^s = \left\{R^s_{(1)}, ..., R^s_{(i)}, ..., R^s_{(N^r)}\right\}$ *where* $R^s_{(i)} = \left\{j \in S \mid ij \in g\right\}$ *is the set of suppliers retailer i is*

[75] The term represents the number of suppliers retailer *i* is connected with.
[76] This definition corresponds to the following condition on pairwise links: $\forall ik, jk \in g$ with $i, j \in R$ and $k \in S$, it must be that also $ij \in g$.

connected with; each element of R^s, S^r has cardinality $\left|S^r_{(j)}\right| = \left|R^s_{(i)}\right| = 1$ *with* $R^s_{(i)} \neq R^s_{(j)} \forall i, j \in R$ *and* $S^r_{(i)} \neq S^r_{(j)} \forall i, j \in S$ [77]*. In other words, both* R^s *and* S^r *must be partitions, respectively* $P(R)$ *and* $P(S)$ *of set R and S with number of parts equal respectively to* N^r *and* N^s *each of them with cardinality equal one.*

In other words, each retailer is linked to one supplier in a perfect matching of the bipartite upstream network formed by the disjoint union of retailers and suppliers nodes.

Definition 8. *The lowest price retailers can extract from consumers is the result of a complete bipartite network architecture* in which the central nodes are not connected (retailers sub-network is empty) and retailers are connected to suppliers through the maximum number of links.

In other words, there are no collaboration links neither buying group nor single-buying power at work. That is, $\mu_i = N^s$ *and* $\eta_i = 0 \forall i \in R$; and both conditions $R^r_{(i)} \supseteq S^r_{(j)} \setminus \{i\}$ and $\left|S^r_{(j)}\right| = \left|R^s_{(i)}\right| = 1$ are not satisfied.

3.3.2 The Subgame Perfect Equilibrium of the U-D Game

We proceed backward in the game and first determine the optimal reactions by retailers (in the last stage) to every previous decision by other retailers (in the previous stages of the game). In other words, we need to find the optimal actions of retailers in each subgame (that here coincide with each stage of the game) induced by every possible strategy of the other retailers.

Proposition 1 *The unique subgame perfect equilibrium of U-D game corresponds to the following profile of strategies for each player in each stage of the game:*

$$\left(1 \leq \mu_i^* \leq N^s; \eta_i^* = N^r - 1; q^* = \frac{a-c}{2(b+d)}\right)$$

Proposition 2 *The strategic interaction among retailers in the two simultaneous linking games leads to a Semi-Bipartite graph configuration and the final consumers' price is the highest retailers can extract from consumers.*

Proposition 3 *Retailers' price and suppliers' price changing, as result of the interaction in different network configurations, shows a bad price transmission pattern.*

Proofs:

[77] This definition corresponds to the following condition on pairwise links: $\forall ik \in g$ *with* $i \in R, k \in S$ it must be that $kj \notin g \ \forall j \neq i \in R$.

Proceeding backward, the optimal action for each retailer in stage three induced by every possible action of other retailers must coincide with the optimal reaction to every possible action of other retailers in the previous stages of the game. The contingent pattern of optimal actions defines the only credible strategy of each retailer that may be part of any subgame-perfect equilibrium.

In the U-D game, a Nash equilibrium of the last stage, is any vector $\tilde{q}(\theta,\xi)=\left(\tilde{q}_1,...,\tilde{q}_i,...,\tilde{q}_{n^r}\right)$ satisfying, for each $i=1,2,...n^r$, the following conditions:

(4) $$\tilde{q}_i \in \arg\max_{q_i} \pi_i\left(q_i(\theta,\xi),\tilde{q}_{-i}(\theta,\xi)\right)$$

To compute the optimal reaction of each retailer in stage three to every previous choices in stage I and stage II we solve the optimization problem:

(5) $$\max_{q_i}\left\{\left[p_c(Q)-p_s(Q)-cmg\right].q_i\right\}$$

Retailers play the simultaneous last stage game knowing the previous choices in stage I and II. In this way they are aware of the market structure in which they are playing in (Oligempory, Monempory, Oligopoly and so on). Therefore, to compute the optimal actions - without specifying the network structure chosen in the previous stages of the game - the profit maximization should include every possible market structure situation (taking into account both downstream and upstream level of the market).

The first order conditions for the generic profit maximization by retailers are:

(6) $$\frac{\partial\pi}{\partial Q}=p_c(Q)+\frac{\partial p_c(Q)}{\partial Q}\frac{\partial Q^c}{\partial q_i}Q-p_s(Q)-\frac{\partial p_s(Q)}{\partial Q}\frac{\partial Q^s}{\partial q_i}Q-cmg=0$$

rearranging,

(6') $$p_c(Q)-P\left(\frac{1}{\varepsilon^r}\right)\theta-p_s(Q)-P\left(\frac{1}{\varepsilon^s}\right)\xi-cmg=0$$

Where θ and ξ, represent respectively the extent of market power and buyer power of retailers (i.e., the influence of their choices on the final output).

The optimal reactions in term of quantity and prices to every action in the previous stages are:

(7) $$\tilde{q}_i(\theta,\xi)=\frac{a-c-cmg}{b(1+\theta)+d(1+\xi)};$$

(8) $$\tilde{p}_c\left(\tilde{q}(\theta,\xi)\right)=\frac{ab\theta+ad(1+\xi)+b(c+cmg)}{b(1+\theta)+d(1+\xi)};$$

(9) $$\tilde{p}_s\left(\tilde{q}(\theta,\xi)\right)=\frac{cd\xi+bc(1+\theta)+d(a-cmg)}{b(1+\theta)+d(1+\xi)}.$$

The payoff functions depend on both θ and ξ in the following way,

(10) $$\tilde{\pi}(\theta,\xi)=\frac{(a-c)^2(b\theta+d\xi)}{\left[b(1+\theta)+d(1+\xi)\right]^2}$$

With $\frac{\partial\pi_i(\theta,\xi)}{\partial\xi}\geq 0$ $\frac{\partial\pi_i(\theta,\xi)}{\partial\theta}\geq 0$.

