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Three Essays in Competition Policy

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Contents

Summary

The thesis comprises three chapters on competition policy.

In the first chapter, entitled *Pricing Strategies and Vertical Contracts in a Monopolistic E-Book Market*, I draw from the e-book antitrust case against Apple and five major international publishers, in order to study the rationality behind the use of different types of vertical contracts in a market with complement products. Apple and five major book publishers have been accused by Antitrust Authorities of price fixing. One of the main argument was that the use of the agency model helped Apple and the publishers to collude. Interestingly, when Amazon was the only retailer for e-books, it chose a wholesale model. When Apple entered the market, it adopted instead an agency model, and soon after Amazon switched to an agency model too. The theoretical question is whether collusion is the only reason behind the use of an agency model or whether using such a vertical contract can be more profitable than a wholesale contract even absent a collusive intent. This chapter contributes to addressing this question by investigating whether the agency model is more profitable for publishers and distributors than the wholesale model in a distributor monopolistic market (thus addressing the question of why Amazon initially chose such a contract). Using a theoretical model, I focus on the effects of two main factors: competition, and complementarity between e-books and e-readers. I consider two upstream firms, each producing e-books, and a downstream firm, selling both e-books and e-readers to final consumers. I show that a downstream monopolist always prefers the wholesale model to the agency model. In fact, once the downstream firm can choose the price of both e-readers and e-books, it can exploit better substitution and complementarity effects

between goods to maximize profits.

The second chapter, co-authored with my supervisor Prof. Lapo Filistrucchi and Dr. Stefan Behringer, is entitled *Price Wars in Two-Sided Markets: the Case of the UK Quality Newspapers in the '90s*. This chapter too starts from a real-word competition policy case: the price war among the UK quality newspapers in the '90s, which was at the time discussed as a possible case of predatory pricing by the Times against the Independent. We observe that in those years — a period of economic boom in the UK — the share of advertising revenues in the financing mix of the newspapers increased drastically. We develop a monopoly, oligopoly and collusive model of the newspaper market to derive conditions under which the optimal share of advertising financing increases when the size of the advertising market increases. Using data on the financing mix of the single newspaper and on GDP in the UK in those years, we conduct some preliminary econometric tests which confirm that, both for each newspaper and overall, there is a significant negative relationship between the observed financing mix and the level of GDP.

Finally, the third chapter, entitled *MFN Clauses and Quality Disclosure on Online Platforms*, also draws from real-world cases, namely those concerning the imposition of Most Favored Nation (MFN) clauses by Expedia and Booking.com. Antitrust Authorities decisions across Europe have made MFN clauses (partly or wholly) illegal. In this chapter I study the effects of a MFN clause imposed by a booking platform on the quality of a hotel. I consider two possible scenarios, depending on whether an MFN clause is allowed or forbidden. I show that when the clause is allowed, a platform will always adopt it to prevent hotel's free riding behaviors and make positive profits. This chapter points out three key findings. Firstly, with MFN clause in place, a hotel facing low costs for quality chooses the maximum quality and sells through the platform, whereas the hotel with high costs for quality chooses the minimum quality and sells only through its own channel. Secondly, a platform prefers to set a percentage fee rather than a non-percentage fee. Finally, when the MFN clause is forbidden, the hotel might choose a listing fee. In such a case, it chooses the minimum quality irrespective of the costs.

CHAPTER 1

Pricing Strategies and Vertical Contracts in a Monopolistic E-Book Market

1.1 Introduction

In recent years the emerging e-book market has been attracting antitrust scrutiny, both in the United States and in European Union. Apple and five publishing companies were suspected of having colluded to limit price competition in the retail market for e-books. The publishers, along with Apple, allegedly acted together to raise retail prices of e-books by taking control of e-book pricing. At the heart of the suits on both sides of the Atlantic is the agency model, under which the retail price of e-books is determined by the publishers themselves and the retailers, who have no power to alter the price, receive a commission. While the agency pricing is not *per se* illegal, the joint adoption of it has been considered by Antitrust Authorities as an outcome of a concerted action between Apple and the publishers with the aim of stopping Amazon, at that time the leader in the market.

The e-books were traditionally distributed to retailers on a wholesale basis. Under the *wholesale model* (or *reseller model*), publishers set a suggested retail price (the so-called "list price") and sell e-books to distributors at a wholesale price—often about half of the cover price for a book. Distributors are then free to establish the retail price charged to consumers. Thus, while publishers might recommend a price, retailers could compete on prices offering discounts to consumers. Until 2010, the e-book wholesale price was approximately 20% below the wholesale price for the corresponding physical book, reflecting the cost savings associated with the distribution and sale of digital books. From 2007 until Spring 2010, Amazon sold newly released and bestselling e-books to consumers for $9.99, a price significantly below the wholesale price. This strategy, presumably implemented to encourage the Kindle e-reader sales, helped Amazon become one of the leading booksellers in the world.

When Apple entered the market with the iPad in 2010, it agreed with the major publishers to sell their e-books in its new iBookstore, adopting the *agency model*. The agency agreements entered into by the five publishers and Apple were more or less identical, containing the same key pricing terms on a global basis. They included in particular a Most Favored Nation (MFN) clause, which required the publishers to match on Apple's platform—the "iBookstore"—the lowest price listed on any other bookstore, "even if the [publisher] did not control that other retailer's ultimate consumer price."[1] On the one hand, it is alleged that the MFN clause acted as a "commitment device" (EC, 2012) and an incentive for the publishers to impose the agency model on Amazon and other major retailers in similar terms as those agreed to by Apple. Otherwise, the MFN clause would have resulted in lower profits and margins for publishers. According to the suits, none of the publishers could individually force Amazon to adopt the new business model. Instead, acting collectively, had Amazon refused to convert to an agency model, the company would have faced the risk of no longer being supplied with e-books from these major publishers. On the other hand, the MFN clause might enable Apple to avoid competition with Amazon. By the end of 2010, also Amazon entered into agency agreements with the five publishers.

It is alleged that the Apple Agency Agreements could have been the result of a common strategy implemented to achieve both the publishers' goal of raising

[1] *See* Complaint, Civil Action No.1:12-CV-2826, *United States vs. Apple Inc., et al.*, ¶65

retail prices above the level set by Amazon, as well as Apple's goal of limiting competition with Amazon. In fact, as the allegations claimed, prior to 2009 the five publishers were concerned about Amazon's strategy, fearing that consumers might grow accustomed to cheaper books as Amazon gained more and more market power, with a consequence of a demand for lower wholesale prices and thus lower publisher profit margins. Publishers also feared that Amazon's discounted strategy could have cannibalized the traditional formats. Finally, publishers were "concerned that Amazon was well positioned to enter the digital publishing business and thereby supplant publishers as intermediaries between authors and consumers.[2]"

Apple would have played a central role in facilitating the conspiracy, since it aimed at having the same retail prices as Amazon's while obtaining profits.

Whereas in the EU all the parties settled and agreed to terminate all on-going agency agreements, and not to enter into agreements that contain MFN clauses for five years, in the US the publishers settled on similar terms as those submitted in the EU, Apple went instead to court and was found guilty.

The chapter is organized as follows. In Section 2, I review some recent contributions dealing with the e-book industry. In Section 3, I present the model. In Section 4, I conclude and discuss some shortcomings of my work and directions for future research.

1.2 Related Literature

Recent theoretical papers analyzing the relation between agency and wholesale models argue that agency model *per se* does not induce higher prices. Indeed, the effects of the agency pricing hinge on several factors, such as the degree of competition at the level where prices are set (Foros et al., 2013), the inclusion of MFN clause in the agreements (Foros et al., 2013; Johnson, 2017), and the presence of

[2] *See* Complaint, Civil Action No.1:12-CV-2826, *United States vs. Apple Inc., et al.*, ¶34

markets for complement goods (Gaudin and White, 2014). Furthermore, in a market with consumer lock-in, as it could be the market for e-books, wholesale model could lead to higher prices than the agency model (Johnson, 2013). However, the empirical work by De los Santos and Wildenbeest (2017) provides evidence that e-book prices are lower under the wholesale model.

Foros et al. (2013) identify the conditions under which it is profitable for retailers to adopt the agency model. They especially focus on the effects on pricing of two factors: the different grade of substitution at the two levels in the distribution chain —upstream and downstream— and the Most Favored Nation (MFN) clause. They consider a market with two upstream firms, producing one good each, and two downstream firms, each selling both upstream firms' products. Upstream firms are publishers for e-books and developers of applications, while downstream firms are platforms such as Apple and Amazon. Foros et al. (2013) suggest that when competition is stronger upstream than downstream, no firm will adopt the agency model, as it would lead to lower retail prices. Moreover, they show that MFN clause might ensure that even if the rival firm does not adopt the agency model, equilibrium prices will be the same as they would be if both firms adopted the agency model, i.e. MFN clause might lead to higher equilibrium retail price.

Johnson (2017) compares the effect of price parity clauses under both a wholesale model and a agency model in a bilateral monopoly and in a bilateral duopoly. He finds that shifting from a wholesale model to an agency model increases retailer profits and consumer surplus, whereas it decreases supplier profits. Indeed, in equilibrium, final prices tend to be higher under the wholesale model. However, the combination of MFN clause with agency pricing drives prices up, thereby harming consumers. In fact, without an MFN clause, under an agency model, the higher the revenue share, the lower the retail price; hence a retailer who offers a larger revenue share receives a larger market share. By contrast, MFN eliminates the incentive to compete on revenue shares, since suppliers charge the same price to all retailers. Consequently, retailers will demand higher revenue shares and retail prices will be higher.

Johnson (2013) sets up a two-period model where consumers are locked-in in order to analyze how switching from a wholesale model to an agency model affects the market equilibrium. Johnson (2013) argues that, although prices may initially be lower under the wholesale model, they tend gradually to increase in the long term, since retailers act as monopolists, once consumers are locked-in. Moreover, consumers are likely to be better off under the agency model than under the wholesale model.

Gaudin and White (2014) study how the complementary market for devices (e-readers and tablets) can influence the e-book pricing. They compare the equilibrium outcomes arising under the agency and the wholesale model, under two different settings: the case in which a retailer controls a complementary market and the case in which it does not. Gaudin and White (2014) show that when a retailer controls the complementary market for reading devices, e-book prices are lower under wholesale, as the retailer will extract consumer surplus using the device price. When the retailer has no such control, the reverse is true.

The aim of my work is to investigate whether it was profitable for Amazon to adopt the wholesale model, when it controlled both the e-book and the e-reader market (prior to Apple's entry in 2010).

The model I present is based on Foros et al. (2013). Their finding suggests that, if Apple aimed at increasing retail prices, it optimally chose the agency model. Nonetheless, the model does not provide any explanation about Amazon's adoption of the wholesale model until 2010. For this reason, I will consider a downstream monopolist which sells both e-books and e-readers in order to investigate how complementary markets influence pricing strategies. In fact, a reasonable conjecture is that Amazon sold e-books under their wholesale cost in order to incentivize the sale of its Kindle e-reader. In the model there are two competing upstream firms, each producing e-books, and one downstream firm, selling both e-books and e-readers. Equilibrium outcomes under two different sale models —wholesale and agency model— are derived and compared.