In the second stage, because profits are increasing in the two parameters, anticipating the reactions of other players in the final stage of the game, retailers will choose the number of horizontal links such that the parameter θ is maximized. That is,

(11) $$\eta_i^* \text{ s.t. } \theta(\eta_i,\eta_{-i})=\sum_1^{N^r}\frac{\eta_i}{N^r\left(N^r-1\right)}=1$$

It is easy to see that $\eta_i^* = \left(N^r - 1\right)\ \forall i \in R$[78]. Retailers choose the number of horizontal links that maximize market power in the downstream market and take into account also the level of the buyer power parameter (that also depend on θ) chosen in the fist stage. We already noticed that parameter ξ depends on the number of vertical links and also on parameter θ. This is due to the fact that the two kinds of powers are interconnected. Indeed, retailers choose horizontal links in order to exert the maximum market power over consumers and to exploit the possibility of buying group power over suppliers in this way:

(12) $$\max_{\theta} \pi\left[\tilde{q}\left(\theta,\xi\right)\right].$$

For every possible given value of ξ, the first order necessary conditions are:

(13) $$b\theta + d\xi\left(\theta\right) = b + d\,,$$

From which we obtain,

(14) $$\tilde{\theta} = \frac{N^s\left(N^r-1\right)\left(b+d\right) - d\left[\sum_{j=1}^{N^s}\left(N^r - \sum_{i=1}^{N^r}\mu_{ij}\right)\right]}{N^s\left(N^r-1\right)b + \left[\sum_{j=1}^{N^s}\left(\sum_{i=1}^{N^r}\mu_{ij} - 1\right)\right]},$$

Equation (14) clearly shows the connection between vertical and horizontal structure. It represents the best response to any vertical strategy decision taken by retailers in the upstream level of the game. It is straightforward to show that it has value equal 1 for every symmetric equilibrium choice taken by retailers in the stage one. That is,

If $\sum_{j=1}^{N^s}\left(N^r - \sum_{i=1}^{N^r}\mu_{ij}\right) = N^s\left(N^r - \sum_{i=1}^{N^r}\mu_i\right)$, then,

(15) $$\tilde{\theta} = \frac{N^s\left(N^r-1\right)\left(b+d\right) - dN^s\left(N^r - \sum_{i=1}^{N^r}\mu_i\right)}{N^s\left(N^r-1\right)b + dN^s\left(\sum_{i=1}^{N^r}\mu_i - 1\right)} = 1$$

This result implies that in equilibrium:

(16) $$\theta^* = 1$$

That in turn implies that the best response to every symmetric strategy chosen in the initial stage is to choose a value of $\eta_i = \left(N^r - 1\right)$ (i.e., to be connected to every retailers).

In the appendix A an example of the second stage linking game has been given. The Strong Nash equilibrium network corresponds to the complete network and results from a strategy profile in which each retailer chooses the maximum number of links.

In the first stage of the game retailers take decisions on the structure of vertical links. Proceeding for backward induction, in the first stage, retailers are aware of the reactions in term of horizontal and quantities strategy in the subsequent stages of the game. Anticipating the reactions of other players in the two subsequent levels of the U-D game, retailers will choose the number of vertical links such that the parameter ξ is maximized. That is,

(17) $$\mu_i^* \text{ s.t. } \xi\left(\mu_i, \mu_{-i}, \tilde{\theta}\right) = 1$$

[78] An example describing the linking game equilibrium is given in the Appendix A.

Parameter ξ is maximum for different values of μ_i. For $\mu_i = 1 \; \forall i \in R$ and with perfect matching, ξ is maximum for every value assumed by $\tilde{\theta}$ in the second stage, where $\mu_i = \sum_{j=1}^{N^s} \mu_{ij}$ [79] represents the number of links each retailer has with suppliers. For every value of $\mu_i \neq 0$ when $\tilde{\theta} = 1$.
It is clear form eq. (17) that for $\theta = 1$ (in the second case), the equilibrium value of $\xi = 1$ as well.

(17)
$$\sum_{j=1}^{N^s} \frac{\left(N^r - \sum_{i=1}^{N^r} \mu_{ij} \right)}{N^s \left(N^r - 1 \right)} + \sum_{j=1}^{N^s} \frac{\left(\sum_{i=1}^{N^r} \mu_{ij} - 1 \right)}{N^s \left(N^r - 1 \right)} = 1$$

This is true for every symmetric strategy μ_i (respect to any supplier j) except $\mu_i = 0$ (it can be shown that this strategy is weakly dominated). Form eq. (15) we derive that since retailers best response in the second stage is to form a complete network of collaboration links for every symmetric equilibrium strategy profile at the first stage, their buyer power will be maximized if the equilibrium strategies profile in the first stage is symmetric. In turn retailers can exploit the buying group power deriving by the shape of the horizontal links they built. In this way, in the first stage of the game, retailers will select every symmetric strategy profile. That is, every symmetric decision taken about vertical strategies will be the best response to $\eta_i = N^r - 1$. In the appendix B a simple example of the retailers linking game in the upstream market is described.

We have shown that ξ depends also on θ in such a way that as $\theta = 1$, also $\xi = 1$. Furthermore, profits are not decreasing both in θ and ξ and they are maximized for $\theta = 1$ and $\xi = 1$ since for these equilibrium values agents operate in a monempory market structure.