1.3 The Model

I here propose an extension of the model presented in Foros et al. (2013). As already discussed, Foros et al. (2013) build a model in which they compare the equilibrium outcomes —retail prices and profits— under the agency model to the equilibrium outcomes under the wholesale model. Whereas they assume competition at both levels, I will consider a market with two competing upstream firms U_j, $j = 1, 2$, and one downstream firm D. By doing so, I aim at studying the rationale behind the use of a wholesale model by a monopolist distributor like Amazon. As in Foros et al. (2013), the upstream firms are e-book publishers and produce one good each. One of the main shortcomings of the work by Foros et al. (2013) is that they consider only the market for e-books, without taking into consideration the complementary market for reading devices. I think that taking into account both the markets for e-books and e-readers could help explain why Amazon sold many e-books under their wholesale cost. For this reason, I extend the analysis provided by Foros et al. (2013) by considering a downstream firm selling both e-books and e-readers. The downstream firm (e.g. Amazon) distributes e-book (good b_j, $j = 1, 2$ hereafter) and produces and sells e-readers (good r).

For simplicity, I do not model the market for traditional books. However, I implicitly assume that e-books and traditional books are substitutes.

I assume the following inverse demand curve for good b_j at downstream firm D:

$$p_{b_j} = 1 - q_{b_j} - \gamma q_{b_{-j}} + \lambda q_r, \tag{1.1}$$

where q_{b_j} is the demanded quantity of e-books produced by the upstream firm U_j, and q_r the demanded quantity of e-readers.

I further assume the following inverse demand curve for good r:

$$p_r = 1 - q_r + \mu(q_{b_j} + q_{b_{-j}}). \tag{1.2}$$

This implies the following direct demand function for good b_j and good r, respectively:

$$q_{b_j} = \frac{(1-\gamma)(1+\lambda) - (1-\lambda\mu)p_{b_j} + (\gamma - \lambda\mu)p_{b_{-j}} - \lambda(1-\gamma)p_r}{(1-\gamma)(1+\gamma-2\lambda\mu)}, \tag{1.3}$$

$$q_r = \frac{(1+\gamma+2\mu) - (1+\gamma)p_r - \mu(p_{b_j} + p_{b_{-j}})}{1+\gamma-2\lambda\mu}. \tag{1.4}$$

$\gamma \in [0,1)$ is a parameter capturing the degree of substitution between e-books. The higher γ, the more the e-books of the two upstream firms are perceived as substitutes. If $\gamma = 0$, the goods are independent, while when $\gamma \to 1$ they tend to become perfect substitutes. The parameter $\lambda \geq 0$ captures the complementarity between e-books and e-readers. If $\lambda = 0$, a change in the e-reader price does not affect the demand of e-books. If $\lambda > 0$, an increase in the e-reader price, on the other hand, will lead to a decrease in the demand of both e-readers and e-books. The parameter $\mu \geq 0$ captures the complementarity between e-readers and e-books. If $\mu = 0$, the demand for e-readers does not depend on the e-book price, while when $\mu > 0$, an increase in the e-book price will lead to a decrease in quantity demanded both for e-reader and for e-book.

I assume that s is chosen by Amazon in order to compensate publishers for the loss in the traditional market. Moreover, I assume that publishers' losses is proportional to Amazon's revenues in the e-book market. Hence, the revenues generated by sales of e-books are split between the publishers and the retailer, with $s \in [0,1]$ denoting the revenues share kept by the latter.

For given revenue shares, downstream firm's profit function is thus defined as follows:

$$\Pi_D = s(p_j q_j + p_{-j} q_{-j}) + p_r q_r, \tag{1.5}$$

whereas the profit function of the upstream firm U_j is given by:

$$\Pi_{U_j} = (1-s)p_j q_j. \tag{1.6}$$

Following Foros et al. (2013), I will compare the equilibrium outcome when prices are set by publishers to the outcome when prices are set by the retailer. In other words, I will compare the outcome when resale price maintenance (RPM) is used and the outcome when it is not (NO RPM). I will focus on symmetric equilibria, in which the downstream firm sells the e-reader and both e-books ($q_{b_j} > 0 \; \forall j = 1, 2$, $q_r > 0$).

1.3.1 Two-way Complementarity when E-books Are Substitutes

I here consider the case in which γ, λ and μ are positive.

No RPM

Without RPM, the downstream firm D sets both e-book retail prices and e-reader prices.

D's optimization problem is given by:

$$\max_{p_{b_1}, p_{b_2}, p_r} \Pi_D.$$

The corresponding first-order conditions are then given by:

$$\frac{\partial \Pi_D}{\partial p_{b_1}} = s\left(q_{b_1} + p_{b_1}\frac{\partial q_{b_1}}{\partial p_{b_1}} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_{b_1}}\right) + p_r\frac{\partial q_r}{\partial p_{b_1}} = 0,$$

$$\frac{\partial \Pi_D}{\partial p_{b_2}} = s\left(p_{b_1}\frac{\partial q_{b_1}}{\partial p_{b_2}} + q_{b_2} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_{b_2}}\right) + p_r\frac{\partial q_r}{\partial p_{b_2}} = 0,$$

$$\frac{\partial \Pi_D}{\partial p_r} = s\left(p_{b_1}\frac{\partial q_{b_1}}{\partial p_r} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_r}\right) + q_r + p_r\frac{\partial q_r}{\partial p_r} = 0.$$

The first-order derivatives of downstream firm D's profit with respect to p_{b_1}, p_{b_2} are, respectively:

$$s[(1-\gamma)(1+\lambda) - 2(1-\lambda\mu)p_{b_1} + 2(\gamma - \lambda\mu)p_{b_2} - (1-\gamma)\lambda p_r] - (1-\gamma)\mu p_r = 0,$$

and

$$s[(1-\gamma)(1+\lambda) - 2(1-\lambda\mu)p_{b_2} + 2(\gamma - \lambda\mu)p_{b_1} - (1-\gamma)\lambda p_r] - (1-\gamma)\mu p_r = 0,$$

which imply that $p_{b_1} = p_{b_2} = p_b$.

The first-order derivative of downstream firm D's profits with respect to p_r is:

$$-(\lambda s + \mu)(p_{b_1} + p_{b_2}) + (1 + \gamma + 2\mu) - 2(1+\gamma)p_r = 0,$$

which implies that:

$$p_r = \frac{(1+\gamma+2\mu) - 2(\mu + \lambda s)p_b}{2(1+\gamma)}.$$

By inserting the expression for p_r in equation $s(1+\lambda-2p_b) - (\mu+\lambda s)p_r = 0$, I obtain the equilibrium prices for e-books and e-readers:

$$p_b^{NO\ RPM} = \frac{(1+\gamma)(2s - \mu + \lambda s) - 2\mu(\mu + \lambda s)}{2[2s(1+\gamma) - (\mu+\lambda s)^2]}, \tag{1.7}$$

and

$$p_r^{NO\ RPM} = \frac{s[1+\gamma+\mu-\lambda s - \lambda(\mu+\lambda s)]}{2s(1+\gamma) - (\mu+\lambda s)^2}. \tag{1.8}$$

Inserting (1.7) and (1.8) into the direct demand functions yields:

$$q_b^{NO\ RPM} = \frac{2s + \mu + \lambda s}{2[2s(1+\gamma) - (\mu+\lambda s)^2]},$$

and

$$q_r^{NO\ RPM} = \frac{s(1+\gamma+\mu+\lambda s)}{2s(1+\gamma) - (\mu+\lambda s)^2}.$$ [3]

The profit for the downstream firm D is given by:

$$\Pi_D^{NO\ RPM} = \frac{s[1+\gamma+2s+2(\mu+\lambda s)]}{2[2s(1+\gamma) - (\mu+\lambda s)^2]}, \tag{1.9}$$

while the profit for the upstream firm U_j is:

$$\Pi_U^{NO\ RPM} = (1-s)\frac{(1+\gamma)[s^2(2+\lambda)^2 - \mu^2] - 2\mu(\mu+\lambda s)[2s - (\mu+\lambda s)]}{4[s(1+\gamma) - (\mu+\lambda s)^2]^2}. \tag{1.10}$$

[3] Note that, in order for quantities to be non-negative, it should be $\gamma > \frac{(\mu+\lambda s)^2}{2s} - 1$.

RPM

When RPM is used, each upstream firm sets the retail price of the e-books, and the downstream firm sets the price for the e-reader. I assume that prices are chosen simultaneously.

Downstream firm D's optimization problem is:

$$\max_{p_r} \Pi_D,$$

while upstream firm U_j's optimization problem is:

$$\max_{p_{b_j}} \Pi_{U_j}.$$

The first-order condition for the downstream firm D is then given by:

$$\frac{\partial \Pi_D}{\partial p_r} = s\left(p_{b_1}\frac{\partial q_{b_1}}{\partial p_r} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_r}\right) + q_r + p_r\frac{\partial q_r}{\partial p_r} = 0, \tag{1.11}$$

and the first-order condition for the upstream firm U_j is given by:

$$\frac{\partial \Pi_{U_j}}{\partial p_{b_j}} = q_{b_j} + p_{b_j}\frac{\partial q_{b_j}}{\partial p_{b_j}} = 0. \tag{1.12}$$

Specifically, the first order derivative of downstream firm D's profit with respect to p_r is given by:

$$-(\lambda s + \mu)(p_{b_1} + p_{b_2}) + (1 + \gamma + 2\mu) - 2(1 + \gamma)p_r = 0, \tag{1.13}$$

while the first-order derivative of upstream firm U_j's profit with respect to p_{b_j} is given by:

$$(1-\gamma)(1+\lambda) - (1-\lambda\mu)p_{b_1} + (\gamma - \lambda\mu)p_{b_2} - \lambda(1-\gamma)p_r - (1-\lambda\mu)p_{b_1} = 0. \tag{1.14}$$

Since the FOCs for the upstream firms are symmetric, it follows that $p_{b_j} = p_{b_{-j}} = p_b$.

By combining the two derivatives, I obtain the optimal prices for e-books and e-readers:

$$p_b^{RPM} = \frac{(1-\gamma)[(2+\lambda)(1+\gamma)-2\lambda\mu]}{2[2(1-\lambda\mu)+(1-\gamma)(\gamma-\lambda^2 s)]}, \tag{1.15}$$

and

$$p_r^{RPM} = \frac{(1+\gamma)(2-\gamma)-2\lambda s(1-\gamma)(1+\lambda)+2\mu(1-\lambda\mu)+\lambda\mu(\gamma-3)}{2[2(1-\lambda\mu)+(1-\gamma)(\gamma-\lambda^2 s)]}. \tag{1.16}$$

By inserting the optimal prices into the direct demand functions, I obtain:

$$q_b^{RPM} = \frac{(1-\lambda\mu)[2(1+\gamma-\lambda\mu)+\lambda(1+\gamma)]}{2(1+\gamma-2\lambda\mu)[2(1-\lambda\mu)+(1-\gamma)(\gamma-\lambda^2 s)]},$$

and

$$q_r^{RPM} = \frac{(1+\gamma^2)(\gamma+2\lambda s-\lambda\mu)+2(1+\gamma)(2+\mu+\lambda\mu^2-\lambda\mu)-4s\lambda\mu(1-\gamma)}{2(1+\gamma-2\lambda\mu)[2(1-\lambda\mu)+(1-\gamma)(\gamma-\lambda^2 s)]}.$$

Since the aim of this work is to investigate whether the wholesale model was profitable for Amazon before Apple entered the market, I will focus on downstream firm's profit.