The equilibrium values of profits and parameters are the followings,

(18)
$$\left(\xi\left(\mu^*\right), \theta\left(\eta^*\right), \pi\left(q^*\right) \right) = \left(1, 1, \frac{\left(a-c\right)^2}{4\left(b+d\right)} \right),$$

induced by a subgame perfect equilibrium of the U-D game corresponding to:

(19)
$$\left(1 \leq \mu_i^* \leq N^s ; \eta_i^* = N^r - 1 ; q^* = \frac{a-c}{2\left(b+d\right)} \right).$$

Moreover, the strategic interaction among retailers in the two simultaneous linking games leads to a Semi-Bipartite graph configuration. The centre is complete and it can be connected to the periphery in different ways, such as with a perfect matching or also with a complete and maximum matching - depending on the equilibrium value of μ_i^*.

[79] Notice that this term is different from the one in eq. (2') in what follows: in eq. (2') $\sum_{i=1}^{N^r} \mu_{ij}$ represents the number of links supplier j has with retailers; in eq. (17) $\sum_{j=1}^{N^s} \mu_{ij}$ represents the number of links retailer i has with suppliers.

The Semi-Bipartite graph configuration corresponds to a monempory case in which suppliers' prices are at their minimum level and consumers' prices at the maximum level.
We also noticed that form eq. (8) and (9),

(8) $$\tilde{p}_c\left(\tilde{q}(\theta,\xi)\right)=\frac{ab\theta+ad\left(1+\xi\right)+b\left(c+cmg\right)}{b\left(1+\theta\right)+d\left(1+\xi\right)};$$

(9) $$\tilde{p}_s\left(\tilde{q}(\theta,\xi)\right)=\frac{cd\xi+bc\left(1+\theta\right)+d\left(a-cmg\right)}{b\left(1+\theta\right)+d\left(1+\xi\right)}.$$

Consumer prices are increasing in the parameter θ and ξ:

$$\frac{\partial\tilde{p}_c\left(\tilde{q}(\theta,\xi)\right)}{\partial\theta}>0;\frac{\partial\tilde{p}_c\left(\tilde{q}(\theta,\xi)\right)}{\partial\xi}>0 \text{ for } \left(a>c\right)$$

On the other hand, supplier prices are decreasing in both parameters:

$$\frac{\partial\tilde{p}_s\left(\tilde{q}(\theta,\xi)\right)}{\partial\theta}<0;\frac{\partial\tilde{p}_s\left(\tilde{q}(\theta,\xi)\right)}{\partial\xi}<0 \text{ for } \left(a>c\right)$$

Therefore we can observe a bad price transmission as result of the strategic interaction of retailers that exercise buyer power over suppliers and market power over consumers. In other words, the reduction in suppliers' prices - due to the exercise of buyer power – (e.g., $\theta=1;\xi=1$) is not transferred to consumers, because retailers gain extra profits reducing the quantity and increasing prices (widening the margins). On the other hand, when $\theta=\xi=0$ - retailers have neither buyer power nor market power - suppliers' prices are higher but consumer' prices are lower because the downstream market is competitive. For instance, we can observe the role of the downstream market in cases in which $\theta=0;\xi\leq1$. Suppliers' prices in these cases are higher even though buyer power is at work and final prices will result in a lower level because of the absence of collaboration links in the downstream market.
Finally, as market power and buyer power decrease the final consumers' price decreases and simultaneously the level of suppliers' price increases. The margins between p_c and p_s narrow down as θ and ξ tend to zero. In other words, an increase in the suppliers' price – due to a decrease in market power - push retailers to reduce profit margin since the latter become almost equal zero when $\theta=\xi=0$.■

3.4 Concluding remarks

The strategic interaction among retailers with respect to an upstream and a downstream market has been studied using game theory and network theory analysis. The purpose of this chapter was to show how the strategic interaction among agents may shape the power structure of the market (described by means of a graph structure) allowing for equilibrium architectures in which retailers have both market power and buyer power (countervailing power against suppliers in the upstream market) and leading to the highest final price for consumers. The U-D game is represented by a three stages game.
In the first stage, retailers choose the number of suppliers that are going to provide them wholesale products (we assume that each producers sells the same good) creating a vertical link with them. The second stage of the game is represented by a simultaneous move game among retailers that independently determine the number of links to

establish with other retailers. In the last stage retailers simultaneously maximize their profits choosing as strategic variable the quantity to buy and sell on the final market given the upstream and downstream market structure resulted from retailers strategic interactions in the previous stages of the game. Using a backward induction procedure for the analysis of the subgame perfect equilibria of the U-D game, we obtain the following results.

Proposition 1 *The subgame perfect equilibria of U-D game correspond to the following profile of strategies for each player in each stage of the game:*

$$\left(1 \le \mu_i^* \le N^s; \eta_i^* = N^r - 1; q^* = \frac{a-c}{2(b+d)}\right)$$

Given the reaction functions of retailers in the last stage of the game, the optimal reaction - in the second stage - by retailers - to every symmetric choice in the first stage - is always to choose a number of links that maximizes parameter θ. In turn, the optimal action in stage two implies that the equilibrium value of ξ is always equal one and the number of vertical links for each retailer may vary from one to N^s.

Proposition 2 *The strategic interaction among retailers in the two simultaneous linking games leads to a Semi-Bipartite graph configuration and the final consumers' price is the highest retailers can extract from consumers.*

Payoffs are increasing in the number of links among retailers, thus, it is clear that the outcome of the linking game in the second stage will be a complete network. This result in turn implies that, for every decision in the first stage, the final network configuration will be represented by a semi-bipartite graph. The semi-bipartite graph corresponds to a market structure that leads to the highest price in the downstream market.