In order to compare the equilibrium outcome under the two formats, I will study the generic profit function, which is the same with and without RPM.

Substituting the expression for equilibrium e-reader price $p_r = \frac{(1+\gamma+2\mu)-2(\mu+\lambda s)p_b}{2(1+\gamma)}$ (which is the same with and without RPM, since it is derived from the downstream firm's FOC in both cases) and the demand functions into the profit function $\Pi_D = 2sp_bq_b + p_rq_r$, I obtain:

$$\Pi_D = \frac{4[(1+\gamma)(2s-\mu+\lambda s)-2\mu(\mu+\lambda s)]p_b-4[2s(1+\gamma)-(\mu+\lambda s)^2]p_b^2+2(1+\gamma+2\mu)(1+\gamma-2\lambda s)}{4(1+\gamma)(1+\gamma-2\lambda\mu)}.$$

Since $\gamma > \frac{(\mu+\lambda s)^2}{2s} - 1$ (see footnote 1.3.1), this function is a parable with downwards concavity. Hence, downstream firm's profit reaches its maximum value at the vertex, when $p_b = \frac{(1+\gamma)(2s-\mu+\lambda s)-2\mu(\mu+\lambda s)}{2[2s(1+\gamma)-(\mu+\lambda s)^2]}$, i.e., when RPM is not used (see equation (1.7)). In fact, as one would reasonably expect, when downstream firm sets both e-book and e-reader prices, it will choose those prices which maximizes its profit. Without RPM, downstream firm can internalize substitutability between e-books and complementarity between e-books and e-readers, taking advantage from them. By contrast, when RPM is used, downstream firm can only control e-reader pricing. Consequently, its profit will be lower than without RPM. This is also due to the fact that without RPM prices are chosen at the downstream level where there is a monopoly and, hence, no competitive pressure. Consequently, a monopolist will always prefer the wholesale model to the agency model.

Proposition 1. *In a market with two competing upstream firms and one downstream firm, when e-books sales boost e-readers sales and, vice versa, e-readers sales boost e-books sales, downstream firm's profits are always higher without RPM than with RPM, regardless of the degree of substitution and complementarity between the goods.*

In contrast, upstream firms' profits are not always higher without RPM. In fact, upstream firms will prefer RPM for certain values of γ, μ and λ. In that case, firms might have an incentive to collude, since downstream firm and upstream firms will prefer different contracts (no RPM and RPM, respectively).

1.4 Conclusions

The goal of this work was to investigate whether it was profitable for Amazon to adopt a wholesale model, when it controlled both the e-book and the e-reader market (prior to Apple's entry in 2010).

I presented a model based on Foros et al. (2013). They consider a market with a duopoly both upstream and downstream. Foros et al. (2013) compare the equilibrium outcomes under the wholesale model and the agency model. Foros et al. (2013) show that the agency model leads to lower e-book prices if competition is stronger among publishers than among retailers. However, the model does not consider the complementary market for reading devices. In the model I proposed, there are two competing upstream firms, each producing e-books, and one downstream firm, selling both e-books and e-readers. Revenues generated by sales of e-books are split between the upstream firms and the downstream firm. Equilibrium outcomes under two different sale models —wholesale and agency model— are derived and compared. The main result suggests that for a monopolist like Amazon is always more profitable to adopt the wholesale model rather than the agency model. In fact, without RPM, the retailer, by taking control over pricing, can internalize and exploit substitution and complementarity effects between goods to maximize profits. In contrast, when RPM is used, the retailer sets only the e-reader price, while each upstream firm sets the e-book price. Hence, retailer's profit will be lower than without RPM. This is also due to the fact that without RPM prices are chosen at the downstream level where there is a monopoly and, hence, no competitive pressure. This result is consistent with Amazon's decision to adopt the wholesale model, when it was the leader in the global e-book market.

However, as it is, my analysis has some limitations. Further work should be devoted to a deeper study of the effects of the parameters of substitution and complementarity on both upstream and downstream firm's profits.

A possible extension of the model could be to consider a duopoly downstream. In fact, while the model I depicted fits well the case before Apple entered the market (through 2009 Amazon controlled 90% of the e-book market and 60% of the e-reader market), a model with a duopoly would better reflect the situation when Apple entered the e-book market and such a model could possibly explain why Amazon switched to the agency model. In fact, even if the wholesale model is preferred in a monopoly, the agency model might be more profitable when there is competition among retailers. The more intense the competition at the downstream level, the more likely retailers will choose to cede control over prices to the upstream firms. Moreover, I found that the higher the complementarity, the greater is the incentive for the downstream firm (in a monopoly) to adopt the wholesale model. However, when competition in the e-reader market increases and e-books are not tied to a specific devices (in fact, e-books became compatible with different devices), it is not obvious that the downstream firm will still prefer the wholesale model. If competition in the market for reading device increases, e-reader prices would probably fall and the downstream firm will have an incentive to induce higher e-book prices. If the competitive pressure is lower upstream, the retailer will probably choose to cede control over e-book retail pricing to the upstream firms (i.e., the retailer will prefer the agency model).

1.A Appendix

1.A.1 No Complementarity

Here I consider the case in which e-books and e-readers are not related, i.e., I set the parameters of complementarity —λ and μ— equal to zero. I thus analyze the effects of γ on the equilibrium outcomes.

When $\lambda = 0$ and $\mu = 0$, the direct demand curve for e-books is given by:

$$q_{b_j} = \frac{1 - \gamma - p_{b_j} + \gamma p_{b_{-j}}}{1 - \gamma^2}, \qquad j = 1, 2, \tag{A-1}$$

while the direct demand curve for e-readers is given by:

$$q_r = 1 - p_r. \tag{A-2}$$

No RPM

I first consider the case in which prices are determined by the downstream firm. Without RPM, downstream firm D's optimization problem is given by:

$$\max_{p_{b_1}, p_{b_2}, p_r} \Pi_D, \tag{A-3}$$

where Π_D is defined by (1.5).

The first-order conditions for downstream firm D are given by:

$$\frac{\partial \Pi_D}{\partial p_{b_1}} = s\left(q_{b_1} + p_{b_1}\frac{\partial q_{b_1}}{\partial p_{b_1}} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_{b_1}}\right) = 0,$$

$$\frac{\partial \Pi_D}{\partial p_{b_2}} = s\left(p_{b_1}\frac{\partial q_{b_1}}{\partial p_{b_2}} + q_{b_2} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_{b_2}} \right) = 0,$$

$$\frac{\partial \Pi_D}{\partial p_r} = q_r + p_r\frac{\partial q_r}{\partial p_r} = 0.$$

By substituting, FOCs become:

$$s(1-\gamma-2p_{b_1}+2\gamma p_{b_2}) = 0,$$

$$s(1-\gamma-2p_{b_2}+2\gamma p_{b_1}) = 0,$$

$$1-2p_r = 0.$$

The first two equations are satisfied when $p_{b_1} = p_{b_2}$. Hence, equilibrium prices are:

$$p_b^{NO\ RPM} = \frac{1}{2}, \tag{A-4}$$

$$p_r^{NO\ RPM} = \frac{1}{2}. \tag{A-5}$$

The equilibrium e-book price does not depend on how the e-books are perceived to be similar (γ), nor on how revenues are split between suppliers and retailers (s). This is due to the fact that without RPM the downstream firm can internalize the substitutability between goods.

Since in this setting e-readers and e-books are independent, the price of the e-reader does not depend on the degree of substitution between e-books (γ), nor on the e-book sales revenue share kept from the downstream firm (s).

Inserting (A-4) and (A-5) into the direct demand functions, yields:

$$q_b^{NO\ RPM} = \frac{1}{2(1+\gamma)},$$

and

$$q_r^{NO\ RPM} = \frac{1}{2}.$$

Less differentiation, i.e. greater substitutability, between e-books reduces the aggregate quantity of e-books.

The profit for the downstream firm D is then:

$$\Pi_D^{NO\ RPM} = \frac{2s+1+\gamma}{4(1+\gamma)},$$

while the profit for the upstream firm U_j is:

$$\Pi_{U_j}^{NO\ RPM} = \frac{1-s}{4(1+\gamma)}.$$

When γ increases, in equilibrium, e-reader quantity and both e-book and e-reader prices hold steady, but e-book quantity decreases. Hence, when γ increases, profits decrease.

Industry profits are:

$$\Pi_I^{NO\ RPM} = \frac{3+\gamma}{4(1-\gamma)}.$$

RPM

I now consider the case when RPM is used. With RPM, the downstream firm sets the price for the e-reader, while each upstream firm sets the retail price of its own e-book.

Downstream firm D's optimization problem is thus defined as follows:

$$\max_{p_r} \Pi_D,$$

while upstream firm U_j's optimization problem is:

$$\max_{p_{b_j}} \Pi_{U_j}.$$

The first-order condition for the downstream firm D with respect to p_r is given by:

$$\frac{\partial \Pi_D}{\partial p_r} = q_r + p_r \frac{\partial q_r}{\partial p_r} = 0,$$

and the first-order condition for the upstream firm U_j with respect to p_{b_j} is given by:

$$\frac{\partial \Pi_{U_j}}{\partial p_{b_j}} = (1-s)\left(q_{b_j} + p_{b_j} \frac{\partial q_{b_j}}{\partial p_{b_j}} \right) = 0.$$

The derivatives with respect to p_r and p_{b_1} are given by, respectively:

$$1 - 2p_r = 0,$$

and

$$1 - \gamma - 2p_{b_1} + \gamma p_{b_2} = 0.$$

Since the FOCs for the upstream firms are symmetric, it follows that $p_{b_1} = p_{b_2} = p_b$. Solving for the Nash equilibrium, I obtain:

$$p_b^{RPM} = \frac{1-\gamma}{2-\gamma}, \tag{A-6}$$

and

$$p_r^{RPM} = \frac{1}{2}. \tag{A-7}$$

Here, the higher γ, the lower the e-book prices. As one would reasonably expect, the more intense the competition, the lower the prices. As in the case without RPM, the e-reader price is independent of both γ and s.

Inserting the optimal prices into the direct demand functions defined by (A-1) and (A-2) yields:

$$q_b^{RPM} = \frac{1}{(2-\gamma)(1+\gamma)},$$

and

$$q_r^{RPM} = \frac{1}{2}.$$

Profits are then given by:

$$\Pi_D^{RPM} = \frac{2s(1-\gamma)}{(2-\gamma)^2(1+\gamma)} + \frac{1}{4},$$

$$\Pi_{U_j}^{RPM} = \frac{(1-s)(1-\gamma)}{(2-\gamma)^2(1+\gamma)}.$$

Industry profits are hence given by:

$$\Pi_I^{RPM} = \frac{12 - 8\gamma - 3\gamma^2 + \gamma^3}{4(2-\gamma)^2(1+\gamma)}.$$

The higher γ, the lower the profits.