Proposition 3 *Retailers' price and suppliers' price changing, as result of the interaction in different network configurations, shows a bad price transmission feature.*

We also observed that changes in suppliers' prices and consumers' prices – due to a change in the market power – behave accordingly to a bad price transmission rule. In particular, when suppliers' price decrease, retailers' margins widen in order to exploit extra-profits instead to convert the reduction of input costs into lower consumers' prices.

3.5 Appendix (A): A three players linking game in the downstream market

Assume $N^r = N^s = 3$. The strategy space of each retailer has size four, that is $2^{R\backslash\{i\}} = \{\{\varnothing\},\{R_j\},\{R_k\},\{R_j,R_k\}\}$. In the second stage, three retailers are involved in a linking game with the strategic form described in Table 3.1 (player i selects rows, player j columns, and player k chooses among the four boxes)

Table (3.1) The three retailers linking game

$s_k = R_i$

i \ j	R_i	R_k	R_jR_k	$\varnothing$
R_j	π^1	π^e	π^1	π^e
R_k	π^1	π^1	π^1	π^1
R_jR_k	π^s	π^1	π^s	π^1
$\varnothing$	π^e	π^e	π^e	π^e

$s_k = R_j$

i \ j	R_i	R_k	R_jR_k	$\varnothing$
R_j	π^1	π^1	π^s	π^e
R_k	π^e	π^1	π^1	π^e
R_jR_k	π^1	π^1	π^s	π^e
$\varnothing$	π^e	π^1	π^1	π^e

$s_k = R_iR_j$

i \ j	R_i	R_k	R_jR_k	$\varnothing$
R_j	π^1	π^1	π^1	π^e
R_k	π^1	π^s	π^1	π^1
R_jR_k	π^s	π^1	π^C	π^1
$\varnothing$	π^e	π^1	π^1	π^e

$s_k = \varnothing$

i \ j	R_i	R_k	R_jR_k	$\varnothing$
R_j	π^1	π^e	π^1	π^e
R_k	π^e	π^e	π^e	π^e
R_jR_k	π^1	π^e	π^1	π^e
$\varnothing$	π^e	π^e	π^e	π^e

The Nash equilibria of the linking game are indicated by the shadowed boxes in the table. Given the payoff order: $\pi^C > \pi^s > \pi^1 > \pi^e$, The Nash equilibria correspond to the following equilibrium strategy profiles:

$$I\left(s_i^*, s_j^*, s_k^*\right) = \left(\{\varnothing\},\{\varnothing\},\{\varnothing\}\right)$$

$$II\left(s_i^*, s_j^*, s_k^*\right) = \left(\{\varnothing\},\{R_k\},\{R_j\}\right)$$

$$III\left(s_i^*, s_j^*, s_k^*\right) = \left(\{R_j\},\{R_i\},\{\varnothing\}\right)$$

$$IV\left(s_i^*, s_j^*, s_k^*\right) = \left(\{R_jR_k\},\{R_i\},\{R_i\}\right)$$

$$V\left(s_i^*, s_j^*, s_k^*\right) = \left(\{R_j\},\{R_iR_k\},\{R_j\}\right)$$

$$VI\left(s_i^*, s_j^*, s_k^*\right) = \left(\{R_k\},\{R_k\},\{R_iR_j\}\right)$$

$$VII\left(s_i^*, s_j^*, s_k^*\right) = \left(\{R_jR_k\},\{R_iR_k\},\{R_iR_j\}\right)$$

The first equilibrium corresponds to the empty network configuration and the value of the parameter θ associated is zero. The correspondent payoff π^e is the lowest for every value of ξ and q chosen in the other stages of the game. Equilibria II and III define configuration with only one link between two retailers. Parameter θ assumes the value of $1/3$ and the payoff π^1 is always greater than the empty network one but always lower than the other possible network outcome.

Equilibria IV, V and VI correspond to a star network[80] configuration, respectively with player i, player j and player k being the hub node. The associated level of parameter $\theta = 2/3$ leads to a payoff function for each player π^s. The payoff function π^s associated to the star network is always lower than the one associated to the complete network π^c for every level of ξ and q chosen in the other stages of the game. For equilibrium VII, $\theta = 1$ and it shapes a complete network configuration. It is straightforward to see that all the other equilibria are Pareto dominated by equilibrium VII and therefore they cannot be Strong Nash equilibria of the game. The equilibrium VII is the unique Strong Nash equilibrium[81] of the three players linking game because there does not exist a joint deviation of the other players that benefits all of them.

3.6 Appendix (B): An example of vertical linking game

Table (1.2): The two retailers vertical linking game

1 \ 2	S_1	S_2	S_1S_2
S_1	$\tilde{\theta}$	1	$(1/2)+(\tilde{\theta}/2)$
S_2	1	$\tilde{\theta}$	$(1/2)+(\tilde{\theta}/2)$
S_1S_2	$(1/2)+(\tilde{\theta}/2)$	$(1/2)+(\tilde{\theta}/2)$	$\tilde{\theta}$

In table (3.2) is depicted the payoff matrix (values of ξ resulting from the linking interaction) of a two retailers game in the vertical linking game. The strategy space of each retailer is equal to the set of suppliers. The value of parameter $\tilde{\theta}$ is defined in the stage II and the payoff matrix describes its relationship with parameter ξ.

If we proceed for backward induction, retailers in the first stage know that $\tilde{\theta} = 1$ for every symmetric decision taken by retailers in stage I.

[80] The star network is characterized by a hub-node of degree (number of links) $(N-1)$ and two spoke-nodes of degree 1.

[81] The Strong Nash equilibrium is defined as a strategic profile for which no subset of players has a joint deviation that strictly benefits all of them.

Tab. (3.3):Payoff matrix with buyer power value range

1 \ 2	S_1	S_2	S_1S_2
S_1	$0<\xi<1$	1	$(1/2)<\xi<1$
S_2	1	$0<\xi<1$	$(1/2)<\xi<1$
S_1S_2	$(1/2)<\xi<1$	$(1/2)<\xi<1$	$0<\xi<1$

It is easy to verify, in this case, that every strategy profile is a Nash equilibrium of the vertical linking game. Table (3.3) represent the payoff matrix for the vertical linking game when retailers take in to account the possible value of parameter ξ given the optimal reactions of retailers in the second stage of the game. We can show that the strategy $\{S_1S_2\}$ is risk dominated by all the others. Assume players are not sure about which strategy the opponent will pick and assign probabilities for each strategy. If each player assigns probabilities $1/3$ to each strategy, then the expected value of ξ from playing strategies $\{S_1\} or \{S_2\}$ exceeds the expected value of ξ from playing $\{S_1S_2\}$.

In this simplified example, in the first stage of the game the risk dominant equilibria correspond to the following equilibrium strategy profiles:

$$I\left(l_1^*,l_2^*\right)=\{S_1,S_2\} \quad II\left(l_1^*,l_2^*\right)=\{S_1,S_1\}$$
$$III\left(l_1^*,l_2^*\right)=\{S_2,S_1\} \quad IV\left(l_1^*,l_2^*\right)=\{S_2,S_2\}$$

That is, equilibria I and III correspond to the *single buyer-buying group* power solution: the perfect matching and the complete downstream network (if $\tilde{\theta}=1$) . The other two equilibria correspond to the case in which the buyer power is completely related to the buyer group power. For every value of $\tilde{\theta}$, $(1/2)+\left(\tilde{\theta}/2\right)$ gives always greater or equal value of ξ (with respect to $\tilde{\theta}$ value) . Therefore when retailers are going to discriminate between them, they will always select the strategy that corresponds to the higher expected value of ξ.

4

Conclusions

Chapter 1 addressed the bilateral free trade agreement formation in a network formation game setting in order to evaluate the impact of bilateralism on the global free trade achievement and the relationship between regionalism and multilateralism. The theory of network formation game is a valid tool to deal with the complex environment of international trade relationships defined by Bhagwati as "a messy maze of preferences as PTAs formed between two countries, with each having bilaterals with other and different countries, the latter in turn bonding with yet others, each in turn having different rules of origin for different sectors" (Bhagwati, 2002). In the current global market a large share of world trade takes place within sixty (or more) overlapping arrangements that reduce barriers to trade on a preferential basis. Thirty percent of world trade takes place within the two largest preferential trading areas: the EU and the NAFTA.

In order to evaluate the impact of regionalism on the multilateral process, two different kinds of stability concepts have been used: *pairwise stability* and *strong stability.* The prediction of the network formation approach in the recent network formation game literature, presented by Goyal-Joshi (2006), is that the Global Free trade network is a pairwise *stable* network. World trade is described by an intra-industry trade model as in Krishna (2005). We have described the relationship between stability and efficiency in a symmetric and asymmetric countries environment. That is, we have distinguished between an asymmetric case in which countries are identical with the same market size and number of firms and an asymmetric case in which two countries have larger market size with respect to a third one.

The results, in the symmetric and in the asymmetric case depend respectively on countries market size and on the degree of asymmetry among them. We distinguish three market size intervals: small ($\gamma < \breve{\gamma}$), medium ($\breve{\gamma} < \gamma < \hat{\gamma}$) and big ($\gamma > \hat{\gamma}$) countries.

When the market size belongs to the interval $\gamma < \breve{\gamma}$ we obtain that the unique *PS*, *SS* (and then also P-efficient) and Efficient network is the empty configuration. Moreover also the one link network is P-efficient.

When the market size belongs to the interval $\breve{\gamma} < \gamma < \hat{\gamma}$ we have two different situations. Either we have that the empty network is the unique PS, SS and Efficient network or the one link network is PS, SS and efficient while the complete network is only PS .

When the market size belongs to the interval $\gamma > \hat{\gamma}$ we obtain that if $\gamma > \hat{\hat{\gamma}}$ (with $\hat{\hat{\gamma}} > \hat{\gamma}$)the unique *PS*, *SS* and Efficient network is the complete configuration while the star and the one link networks are P-efficient. If $\hat{\gamma} < \gamma < \hat{\hat{\gamma}}$ then the Efficient network is the star configuration.

In general, when countries are symmetric and large the global free trade outcome may emerge in equilibrium both via bilateral agreements expansion and multilateral trade rounds. This result is consistent with recent findings on the empirical properties of world trade web by Fagiolo, Reyes and Schiavo (2009). They show that the world trade web is an extremely symmetric network, where almost all trade relationships tend to be reciprocated with similar intensities. Therefore, they studied its characteristics as if it were a weighted[82] undirected network. They pointed out that richer countries tend to be more clustered (and increasingly so over the years) supporting with their finding the "rich club phenomenon".

On the other hand, with medium-size countries we see a potential adverse effect of bilateralism on multilateralism. In other words, even though global free trade agreement may still be reached through bilaterals, multilateral negotiation can be blocked by the existence of profitable deviations to the one link network (i.e. bilateralism). Moreover, because of the trade diverting feature of Ptas, when emerging only through bilateral agreements, the complete network, is not the efficient configuration.

When countries are asymmetric we obtain different results on efficiency and stability depending both on the degree of asymmetry and the market size intervals.

When the market size of the symmetric countries belongs to the interval $\gamma < \breve{\gamma}$ we obtain that the unique *PS*, *SS* (and then also P-efficient) is the empty network while the one link network g^1_{xy} is Efficient.