By comparing the equilibrium outcome without RPM and the equilibrium outcome with RPM, I obtain the following result:

Proposition 2. *In a market with two competing upstream firms and one downstream firm, when demand for e-readers is independent of demand for e-books and vice versa, e-book prices, downstream firm's profits, upstream firms' profits, and industry profits are always higher without RPM than with RPM, regardless of the degree of substitution between the goods. E-reader prices are the same with and without RPM.*

It follows that each firm will prefer the wholesale model. In particular, the upstream firm will choose to cede control over retail pricing to the downstream firm, since it will induce higher prices. The reason is that with RPM prices are determined at the level where the competitive pressure is higher (the upstream level). By contrast, without RPM, prices are determined by a monopolistic firm, which will thus set industry-maximizing prices. Since both downstream firm and upstream firms earn higher profits without RPM, no firm would prefer RPM. Hence, where there is no complementarity, firms have no incentive to collude when they contract the sale model.

1.A.2 One-way Complementarity When E-books Are Independent

In the following sections, I will present the case when e-books are independent, and e-books and e-readers are complement products. In particular, I will first consider the case in which e-books are independent ($\gamma = 0$) and changes in e-reader prices do not have effects on the demanded quantity of e-books ($\lambda = 0$). In this way, I can analyze the effects of changes in e-book prices on the demanded quantity of e-readers. Thereafter, I will study the case in which e-books are still independent ($\gamma = 0$), and the demanded quantity of e-readers does not depend on the e-book price ($\mu = 0$). Thus, I can investigate the effects of changes in e-reader prices on demand of e-books.

The Effect of Changes in E-book Prices on E-readers Demand

Now I consider the case when e-books are independent ($\gamma = 0$) and the demand of e-books does not depend on the e-reader price ($\lambda = 0$), but e-books sales boost e-reader sales.

When $\gamma = 0$ and $\lambda = 0$, the direct demand curve for e-books is given by:

$$q_{b_j} = 1 - p_{b_j}, \tag{A-8}$$

and the direct demand curve for e-readers is given by:

$$q_r = 1 + 2\mu - p_r - \mu(p_{b_j} + p_{b_{-j}}). \tag{A-9}$$

No RPM

Again, I first consider the case in which the downstream firm sets both the e-book and e-reader price.

Downstream firm D's optimization problem is given by:

$$\max_{p_{b_1}, p_{b_2}, p_r} \Pi_D, \tag{A-10}$$

with Π_D defined by (1.5).

The corresponding first-order conditions for D with respect to p_{b_1}, p_{b_2} and p_r are given by:

$$\frac{\partial \Pi_D}{\partial p_{b_1}} = s\left(q_{b_1} + p_{b_1}\frac{\partial q_{b_1}}{\partial p_{b_1}}\right) + p_r\frac{\partial q_r}{\partial p_{b_1}} = 0,$$

$$\frac{\partial \Pi_D}{\partial p_{b_2}} = s\left(q_{b_2} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_{b_2}}\right) + p_r\frac{\partial q_r}{\partial p_{b_2}} = 0,$$

$$\frac{\partial \Pi_D}{\partial p_r} = q_r + p_r\frac{\partial q_r}{\partial p_r} = 0.$$

By substituting, FOCs become:

$$s(1 - 2p_{b_1}) - \mu p_r = 0,$$

$$s(1 - 2p_{b_2}) - \mu p_r = 0,$$

$$1 + 2\mu - 2p_r - \mu(p_{b_1} + p_{b_2}) = 0.$$

The first two equations are satisfied when $p_{b_1} = p_{b_2} = p_b$, while the third derivative implies that:

$$p_r = \frac{1 + 2\mu - \mu(p_{b_1} + p_{b_2})}{2}.$$

Inserting the expression for p_r into $s(1 - 2p_b) - \mu p_r = 0$ yields the equilibrium prices:

$$p_b^{NO\ RPM} = \frac{2s - 2\mu^2 - \mu}{2(2s - \mu^2)}, \tag{A-11}$$

and

$$p_r^{NO\ RPM} = \frac{s(1 + \mu)}{2s - \mu^2}. \tag{A-12}$$

The price of e-books is always decreasing in μ, while the price of e-readers is increasing in μ. Substituting (A-11) and (A-12) into the direct demand functions yields:

$$q_b^{NO\ RPM} = \frac{2s+\mu}{2(2s-\mu^2)},$$

and

$$q_r^{NO\ RPM} = \frac{s(1+\mu)}{2s-\mu^2}.$$

A higher degree of complementarity between e-books and e-readers (i.e., a higher μ) increases the size of both the e-book market and the e-reader market.

The profit for the downstream firm D is then:

$$\Pi_D^{NO\ RPM} = \frac{s(2s+2\mu+1)}{2(2s-\mu^2)},$$

while the profit for the upstream firm U_j is given by:

$$\Pi_{U_j}^{NO\ RPM} = \frac{(1-s)(2s+\mu)(2s-2\mu^2-\mu)}{4(2s-\mu^2)^2}.$$

Both the downstream firm's profit and the upstream firms' profit are increasing in μ.[4]

Finally, industry profits are:

$$\Pi_I^{NO\ RPM} = \frac{(2s+\mu)(2s-2\mu^2-\mu)+2s^2(1+\mu)^2}{2(2s-\mu^2)^2}.$$

The higher μ, the higher the profits.

RPM

When RPM is used, the downstream firm sets the price for the e-reader, and the upstream firms choose the prices for the e-books.

[4]Note that the participation constraint of upstream firms is satisfied if and only if $\frac{-1+\sqrt{1+16s}}{4} \leq \mu < \sqrt{2s}$. Once this condition holds, also the participation constraint of the downstream firm is satisfied.

Downstream firm D's optimization problem is:

$$\max_{p_r} \Pi_D,$$

while upstream firm U_j's maximization problem is:

$$\max_{p_{b_j}} \Pi_{U_j}.$$

The first-order condition for the downstream firm D is then given by:

$$\frac{\partial \Pi_D}{\partial p_r} = q_r + p_r \frac{\partial q_r}{\partial p_r} = 0,$$

and the first-order condition for the upstream firm U_j is given by:

$$\frac{\partial \Pi_{U_j}}{\partial p_{b_j}} = (1-s)\left(q_{b_j} + p_{b_j} \frac{\partial q_{b_j}}{\partial p_{b_j}} \right) = 0$$

Specifically, the derivatives with respect to p_r and p_{b_j} are:

$$1 + 2\mu - 2p_r - \mu(p_{b_1} + p_{b_2}) = 0,$$

and

$$1 - 2p_{b_1} = 0$$

Setting $p_{b_1} = p_{b_2} = p_b$ (because upstream firms are symmetric), and solving for the Nash equilibrium, yields:

$$p_b^{RPM} = \frac{1}{2}, \tag{A-13}$$

and

$$p_r^{RPM} = \frac{1+\mu}{2}. \tag{A-14}$$

In contrast to the case without RPM, the price of e-books does not depend on μ. Indeed, since the upstream firm earns profits from e-book sales only, when it sets the e-book price, it does not take into consideration the effects of its pricing strategy on the demand for e-readers. By contrast, the e-reader price, set by the downstream firm, is increasing in μ. In fact, an increase in μ shifts the demand function rightwards (see equation (1.2)), and, hence, prices increase.

By inserting (A-13) and (A-14) into the direct demand functions, I obtain:

$$q_b^{RPM} = \frac{1}{2},$$

and

$$q_r^{RPM} = \frac{1+\mu}{2}.$$

Since the demand of e-books and the equilibrium e-book price do not depend on μ, the quantity in equilibrium is independent of μ as well. By contrast, the demanded quantity of e-readers is increasing in μ.

The upstream firm's profit function in equilibrium is then given by:

$$\Pi_D^{RPM} = \frac{2s + (1+\mu)^2}{4}, \tag{A-15}$$

and the profit for the upstream firm U_j is given by:

$$\Pi_{U_j}^{RPM} = \frac{(1-s)}{4}. \tag{A-16}$$

Since prices and quantities of e-books are independent of μ, the upstream firm's profit is independent of μ as well. By contrast, the downstream firm's profit is increasing in μ.

The industry profits are given by:

$$\Pi_I^{RPM} = \frac{\mu^2 + 2\mu + 3}{4}.$$

By comparing the equilibrium outcome without RPM and the equilibrium outcome with RPM, I obtain the following result:

Proposition 3. *In a market with two upstream firms and one downstream firm, when e-books are independent but more sales of e-books increase demand for e-readers, e-reader prices, downstream firm's profits and industry profits are higher without RPM than with RPM. E-book prices and upstream firms' profits are lower without RPM than with RPM.*

The intuition for this result is that the upstream firm cares only about the demand for e-books, while the downstream firm cares also about the demand for e-readers. Therefore, e-book prices will be higher when they are set by the upstream firm, i.e., when RPM is used. In contrast, when the downstream firm sets both the e-book prices and the e-reader price, it internalizes the complementarity between the goods. Since —with $\mu > 0$— an increase in the price of e-books leads to a decrease in both the price and the quantity of e-readers (and hence to a decrease in e-readers revenues), the downstream firm, without RPM, will set a lower e-book price than the upstream firm with RPM (whose profit does not depend on μ). Therefore, while e-book prices are lower, e-reader prices are greater without RPM.

Moreover, in this case, firms might have an incentive to collude. More into details, since downstream firm and upstream firms will prefer different contracts (no RPM and RPM, respectively), once we allow for profit transfers, the downstream firm will offer the upstream firms an amount for which not using RPM will be profitable also for the upstream firms. Such an amount will indeed be higher than the loss deriving from not using RPM.

The Effect of Changes in E-reader Prices on E-books Demand

Here I study the case in which e-books are independent ($\gamma = 0$) and the demand of e-readers does not depend on the e-book price ($\mu = 0$), but, vice versa, the demand of e-books depends on the e-reader price ($\lambda > 0$).

When $\gamma = 0$ and $\mu = 0$, the direct demand curve for e-books is given by:

$$q_{b_j} = 1 + \lambda - p_{b_j} - \lambda p_r, \tag{A-17}$$

and the direct demand curve for e-readers is given by:

$$q_r = 1 - p_r. \tag{A-18}$$

No RPM

Here I consider the case in which the e-book and the e-reader price are set by the downstream firm. Downstream firm D's optimization problem is given by:

$$\max_{p_{b_1}, p_{b_2}, p_r} \Pi_D,$$

with Π_D defined by (1.5).