When the market size of the symmetric countries belongs to the interval $\breve{\gamma} < \gamma < \bar{\gamma}$ we have two different situations. First, if $\breve{\gamma} < \gamma < \tilde{\gamma}$ then either the empty network or the one link network g^1_{xy} are PS and SS depending on the market Z size. If $\gamma_z < \gamma_z^5(\gamma, n)$ then the empty network is SS and since $\gamma_z^5(\gamma, n)$ is not increasing in γ, when $\breve{\gamma} < \gamma < \tilde{\gamma}$ the greater is the asymmetry higher is the possibility that the empty network is SS. Second, if $\tilde{\gamma} < \gamma < \bar{\gamma}$ then the one link network g^1_{xy} is *PS*, *SS* and Efficient.

When the market size of the symmetric countries belongs to the interval $\bar{\gamma} < \gamma < \gamma^B$ we obtain that the one link network g^1_{xy} is *PS*, *SS* and Efficient while the complete network and the star network $g^S_{x,y}$ are *PS* (for higher values of γ_z).

When the market size of the symmetric countries is $\gamma > \gamma^B$ we obtain that the one link network g^1_{xy} is the unique *SS* network. Moreover, g^1_{xy} is also the unique PS network for

[82] Each directed link from node i to j is weighted by total exports of country i to country j and then divided by the country i's GDP (i.e., the exporter country). Such a weighting setup allows one to measure how much economy i depends on economy j as a buyer (Fagiolo et al, 2009).

small values of γ_z. The complete network is the unique PS, Efficient (and P-efficient) network for large values of γ_z and the star network $g^s_{x,y}$ is PS otherwise.

In the asymmetric case the global free trade outcome (also efficient for large countries) is never obtained as result of a multilateral trade round since the complete network is not strongly stable and the one link network is immune to multilateral deviations (except in the case of small countries). Moreover, when countries are asymmetric, another form of link architecture emerges in equilibrium: the hub and spoke configuration. Hub and spoke agreements take place more often among a developed country as a hub and more smaller spokes. Thus, the presence of such agreements in equilibrium is justified by the extent of asymmetry among countries. Indeed, empirical recent studies in which the relative weight of trade flows towards and from countries is considered, describe the world trade web as a very high density network but with the average strength of nodes being rather poor (Fagiolo et al, 2009). In other words, most countries hold mainly weak relationships, whereas only a selected core on nodes combine high degree and high strength. Indeed, the world trade web, according to Fagiolo, Reyes and Schiavo has a dis-assortative nature. That is, countries holding many (and more intense) trade relationships preferably trade with poorly connected countries.

The model we presented could be further extended including increasing economies of scale in order to underline the role of the intra-industry trade and its effect on regionalism and multilateralism. Moreover, many North-South preferential agreement are asymmetric in trade liberalization. That is, these types of agreements often allow "one- way" free access to developing countries in developed market such as agreement between UE and ACP countries (Africa, Caribbean, Pacific). Therefore, it would be interesting also to take into account the one-way access among asymmetric countries describing these agreements by means of digraphs.

In chapter 2 we have analyzed the efficiency and stability of the bilateral free trade agreement formation in a dynamic network formation game setting. Making use of a dynamic formation process and the Pareto efficiency concept we have described the relationship between stability and efficiency in a symmetric and asymmetric countries environment. We considered two cases. First, a symmetric case in which each country has the same market size α and the same number of firms n. Second, an asymmetric case in which two countries have bigger market sizes α and one smaller market size α_3.

From the analysis of the value functions of each network configuration and from the associated payoff functions we obtained the following results on Pareto efficiency:

1) If the size of countries is small and $\alpha < n/2$ (when countries are symmetric and small) the protectionism architecture (the empty network) gives always higher payoffs for each node. Therefore, the Pareto efficient network is the empty one.
2) If $\alpha > (10n^2 + 6n + 1)/2(n+1)$ we have that the Pareto efficient network is the complete one.
3) Assuming that $n^* = \alpha_1 = \alpha_2 = \alpha$ and $\alpha > \alpha_3$ (countries are asymmetric) we found out that the empty network is not Pareto efficient.

From the study of the dynamic process of free trade agreement formation together with the efficiency analysis we found that,

Proposition 1. *If $\alpha < n/2$, then no links ever form. That is, when countries are symmetric and small the empty network is stable and Pareto efficient.*

Proposition 2. *If* $\alpha > \left(10n^2 + 6n + 1\right) / 2\left(n+1\right)$ *then every link forms and remains (no links are ever broken). That is, when countries are symmetric and big the complete network is stable and Pareto efficient.*

Proposition 3. *if* $\alpha > 3\alpha_3$ *, then no links ever form. With symmetric countries the empty network is stable but not Pareto efficient.*

Thus, whenever we assume that countries are symmetric both in market sizes and number of firms we always obtain that the stable network configuration is also the Pareto efficient one. On the other hand, if we assume a degree of asymmetry among countries we end up with a network configuration that is not the efficient one and that leads to a network architecture in which bigger countries are better off and small countries are worse off.

Finally, in chapter 3 the strategic interaction among retailers with respect to an upstream and a downstream market has been studied using game theory and network theory analysis. We described a model in which the strategic interaction among agents shapes the distribution of power in the market (described by means of a graph structure). The interaction results in equilibria architectures in which retailers have both market power and buyer power (countervailing power against suppliers in the upstream market) and leads to the highest final price for consumers. The U-D game is represented by a three stages game. In the first stage, retailers choose the number of suppliers are going to provide them wholesale products (we assume that each producers sells the same good) building a vertical link with them. The second stage of the game is represented by a simultaneous move game among retailers that independently determine the number of links to establish with other retailers. In the last stage retailers maximize their profits choosing the optimal quantity given the upstream and downstream market structure resulted from retailers strategic interactions in the previous stages of the game. Using a backward induction procedure for the analysis of the subgame perfect equilibria of the U-D game we obtain the following results.

Proposition 1 *The unique subgame perfect equilibrium of U-D game corresponds to the following profile of strategies for each player in each stage of the game:*

$$\left(\mu_i^* = \left(0, N^s\right]; \eta_i^* = N^r - 1; q^* = \frac{a-c}{2\left(b+d\right)}\right)$$

Given the reaction functions of retailers in the last stage of the game, the optimal reaction - in the second stage - by retailers - to every symmetric choice in the first stage - is always to choose a number of links that maximizes parameter θ. In turn, the optimal action in stage two implies that the equilibrium value of ξ is always equal one and the number of vertical links for each retailer may vary from one to N^s.

Proposition 2 *The strategic interaction among retailers in the two simultaneous linking games leads to a Semi-Bipartite graph configuration and the final consumers' price is the highest retailers can extract from consumers.*

Payoffs are increasing in the number of links among retailers, thus, it is clear that the outcome of the linking game in the second stage will be a complete network. This result in turn implies that, for every decision in the first stage, the final network configuration will be represented by a semi-bipartite graph. The semi-bipartite graph corresponds to a market structure that leads to the highest price in the downstream market.

Proposition 3 *Retailers' price and suppliers' price changing as result of the interaction in different network configurations, showing a bad price transmission feature.*

We also observed that changes in suppliers' prices and consumers' prices – due to a change in the market power – behave accordingly to a bad price transmission rule. In particular, when suppliers' price decrease, retailers' margins widen in order to exploit extra-profits instead to convert the reduction of input costs into lower consumers' prices.

Bibliography

ASIEH KARAMI (2007): *Characteristics of 'semi-bipartite' networks.* Phys. Scr. 75 316-319).

BAIER S.L. and BERGSTRAND J.H., (2007): *Do Free Trade Agreements actually increase members' international trade?*. Journal of International Economics, 71, issue 1, pp. 72-95.

BAILEY D. AND BRORSEN B.W., (1989): *Price Asymmetry in Spatial Fed Cattle Markets*, Western Journal of Agricultural Economics, Vol. 14(2), pp. 246-252.

BAIN J. S., (1956): *Barriers to New Competition*, Cambridge MA: Harvard University Press.

BAKUCS AND FERTO (2006): *Farm to Retail Price Transmission on Pork Market: A German - Hungarian Comparison*. Manuscript.

BALA V. AND GOYAL S., (2000): *A Non-Cooperative Model of Network Formation*. Econometrica 68, 1181-1231 (2000)

BALDWIN R., (1993): *A Domino Theory of Regionalism.* Centre for Economic Policy research (London). Working paper n.857, November.

BARABASI, A.L., (2003):. *Linked: How Everything Is Connected to Everything Else and What It Means for Business, Science, and Everyday Life*. Plume Books.

BEHAR J., (1991): *Economic Integration and Intra-Industry Trade: the case of the Argentine-Brazilian Free Trade Agreement*. Journal of Common Market Studies, 29, No. 4, June, pp. 527-552.

BELLEFLAMME P. AND F. BLOCH (2002): *Market Sharing Agreements and Stable Collusive Networks*. Mimeo: University of London and GREQAM.

BHAGWATI J. (2002), *Free Trade Today*, Princeton University Press.

BHAGWATI J. (1993): *Regionalism and Multilateralism: An Overview,* in J. de Melo and A. Panagariya, eds., New Dimensions in Regional Integration, World Bank and Cambridge University Press, Cambridge, UK, 22-51.

CALVO' A. AND İLKILIÇ R., (2005): *Pairwise Stability and Nash Equilibria in network formation*, Working Papers 34, Fondazione Eni Enrico Mattei.

CAULIER J.F., MAULEON A., VANNETELBOSCH V., (2007): Contractually Stable Networks. NajEcon Working Paper Reviews 843644000000000084, www.najecon.org.

CONNOR, J. M., (2003): *The Changing Structure of Global Food markets: Dimensions, Effects. And Policy Implications,* Working Papers 03-02, Purdue University, College of Agriculture, Department of Agricultural Economics.

CONNOR, J. M., (1999): *Evolving Research on Price Competition in the Grocery Retailing Industry*. Agricultural and Resource Economics Review. 28119-127.

CURRARINI S., and MORELLI M., (2000): *Network Formation with Sequential Demands*. Review of Economic Design, 5, pp 229-250.

DEMANGE, G., and WOODERS, M. H., (2005): *Group Formation in Economics: Networks, Clubs and Coalitions.* Cambridge University Press.

DOBSON CONSULTING (1999): *Buyer Power and its impact on competition in the food retail distribution sector of the European Union.* Commission – DGIV Study contract No. IV/98/ETD/07.UK.

DOBSON P., WATERSON M., CHU A., (1998): *The Welfare Consequences of the Exercise of Buyer Power.* Office of Fair Trading. Research paper n16, UK.

DOBSON P., WATERSON M., (1997): *Countervailing Power and Consumer Prices*, Economic Journal, Vol. 107, pp. 418-430.

DUTTA B. AND MUTUSWAMI S., (1997): *Stable Networks*. Journal of Economic Theory, Elsevier, vol. 76(2), pages 322-344, October.

FAGIOLO, G., REYES, J. and SCHIAVO, S. (2009): *The Evolution of the World Trade Web.* Journal of Evolutionary Economics, forthcoming.

FURUSAWA T. AND H. KONISHI (2005): *Free Trade Networks with Transfers*. Japanese Economic Review 56, 144-16.