The corresponding first-order conditions for D with respect to p_{b_1}, p_{b_2} and p_r are given by:

$$\frac{\partial \Pi_D}{\partial p_{b_1}} = s\left(q_{b_1} + p_{b_1}\frac{\partial q_{b_1}}{\partial p_{b_1}}\right) = 0,$$

$$\frac{\partial \Pi_D}{\partial p_{b_2}} = s\left(q_{b_2} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_{b_2}}\right) = 0,$$

$$\frac{\partial \Pi_D}{\partial p_r} = s\left(p_{b_1}\frac{\partial q_{b_1}}{\partial p_r} + p_{b_2}\frac{\partial q_{b_2}}{\partial p_r}\right) + q_r + p_r\frac{\partial q_r}{\partial p_r} = 0.$$

The derivatives with respect to p_{b_1}, p_{b_2} are given by, respectively:

$$1 + \lambda - 2p_{b_1} - \lambda p_r = 0,$$

and

$$1 + \lambda - 2p_{b_2} - \lambda p_r = 0,$$

which imply that $p_{b_1} = p_{b_2} = p_b$.

The derivative with respect to p_r is given by:

$$1 - 2p_r - \lambda s(p_{b_1} + p_{b_2}) = 0,$$

which implies that:

$$p_r = \frac{1 - \lambda s(p_{b_1} + p_{b_2})}{2}.$$

Inserting the expression for p_r into $1 + \lambda - 2p_b - \lambda p_r = 0$ yields the equilibrium prices:

$$p_b^{NO\ RPM} = \frac{2+\lambda}{2(2-\lambda^2 s)}, \tag{A-19}$$

and

$$p_r^{NO\ RPM} = \frac{1-\lambda s-\lambda^2 s}{2-\lambda^2 s}. \tag{A-20}$$

The price of e-books is always increasing in λ, while the price of e-readers is decreasing in λ. Substituting (A-19) and (A-20) into the direct demand functions yields:

$$q_b^{NO\ RPM} = \frac{2+\lambda}{2(2-\lambda^2 s)},$$

and

$$q_r^{NO\ RPM} = \frac{1+\lambda s}{2-\lambda^2 s}.$$

In equilibrium, both e-book quantity and e-reader quantity are increasing in λ.

The profit for the downstream firm D is then:

$$\Pi_D^{NO\ RPM} = \frac{2s+2\lambda s+1}{2(2-\lambda^2 s)},$$

while the profit for the upstream firm U_j is given by:

$$\Pi_{U_j}^{NO\ RPM} = \frac{(1-s)(2+\lambda)^2}{4(2-\lambda^2 s)^2}.$$

Profits are increasing in λ.[5]

Hence, industry profits are:

$$\Pi_I^{NO\ RPM} = \frac{6+4\lambda+\lambda^2-2\lambda^2 s(1+s+\lambda s)]}{2(2-\lambda^2 s)^2}.$$

[5]In order for quantities and profits to be non-negative, it should be $0 \leq \lambda < \sqrt{\frac{s}{2}}$.

RPM

When RPM is used, the downstream firm sets the price for the e-reader, and the upstream firms choose the prices for the e-books.

Downstream firm D's optimization problem is:

$$\max_{p_r} \Pi_D,$$

while upstream firm U_j's problem is:

$$\max_{p_{b_j}} \Pi_{U_j}.$$

The first-order condition for the downstream firm D is then given by:

$$\frac{\partial \Pi_D}{\partial p_r} = q_r + p_r \frac{\partial q_r}{\partial p_r} = 0$$

and the first-order condition for the upstream firm U_j is given by:

$$\frac{\partial \Pi_{U_j}}{\partial p_{b_j}} = (1-s)\left(q_{b_j} + p_{b_j} \frac{\partial q_{b_j}}{\partial p_{b_j}} \right) = 0$$

The derivatives with respect to p_{b_j} and p_r are given by:

$$1 + \lambda - 2p_{b_j} - \lambda p_r = 0,$$

and

$$1 - 2p_r - \lambda s(p_{b_1} + p_{b_2}) = 0.$$

Since the first-order conditions for the upstream firms U_1 and U_2 are symmetric, it follows that $p_{b_1} = p_{b_2} = p_b$.

By combining the two first-order derivatives, I obtain the optimal prices:

$$p_b^{RPM} = \frac{2+\lambda}{2(2-\lambda^2 s)}, \tag{A-21}$$

and

$$p_r^{RPM} = \frac{1-\lambda s-\lambda^2 s}{2-\lambda^2 s}. \tag{A-22}$$

Equilibrium prices are the same with and without RPM. The reason is that, in this settings, e-books are independent and their price does not affect the demand of e-readers. However, the e-reader price does affect the demand for e-books. As a consequence, e-book prices will be the same under the two business formats, independently of who —upstream or downstream firm— sets the prices.

Consequently, also the equilibrium quantity of e-books and e-readers —obtained by substituting (A-21) and (A-22) into the direct demand functions —are equivalent to those without RPM:

$$q_b^{RPM} = \frac{2+\lambda}{2(2-\lambda^2 s)},$$

and

$$q_r^{RPM} = \frac{1+\lambda s}{2-\lambda^2 s}.$$

Hence, also downstream firm's and upstream firms' profits are the same under the two different formats, that is:

$$\Pi_D^{RPM} = \frac{2s+2\lambda s+1}{2(2-\lambda^2 s)},$$

and

$$\Pi_{U_j}^{RPM} = \frac{(1-s)(2+\lambda)^2}{4(2-\lambda^2 s)^2}.$$

Finally, industry profits are:

$$\Pi_I^{RPM} = \frac{8+\lambda[(\lambda-4)-4\lambda s(1+s-\lambda s)]}{4(2-\lambda^2 s)^2}.$$

Proposition 4. *In a market with two upstream firms and one downstream firm, when e-books are independent and demand for e-readers is independent of demand for e-books, but e-readers sales boost e-books sales, e-book prices are the same with and without RPM, and this is true for e-reader prices, downstream firm's profits, upstream firms' profits and industry profits as well, regardless of the degree of complementarity between e-books and e-readers (λ).*

In this case, firms are indifferent between NO RPM and RPM, since their profits are the same in both cases. Hence, they have no incentive to collude when they contract the sale model.

CHAPTER 2

Price Wars in Two-Sided Markets: The Case of the UK Quality Newspapers in the '90s

Co-authored with Lapo Filistrucchi (University of Florence) and Stefan Behringer (Universität Duisburg-Essen)

2.1 Introduction

In this work we investigate the price war in the UK weekly quality newspaper industry in the 1990s. At the time this was discussed as a possible case of predatory pricing. However, recent theoretical advances in economics suggest that the two-sidedness of the newspaper market should also be take into account.

A two-sided market (see Armstrong, 2006, Rochet and Tirole, 2006) is a market where two distinct groups of consumers interact via platforms. The benefit for each group joining the platform depends on the size of the other group. In other words, indirect network effects arise; however, they are not internalized by end-users. The intermediary (i.e., the platform) does internalize such effects, even if it does not necessarily emerge to solve this externality problem (Evans, 2003).

Because of such network effects, games might have multiple Nash equilibria, and obtaining robust predictions about comparative statics might be difficult. The solution concept of *insulating equilibrium* proposed by White and Weyl (2016) substantially simplifies this problem. Similarly to the work by Shaked and Sutton (1982), in White and Weyl (2016) firms are assumed to take other firms' quality choice as given.

The media market is a typical two-sided market (see Anderson and Gabszewicz, 2006), as a media firm sells content to consumers—i.e. readers, viewers, or listeners—, and advertising space to advertisers. The firm knows that the number (and characteristics) of consumers of content influence the demand for advertising space whereas, vice versa, depending on the media product, the number (or concentration) of advertising spaces may influence the demand from consumers. Weyl (2010)'s monopoly analysis is also accompanied by a motivation from the newspaper industry.

In the case of newspapers, a newspaper with a higher market share will face a higher demand for its advertising slots for any given advertising tariff. Whether readers like or dislike advertising will rather depend on the particular publication (see Chandra and Kaiser, 2015).

Some previous theoretical works have modeled newspaper competition as taking place on the political line using the Hotelling (1929) model of horizontal product differentiation. Among them, Gabszewicz et al. (2002) endogenize the location (i.e., the differentiation) choice at a first stage, whereas in most models location is only exogenous. Such endogenous political locations represent a crucial factor to understand the events in the UK quality newspaper market, and in Behringer and Filistrucchi (2015b) we extend the model to more than two firms.

Due to the complexity of the theoretical modeling and the substantial data requirements, structural econometric literature on the media as two-sided markets is still quite scarce. Rysman (2004) analyzes the market for yellow pages in the U.S., and shows that network effects between advertisers and readers are indeed present. He also considers whether the market benefits from monopoly (which takes advantage of network effects) or oligopoly (which reduces market power), and finds that a more competitive market is preferable.

Kaiser and Wright (2006) estimate an adapted version of Armstrong (2006) model of competition in a two-sided market where magazines compete as Hotelling duopolists, and find that, due to the presence of indirect network effects, in Germany the readers' side of the market is subsidized by the advertisers. Argentesi and Filistrucchi (2007) test for market power in the national daily newspaper market in Italy, concluding that the four main national daily newspapers have been colluding on the cover price but not on the advertising one. Fan (2013) analyzes the market for daily newspapers in the U.S., and simulates some proposed mergers among them.

The candidate explanation for the observed price war we look at is a change in the optimal financing mix of newspapers that followed a steady increase in the demand for advertising.[1]

In our models we consider demand for products on both sides of the platforms, and profit maximization in a monopolistic, oligopolistic, and collusive setting. We show that the observed increase in the share of advertising revenues relative to readership revenue can be explained by an exogenous increase in the advertising demand.

2.2 The UK Newspaper Industry in the 1990s

The labor force of the UK newspaper industry was heavily unionized when in February 1981 Rupert Murdoch's News International Newspaper Ltd. (NIN) purchased The Times. During the 1980s, production was moved from Fleet Street to a facility in the London district of Wapping, where newspapers could be produced using modern offset lithography instead of the hot-metal and labor-intensive linotype method.

[1] See Anderson and Jullien, 2016 for a survey of advertising-financed business models in two-sided media markets, and Angelucci and Cagé, 2016 for closely related work.

At the time, NIN owned The Times, the Sunday Times, the Sun, and the News of the World. When the print unions announced a strike, NIN activated the new plant, and hired members of the Electrical, Electronic, Telecommunications, and Plumbing Union (EETPU). This led to the "Wapping dispute" from January 1986 to February 1987 which changed the history of UK industrial relations and of the newspaper industry in the UK.

Despite these events, during the early 1990s, cover prices in the UK quality broadsheet newspaper market, composed of The Times, The Independent, The Guardian, and The Daily Telegraph, were relatively stable.

Then, on 6 September 1993, The Times cut its price from 45p to 30p, while the Guardian and The Independent were still priced at 45p, and the Daily Telegraph at 48p. On 12 October 1993 The Independent, which had experienced a significant decrease in circulation revenues, was forced to increase its price from 45p to 50p. On 24 June 1994 The Times reduced its price again from 30 to 20p, followed by The Telegraph which lowered its price from 48p to 30p. On 1 August 1994 also The Independent decreased its price from 50 to 30p, in order to boost circulation, which had dropped by 20% since September 1993. The Office of Fair Trading (OFT) opened an investigation after a complaint by The Independent, which argued that the price cut was targeted against it[2]. According to The Independent, this alleged predatory pricing cost The Times about £ 50,000 per day.