FURUSAWA T. and KONISHI H., (2007): *Free Trade Networks*. Journal of International Economics, 72, pp. 310-335.

GALBRAITH J.K., (1952): *American Capitalism: The Concept of Countervailing Power.* Boston: Houghton Mifflin.

GLOBAL ECONOMIC PROSPECTS (GEP)2005: *Trade, Regionalism and Development*. The World Bank's annual Report. (www.worldbank.org)

GOYAL, S., (2007): *Connections: An Introduction to the Economics of Networks*. Princeton University Press.

GOYAL S. AND JOSHI S., (2006): *Bilateralism and Free Trade*. International Economic Review.

GROSSMAN G., HELPMAN E., (1995): *The Politics of Free Trade Agreements*. American Economic Review. 85:4. pp.667-690.

HUIZINGA H., (1993): *International Market Integration and Union Wage Bargaining*. Scandinavian Journal of Economics 95, 249-255.

JACKSON M.O., (2008): *Social and Economic Networks*. Princeton University Press.

JACKSON M. O. AND A. VAN DEN NOUWELAND (2005): *Strongly Stable Networks*, Games and Economic Behavior 51, 420-444.

JACKSON M.O. (2005): *A Survey of Models of Network Formation: Stability and Efficiency* in Group Formation in Economics: Networks, Clubs and Coalitions, edited by G. Demange and M. Wooders, Cambridge University Press.

JACKSON M.O., (2003) *The Stability and Efficiency of Economic and Social Networks*, in Networks and Groups: Models of Strategic Formation, edited by B. Dutta and M.O. Jackson, Springer-Verlag: Heidelberg .

JACKSON M.O. AND WATTS A., (2002): *The Evolution of Social and Economic Networks*, Journal of Economic Theory 106, 265-295.

JACKSON M.O. AND WATTS A., (2001): *The Existence of Pairwise Stable Networks*. Seoul Journal of Economics 14, 299-321.

JACKSON M.O. AND WOLINSKY A., (1996): *A Strategic Model of Social and Economic Networks*. Journal of Economic Theory 71, 44-74.

KRANTON, MINEHART (1998): *A theory of Buyer-Seller Networks*, Mimeo.

KRISHNA P., (1998): *Regionalism and Multilateralism: A Political Economy Approach*. The Quarterly Journal of Economics 113, 227-251.

KRUGMAN P.R., (1991): *Is Bilateralism Bad?*, in E. Helpman and A. Razin, eds., International Trade and Trade Policy MIT Press, Cambridge, 9-23.

LEVY P.I., (1997): *A Political-Economic Analysis of Free-Trade Agreements*. American Economic Review 87, 506-519.

LLOYD P. J. and MACLAREN D., (2003): *The Case of Free Trade and the Role of RTAs.* Seminar on Regionalism and the WTO. Geneva.(www.wto.org)

MCDOWELL S. MUDAMBI, (1994): *A topology of Strategic Choice in Retailing* . International Journal of Retail & Distribution Management. Vol. 22-4.pg.32-40.

MEYER J., AND VON CRAMON-TAUBADEL S., (2004): *Asymmetric Price Transmission: a Survey*, Journal of Agricultural Economics, 50, pp. 581-611.

MILGRAM−BALEIX J., and MORO−EGIDO A.I., (2005) : *Intra−industry trade with emergent countries: what can we learn from spanish data?*. Economics Bulletin, Vol. 6, No. 12, pp.1−17

MYERSON R.B., (1991). *Game Theory: Analysis of Conflict*, Harvard University Press, Cambridge Massachusetts.

MYERSON R.B., (1977): *Graphs and cooperation in games*. Mathematics of Operations Research 2: 225-229.

ORNELAS, E., (2005): *Endogenous Free Trade Agreements and the Multilateral Trading System*. Journal of International Economics 67, 471-497

PELTZMAN, S., (2000): *Prices Rise Faster than they fall*, Journal of Political Economy, Vol. 108, No. 3, pp. 466-502.

SELTEN R., (1965): *Spieltheoretische Behandlung eines Oligopolmodells mit Nachfragentragheit*, Zeitschrift fur die gesamte Staatswissenschaft, 12, 201-324.

STEINBERG F., (2007):*The Future of World Trade: Doha or Regionalism and Bilateralism? (ARI).* (Analyst in International Economy at the Elcano Royal Institute and Professor at the Autonomous University of Madrid).

VEGA-REDONDO F., (2007): *Complex Social Networks*. Econometric Society Monograph Series, Cambridge University Press.

VETTAS N., (2007): *Market Control and Competition issues along the Commodity Value Chain*, pp. 9-26, in Governance, Coordination, and Distribution along Commodity Value Chains, Food and Agriculture Organization of the United Nations, Rome, 2007.

WARD, R.W., (1982): *Asymmetry in Retail, Wholesale and Shipping Point Pricing for fresh Vegetables*, American Journal of Agricultural Economics, Vol. 62, pp. 205-212.

WASSERMAN, S., FAUST, K., and IACOBUCCI, D., (1994): *Social Network Analysis : Methods and Applications (Structural Analysis in the Social Sciences).* Cambridge University Press.

WATTS, A., (2001): *A Dynamic model of Network Formation*. Games and Economic behaviour 34,331-341.

WELDEGEBRIEL TADESSE W., (2004): *Imperfect Price Transmission: Is Market Power really to Blame?* Journal of Agricultural Economics 55 101-114.

WILSON, ROBIN J., (1996): *Introduction to graph theory* 4th ed. Longman.

WONNACOTT R. J. (1996): *Trade and Investment in a Hub-and-spoke System versus a Free Trade Area.* The World Economy, 19 (3), pp. 237-252.