On 21 October 1994 the OFT closed the investigation, holding that it could not be considered a case of predatory pricing. Afterward newspaper cover prices started to increase: on 3 July 1995 both The Times and The Telegraph increased cover prices by 5p, to 25p and 35p respectively. Soon after, on 17 July, also The Independent raised its price to 35p. In November 1995 The Times and The Telegraph increased their prices by other 5p. From January 1996, when also The Independent raised

[2] "When the Independent was launched in 1986, it took more readers from The Times than The Guardian or The Telegraph [...] It has been the Independent holding back The Times ever since"("Media analysts say Times' cut is commercial madness", The Independent, 3 September, 1993).

its prices again, the market for newspaper experienced again a period of stable prices.

Between August 1993 and January 1996 The Times increased its circulation market share from about 17% to 28%, whereas The Independent and The Daily Telegraph decreased their market share from 16% to 12%, and from 49% to 43%, respectively. The market share of The Guardian decreased a little too.

2.3 Predatory Pricing

The evidence brought forward at the time is not sufficient to establish a case of predatory pricing, as it has neglected the critical two-sidedness of the firms. The standard empirical test for predatory pricing in a single-sided market is the Areeda-Turner rule, according to which a price is predatory if it is below the short-run marginal cost. However, this condition is only necessary but not sufficient. It is also necessary to check whether the pricing strategy is likely to lead to the exit from the market of the targeted competitor and whether the predator can expect to recoup the short run losses in the long run.

Yet, as discussed in Evans (2003), in a two-sided market the Areeda-Turner rule is not even a necessary condition, and therefore cannot be applied. The reason is that a firm in a two-sided market acts as a platform, and sells two products or services to two distinct groups of consumers, and recognizes that the demand from one type of consumers depends on the demand from the other type of consumers, and vice versa. It is therefore conceivable that, by pricing below marginal cost on one side of the market, a firm is increasing demand on that side, and thus boosting demand on the other side, with an overall positive effect on its profits. Indeed, depending on the size of the price elasticity in the two-sides of the market, and on the size of the network effects, even a monopolist platform might find it profitable to lower the price below marginal cost on one side of the market. Testing for predatory pricing in two-sided markets should therefore take into account the presence of the critical network effects between the two sides.

In Behringer and Filistrucchi (2015a) we propose such an extension of the two-sided market predation definition. We argue that, despite the huge price cuts, the pricing strategy of The Times in 1993 and 1994 could not be presumed predatory according to an Areeda-Turner rule properly modified to take into account two-sidedness of the market. Accepting the claim of The Independent that the average variable cost of The Times was 32.5p, we calculate the overall markups of the Times from 1991 to 1997 per copy sold, taking into account both the cover price and the revenues from advertising, as suggested by the two-sided Areeda-Tuner rule.

2.4 An Empirical Finding

We set out to shed some light on issues of this price war using a model of a two-sided market. We observe that the share of readership revenue over total revenue is steadily declining during the 1990 for all firms (see Figure 2.1). For The Independent this ranges from over 70% to just above 20%, and even for The Guardian, who did not adjust cover prices to readers, from over 70% to 30%.

Hence, a candidate explanation of the observed price war is that The Times was first to react to this increase in advertising demand, and was willing to sacrifice readership revenue, thereby generating even higher indirect network benefits for advertisers.

In what follows we build a theoretical model to derive the conditions under which the optimal share of advertising financing increases when the size of the advertising market increases.

2.5 A Theoretical Model

Let

$$u = \bar{u} - p - \gamma a$$

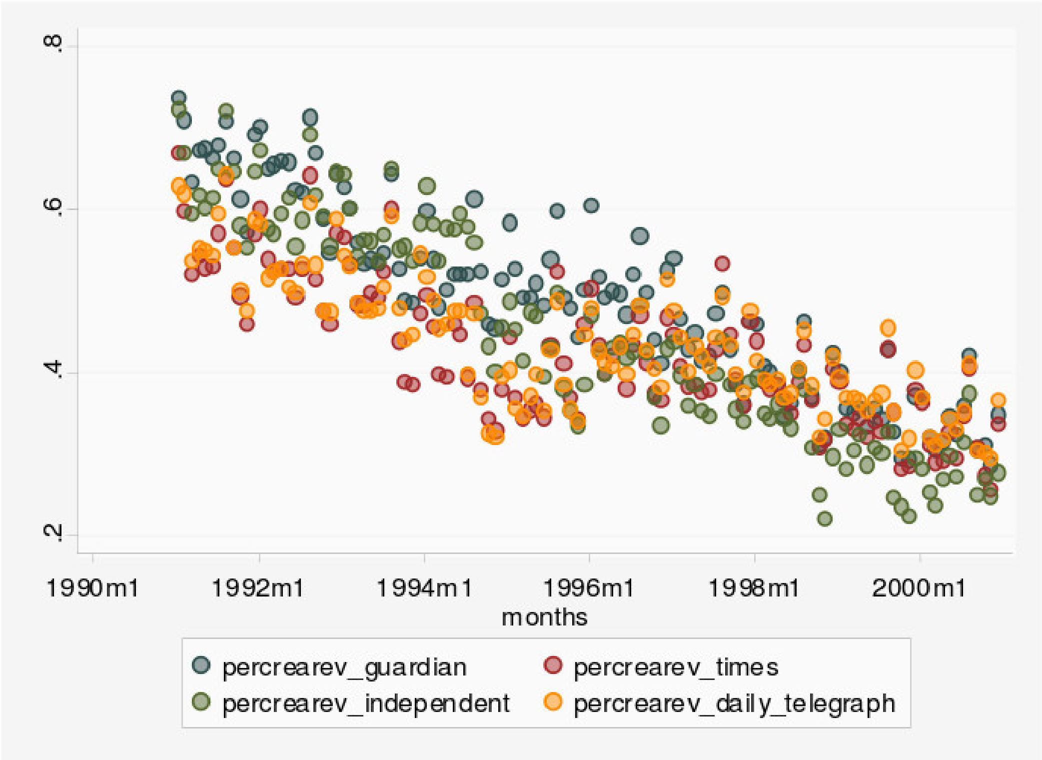

Figure 2.1: Percentage of revenues from readers over total revenues

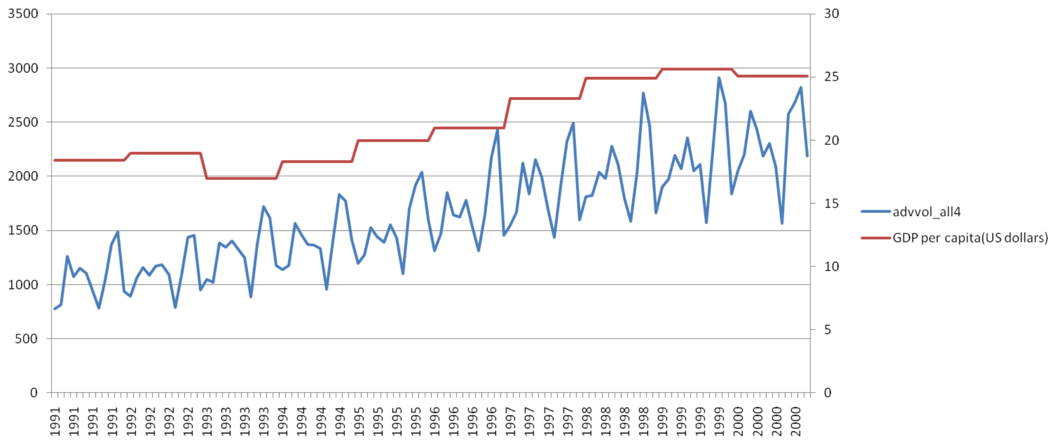

Figure 2.2: Advertising volume and per capita GDP

be the total monetary utility of reading a newspaper with cover price $p \geq 0$ and some quantity of advertising $a \geq 0$. The scalar γ measures the marginal effect of advertising.

We assume that the *per copy revenue* from advertising is a function $R(a, m)$, where $m > 0$ is an exogenous parameter shifting advertising demand outward such that $\frac{\partial R}{\partial m} > 0$. We also assume that $\frac{\partial R}{\partial a} > 0$, and $\frac{\partial^2 R}{\partial a^2} < 0$, (i.e. R is increasing and strictly concave in a). Moreover, we assume that $\frac{\partial^2 R}{\partial a \partial m} > 0$ (i.e. marginal revenue from ads is increasing in the exogenous shift parameter m).

We want to find the conditions for the optimal Revenue Ratio

$$RR = \frac{pN}{R(a, m)N} = \frac{p^*(m)}{R(a^*, m)} \tag{2.1}$$

to be a decreasing function of m.

2.5.1 Monopoly

A monopolist sets p and a so as to maximize

$$\Pi = [R(a, m) + p - c]N(p + \gamma a) \tag{2.2}$$

where $N(\cdot)$ is the number of copies sold of the newspaper, which depends on both the cover price and the ads quantity. We assume $N'(\cdot) < 0$.

To find the optimal advertising quantity a^*, the profit maximization problem of the monopolist can be rewritten as:

$$\max_a \widetilde{\Pi} = [R(a, m) + f - \gamma a - c]N(f). \tag{2.3}$$

where $f = p + \gamma a$ is the monetary cost due to both the cover price and the disutility from advertising.

The first order condition w.r.t. a is

$$F_a \equiv \frac{\partial R(a, m)}{\partial a} - \gamma = 0, \tag{2.4}$$

which implicitly defines the optimal advertising level $a^* = a^*(m)$.

Lemma 2.1. *The optimal level of advertising* a^* *is increasing in the exogenous shift parameter* m.

Proof. By the implicit function theorem

$$\frac{da^*}{dm} = -\frac{\frac{\partial F_a}{\partial m}}{\frac{\partial F_a}{\partial a^*}} = -\frac{\frac{\partial^2 R(a,m)}{\partial a \partial m}}{\frac{\partial^2 R(a,m)}{\partial a^2}},$$

which is positive since $\frac{\partial^2 R(a,m)}{\partial a \partial m} > 0$ and $\frac{\partial^2 R(a,m)}{\partial a^2} < 0$ by assumption. □

Corollary 1. *The revenue from advertising* $R(a^*(m), m)$ *is increasing in the exogenous shift parameter* m.

Proof.

$$\frac{dR(a^*(m), m)}{dm} = \frac{\partial R(a^*)}{\partial a}\frac{\partial a^*(m)}{\partial m} + \frac{\partial R(m)}{\partial m}$$

is positive since $\frac{\partial R(a^*)}{\partial a} > 0$ and $\frac{\partial R(m)}{\partial m} > 0$ by assumption, and $\frac{\partial a^*(m)}{\partial m} > 0$ by Lemma 2.1. □

The first order condition w.r.t. p is instead

$$F_p \equiv N(p + \gamma a) + [R(a, m) + p - c]\frac{\partial N\left(p + \gamma a\right)}{\partial p} = 0 \qquad (2.5)$$

which implicitly defines the optimal price $p^* = p^*\left(a\left(m\right), m\right)$.

We assume that the second order condition for profit maximization (SOC) is satisfied (see Appendix 2.A).

Lemma 2.2. *The optimal cover price* p^* *is decreasing in the exogenous shift parameter* m *if and only if* $\gamma > \gamma_m = -\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}}{\frac{\partial a^*}{\partial m}\frac{\partial F_p}{\partial p^*}}$.
In particular, the optimal cover price p^* *is decreasing in* m *if* $\gamma \geq 0$.

Proof. By substituting $a^* = a^*(m)$ into (2.5) we obtain:

$$F_p \equiv N(p+\gamma a^*(m)) + [R(a^*(m),m)+p-c]\frac{\partial N\left(p+\gamma a^*(m)\right)}{\partial p} = 0.$$

By the implicit function theorem

$$\frac{dp^*}{dm} = -\frac{\frac{\partial F_p}{\partial m}}{\frac{\partial F_p}{\partial p^*}} = -\frac{\frac{\partial F_p}{\partial a}\frac{\partial a^*}{\partial m} + \frac{\partial F_p}{\partial m}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}}$$

$$= -\frac{\left(\frac{\partial N(p+\gamma a)}{\partial a} + \frac{\partial R(a,m)}{\partial a}\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p \partial a}\right)\frac{\partial a^*}{\partial m}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}}$$

$$+ \frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}} \tag{2.6}$$

Note that $\frac{\partial N(p+\gamma a)}{\partial p} = \frac{dN(f)}{df}\frac{df}{dp} = \frac{dN(f)}{df}$. Hence, $\frac{\partial^2 N(p+\gamma a)}{\partial p \partial a} = \frac{d^2 N(f)}{df^2}\frac{df}{da} = \gamma\frac{d^2 N(f)}{df^2}$ as then the term in brackets in (2.6) is positive.

By rearranging terms in (2.6) we obtain:

$$\frac{dp^*}{dm} = -\left(\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p} + \left(\frac{\partial R(a,m)}{\partial a} - \gamma\right)\frac{\partial N(p+\gamma a)}{\partial p}\frac{\partial a^*}{\partial m}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}} + \gamma\frac{\partial a^*}{\partial m}\right) \tag{2.7}$$

Consider the first term. The denominator is negative by the SOC. $\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p} < 0$ by assumption, and $\left(\frac{\partial R(a,m)}{\partial a} - \gamma\right)\frac{\partial N(p+\gamma a)}{\partial p}\frac{\partial a^*}{\partial m} = 0$ by the FOC in (2.4). Hence, the sign of the first term in (2.7) is positive. If $\gamma \geq 0$, then also the second term is non-negative as $\frac{\partial a^*}{\partial m} > 0$ by Lemma 2.1. Hence $\frac{dp^*}{dm}$ is negative. (see Appendix 2.A for a more detailed proof). □

Note that $\frac{dp^*}{dm}$ is negative also if $0 > \gamma > \gamma_m = -\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}}{\frac{\partial a^*}{\partial m}\frac{\partial F_p}{\partial p^*}}$.

Lemma 2.1 and Lemma 2.2 lead us to the following result:

Proposition 2.1. *The optimal Revenue Ratio is decreasing in the exogenous shift parameter m if and only if $\varepsilon_m^{p^*} < \varepsilon_m^{R^*}$.*

The optimal Revenue Ratio is decreasing in the exogenous shift parameter m also if $\gamma > \gamma_m = -\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}}{\frac{\partial a^*}{\partial m}\frac{\partial F_p}{\partial p^*}}$.

Proof. The optimal Revenue Ratio $RR^*(a^*(m), p^*(m), R(a^*, m)) = \frac{p^*(m)}{R(a^*,m)}$ declines as m increases if and only if

$$\frac{dRR^*}{dm} = \frac{\frac{dp^*(m)}{dm}R(a^*(m), m) - \frac{dR(a^*(m),m)}{dm}p^*(m)}{R^2(a^*(m), m)} < 0. \tag{2.8}$$

Since the denominator is always positive, the condition in (2.8) can be rewritten as $\frac{dp^*(m)}{dm} < \frac{p^*(m)}{R(a^*(m),m)}\frac{dR(a^*(m),m)}{dm}$, or, alternatively, as $\frac{dp^*(m)}{p^*m}\frac{m}{dm} < \frac{dR(a^*(m),m)}{R(a^*(m),m)}\frac{m}{dm}$. Hence, $\varepsilon_m^{p^*} < \varepsilon_m^{R^*}$ is a necessary and sufficient condition for RR to be a decreasing function of m.

Furthermore, according to Lemma 2.2, when $\gamma \geq \gamma_m$, $\frac{dp^*(m)}{dm} < 0$. Moreover, $\frac{dR(a^*(m),m)}{dm} > 0$ by Corollary 2. Hence, $\frac{dRR}{dm}$ is negative, i.e., the optimal Revenue Ratio is decreasing in m.

□

2.5.2 Competition

The profit of a firm in *oligopolistic competition* with differentiated products is:

$$\Pi_i = [R_i(a_i, m) + p_i - c_i]N_i(\mathbf{p} + \gamma\mathbf{a}) \tag{2.9}$$

where

$$N_i(\mathbf{p} + \gamma\mathbf{a}) = N_i\big((p_i, \mathbf{p}_{-i}) + \gamma(a_i, \mathbf{a}_{-i})\big)$$

and firm i's residual demand depends on prices and advertising quantities of *all* n firms (defined by vectors $\mathbf{p}_{-i}$ and $\mathbf{a}_{-i}$).

We assume that parameter m shifts advertising demand outward such that $\frac{\partial R}{\partial m} > 0$. We also assume that R is increasing and strictly concave in a (i.e., $\frac{\partial^2 R}{\partial a^2} < 0$).

Moreover, we assume that marginal revenue from ads is increasing in m (i.e., $\frac{\partial^2 R}{\partial a \partial m} > 0$). Finally, R_i does not depend on the advertising quantities on competing newspapers, $\mathbf{a}_{-i}$ (i.e., $\frac{\partial R_i}{\partial a_{-i}} = 0$).

The profit maximization problem of firm i can be rewritten as:

$$\max_{a_i, f_i} \widetilde{\Pi} = [R_i(a_i, m) + f_i - \gamma a_i - c_i] N_i(\mathbf{f}) \tag{2.10}$$

where $f_i = p_i + \gamma a_i$ and $\mathbf{f} = (f_i, \mathbf{f_{-i}})$.

The first order condition w.r.t. a_i is

$$F_{a_i} \equiv \frac{\partial R_i(a_i, m)}{\partial a_i} - \gamma = 0, \tag{2.11}$$

which implicitly defines the optimal advertising level $a_i^* = a_i^*(\gamma, m)$.

Lemma 2.3. *The optimal level of advertising a_i^* is increasing in the exogenous shift parameter m.*

Proof. By the implicit function theorem

$$\frac{da_i^*}{dm} = -\frac{\frac{\partial F_{a_i}}{\partial m}}{\frac{\partial F_{a_i}}{\partial a_i^*}} = -\frac{\frac{\partial^2 R_i(a_i,m)}{\partial a_i \partial m}}{\frac{\partial^2 R_i(a_i,m)}{\partial a_i^2}}, \tag{2.12}$$

which is positive since $\frac{\partial^2 R_i(a_i,m)}{\partial a_i \partial m} > 0$ and $\frac{\partial^2 R_i(a_i,m)}{\partial a_i^2} < 0$ by previous assumptions. □

Corollary 2. *The revenue from advertising $R_i(a_i^*(m), m)$ is increasing in the exogenous shift parameter m.*

Proof.

$$\frac{dR_i(a_i^*(m), m)}{dm} = \frac{\partial R_i(a_i^*)}{\partial a_i} \frac{\partial a_i^*(m)}{\partial m} + \frac{\partial R_i(m)}{\partial m}$$

is positive since $\frac{\partial R_i(a_i^*)}{\partial a_i}$ and $\frac{\partial R_i(m)}{\partial m}$ are positive by assumption, and $\frac{\partial a_i^*(m)}{\partial m}$ is positive by Lemma 2.3. □

The first order condition w.r.t. p_i is

$$F_{p_i} \equiv N_i(\mathbf{p}+\gamma\mathbf{a}) + [R_i(a_i,m)+p_i-c_i]\frac{\partial N_i(\mathbf{p}+\gamma\mathbf{a})}{\partial p_i} = 0, \tag{2.13}$$

which implicitly defines $p_i^* = p_i^*\big(a_i^*(m,\gamma), c_i, \gamma, m, \mathbf{p}_{-i}, \mathbf{a}_{-i}\big)$.

Proposition 2.2. *In a duopoly the optimal Revenue Ratio is decreasing in the exogenous shift parameter m if and only if $\varepsilon_m^{p_i^*} < \varepsilon_m^{R_i^*}$.*

Proof. The optimal Revenue Ratio $RR^*(a^*(m), p^*(m), R(a^*,m)) = \frac{p^*(m)}{R(a^*,m)}$ declines as m increases if and only if

$$\frac{dRR^*}{dm} = \frac{\frac{dp^*(m)}{dm}R(a^*(m),m) - \frac{dR(a^*(m),m)}{dm}p^*(m)}{R^2(a^*(m),m)} < 0. \tag{2.14}$$

Since the denominator is always positive, the condition in (2.14) can be rewritten as $\frac{dp_i^*(m)}{dm} < \frac{p_i^*(m)}{R_i(a_i^*(m),m)}\frac{dR(a_i^*(m),m)}{dm}$, or, alternatively as $\frac{dp_i^*(m)}{dm}\frac{m}{p_{i\,m}^*} < \frac{dR(a_i^*(m),m)}{dm}\frac{m}{R_i(a_i^*(m),m)}$. Hence, $\varepsilon_m^{p_i^*} < \varepsilon_m^{R_i^*}$ is a necessary and sufficient condition for RR to be a decreasing function of m.

□

The Revenue Ratio RR is decreasing in m also if the cover price is decreasing in m. In fact, as we showed above, the Revenue Ratio is decreasing in m if and only if:

$$\frac{dp_i^*(m)}{dm} < \frac{p_i^*(m)}{R_i(a_i^*(m),m)}\frac{dR(a_i^*(m),m)}{dm} \tag{2.15}$$

.

Since the right-hand side is positive by Corollary 2, the inequality always holds if $\frac{dp_i^*(m)}{dm} < 0$.

In the following sections we conduct a deeper analysis to identify the conditions under which the RR is decreasing in m. We first assume symmetry among firms; then we relax the assumption.

Symmetric duopoly

Lemma 2.4. *In a symmetric duopoly the optimal cover price p^* is decreasing in the exogenous shift parameter m if and only if* $\gamma > \bar{\gamma}_{SO} = -\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N}{\partial p}}{\frac{\partial a}{\partial m}\left(\frac{\partial F_p}{\partial p}+\frac{\partial F_{p_1}}{\partial p_2}\right)}$

In particular, the optimal cover price p^* is decreasing in m if $\gamma \geq 0$.

(The proof of this lemma is presented in Appendix 2.A.)

Proposition 2.3. *In a symmetric duopoly the optimal Revenue Ratio is decreasing in m if* $\gamma > \bar{\gamma}_{SO} = -\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N}{\partial p}}{\frac{\partial a}{\partial m}\left(\frac{\partial F_p}{\partial p}+\frac{\partial F_{p_1}}{\partial p_2}\right)}$.

Proof. According to Lemma 2.4, if $\gamma > \bar{\gamma}_{SO}$, then $\frac{dp^*(m)}{dm} < 0$. Moreover, $\frac{dR(a^*(m),m)}{dm} > 0$ by Corollary 2. Hence, $\frac{dRR}{dm}$ is negative, i.e., the optimal Revenue Ratio is decreasing in m.

□

Asymmetric duopoly

As we proved before, a sufficient condition for the Revenue Ratio to be decreasing in m is that the optimal cover price is decreasing in m.

Lemma 2.5. *In an asymmetric duopoly the optimal cover price p^* is decreasing in m if and only if* $\gamma > \bar{\gamma}_{AO} = \frac{\frac{\partial R_2(a_2,m)}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2}\frac{\partial \Pi_1}{\partial p_1 \partial p_2} - \frac{\partial R_1(a_1,m)}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2}}{\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}\right]}$.

Moreover, the optimal cover price p^* is decreasing in m if $\gamma \geq 0$, and prices are strategic complements (i.e., $\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} > 0$).

(The proof of this lemma is presented in Appendix 2.A.)

Proposition 2.4. *In an asymmetric duopoly the optimal Revenue Ratio is decreasing in m if* $\gamma > \bar{\gamma}_{AO} = \frac{\frac{\partial R_2(a_2,m)}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2}\frac{\partial \Pi_1}{\partial p_1 \partial p_2} - \frac{\partial R_1(a_1,m)}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2}}{\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}\right]}$.

(See proof of Proposition 2.3).

2.5.3 Collusion

When firms can collude, they will maximize the joint profit by acting as a multi-product monopolist. The profit of a monopolist m selling n newspapers is

$$\Pi_m = \sum_{i=1}^{n} [R_i(a_i, m) + p_i - c_i] N_i(\mathbf{p} + \gamma \mathbf{a}) \tag{2.16}$$

The profit maximization problem can be rewritten as:

$$\max_{a_i, f_i} \widetilde{\Pi}_m = \sum_{i=1}^{n} [R_i(a_i, m) + f_i - \gamma a_i - c_i] N_i(\mathbf{f}) \tag{2.17}$$

where $f_i = p_i + \gamma a_i$ and $\mathbf{f} = (f_i, \mathbf{f}_{-i})$.

The first order condition w.r.t. a_i is

$$F_{a_i} \equiv \left(\frac{\partial R_i}{\partial a_i} - \gamma \right) = 0, \tag{2.18}$$

which implicitly defines the optimal advertising level $a_i^* = a_i^*(\gamma, m)$.

Lemma 2.6. *The optimal level of advertising a_i^* is increasing in the exogenous shift parameter m.*

Proof. By the implicit function theorem

$$\frac{da_i^*}{dm} = -\frac{\frac{\partial F_{a_i}}{\partial m}}{\frac{\partial F_{a_i}}{\partial a_i^*}} = -\frac{\frac{\partial^2 R_i(a_i, m)}{\partial a_i \partial m}}{\frac{\partial^2 R_i(a_i, m)}{\partial a_i^2}}, \tag{2.19}$$

which is positive since $\frac{\partial^2 R_i(a_i,m)}{\partial a_i \partial m} > 0$ and $\frac{\partial^2 R_i(a_i,m)}{\partial a_i^2} < 0$ by previous assumptions. □

Corollary 3. *The revenue from advertising $R_i(a_i^*(m), m)$ is increasing in the exogenous shift parameter m.*

Proof.

$$\frac{dR_i(a_i^*(m), m)}{dm} = \frac{\partial R_i(a_i^*)}{\partial a_i}\frac{\partial a_i^*(m)}{\partial m} + \frac{\partial R_i(m)}{\partial m}$$

is positive since $\frac{\partial R_i(a_i^*)}{\partial a_i}$ and $\frac{\partial R_i(m)}{\partial m}$ are positive by assumption, and $\frac{\partial a_i^*(m)}{\partial m}$ is positive by Lemma 2.6. □

The first order condition w.r.t. p_i is

$$\begin{aligned} F_{p_i} \equiv & N_i(\mathbf{p} + \gamma\mathbf{a}) + [R_i(a_i, m) + p_i - c_i]\frac{\partial N_i(\mathbf{p} + \gamma\mathbf{a})}{\partial p_i} \\ & + \sum_{i=1}^{n}[R_{-i}(a_{-i}, m) + p_{-i} - c_{-i}]\frac{\partial N_{-i}(\mathbf{p} + \gamma\mathbf{a})}{\partial p_i} = 0 \end{aligned} \tag{2.20}$$

which implicitly defines $p_i^* = p_i^*\big(a_i^*(m, \gamma), c_i, \gamma, m, p_{-i}, a_{-i}\big)$.

We assume that the second order condition for profit maximization (SOC) is satisfied (see Appendix 2.A).

Proposition 2.5. *In a duopoly cartel the optimal Revenue Ratio is decreasing in the exogenous shift parameter m if and only if $\varepsilon_m^{p_{ic}^*} < \varepsilon_m^{R_{ic}^*}$.*

(For the proof, see proof of Proposition 2.2).

As in the previous section, we now consider a duopoly cartel with symmetric firms and then a duopoly cartel with asymmetric firms.

Symmetric duopoly

Lemma 2.7. *In a symmetric duopoly, the optimal collusive cover price p^* is decreasing in the exogenous shift parameter m if and only if $\gamma > \bar{\gamma}_{SC} = -\frac{\frac{\partial R(a,m)}{\partial m}\left(\frac{\partial N}{\partial p} + \frac{\partial N_2}{\partial p_1}\right)}{\frac{\partial a}{\partial m}\left(\frac{\partial F_p}{\partial p} + \frac{\partial F_{p_1}}{\partial p_2}\right)}$*

(The proof of this lemma is presented in Appendix 2.A.)

Proposition 2.6. *In a symmetric duopoly cartel, the optimal Revenue Ratio is decreasing in m if $\gamma > \gamma_{SC} = -\frac{\frac{\partial R(a,m)}{\partial m}\left(\frac{\partial N}{\partial p} + \frac{\partial N_2}{\partial p_1}\right)}{\frac{\partial a}{\partial m}\left(\frac{\partial F_p}{\partial p} + \frac{\partial F_{p_1}}{\partial p_2}\right)}$.*

Proof. According to Lemma 2.7, when $\gamma > \bar{\gamma}_{SC}$, $\frac{dp^*(m)}{dm} < 0$. Moreover, $\frac{dR(a^*(m),m)}{dm} > 0$ by Corollary 3. Hence, $\frac{dRR}{dm}$ is negative, i.e., the optimal Revenue Ratio is decreasing in m. □

Asymmetric duopoly

Lemma 2.8. *In an asymmetric duopoly the optimal collusive cover price p^* is decreasing in m if and only if*

$$\gamma > \bar{\gamma}_{AC} = -\frac{\frac{\partial R_1}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_1(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_1(\cdot)}{\partial p_2}\right) + \frac{\partial R_2}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_2(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_2(\cdot)}{\partial p_2}\right)}{\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \left(\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\right)^2\right]}.$$

(The proof of this lemma is presented in Appendix 2.A.)

Proposition 2.7. *In an asymmetric duopoly the optimal Revenue Ratio is decreasing in m if*

$$\gamma > \bar{\gamma}_{AC} = -\frac{\frac{\partial R_1}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_1(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_1(\cdot)}{\partial p_2}\right) + \frac{\partial R_2}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_2(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_2(\cdot)}{\partial p_2}\right)}{\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \left(\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\right)^2\right]}$$

(See proof of Proposition 2.6).

2.6 A Simple Empirical Test

In the previous section we built a model to show that the optimal Revenue Ratio $\frac{p^*}{R(a^*,m)}$ is decreasing in the shifting parameter m. In this section we empirically test our model[3].

We use level data on revenues of the four national newspapers in the UK (The Guardian, The Times, The Independent, and The Daily Telegraph) with monthly observations from 1991 to 2000 from Nielsen Media Research UK. In particular,

[3] This is just a preliminary test of the positive correlation between advertising demand and revenue ratio.

we calculate the ratio between readership revenues and advertising revenues. We acquired data on UK real GDP for the same period from Federal Reserve Economic Data (FRED). Since data on GDP are available only on quarterly basis, we calculate the average newspaper revenue ratio for each quarter.

We use GDP as a proxy of the UK advertising market. Our prediction is that an increase of the GDP (i.e., a rightwards shifting of the demand in the advertising market) leads to a decrease of the revenue ratio.

The first estimates are presented in Table 2.1, and suggest that variables are indeed related. The coefficient is statistically significant with the expected negative sign (both for each newspaper and overall)[4].

Table 2.1: OLS regression of newspapers revenue ratio on GDP

	(All)	(The Guardian)	(The Independent)	(The Times)	(The Daily Telegraph)
GDP	-0.00116***	-0.00156***	-0.00157***	-0.000785***	-0.000746***
	(-16.84)	(-11.67)	(-13.62)	(-8.78)	(-8.84)
_cons	431.6***	564.8***	551.7***	309.1***	300.7***
	(21.26)	(14.41)	(16.33)	(11.77)	(12.14)
N	160	40	40	40	40
R^2	0.6422	0.7819	0.8300	0.6697	0.6730

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

We also run an OLS regression omitting the period of the price war. Results are similar, even though, as expected, the effect of GDP on revenue ratio is slightly greater (see Table 2.2).

[4]We obtain the same result when we control for fixed effects. This result might depend on the fact that, as for now, we are only considering one independent variable.

Table 2.2: OLS regression of newspapers revenue ratio on GDP (October 1993 – March 1996 omitted)

	(All Newspapers)	(The Guardian)	(The Independent)	(The Times)	(The Daily Telegraph)
GDP	-0.00124***	-0.00167***	-0.00159***	-0.000879***	-0.000811***
	(-17.95)	(-12.42)	(-13.42)	(-11.71)	(-11.17)
_cons	459.0***	605.0***	563.2***	342.7***	325.2***
	(22.34)	(15.07)	(15.96)	(15.33)	(15.03)
N	120	30	30	30	30
R^2	0.7320	0.8463	0.8654	0.8304	0.8166

t statistics in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

2.7 Conclusions

In this work we propose a monopoly, oligopoly and collusive model of the newspaper market to derive conditions under which the optimal share of advertising financing increases when the size of the advertising market increases.

Our candidate explanation for the events in the UK in the 1990s is that the observed changes in market structure resulted from an expected positive shock on the demand side for advertising. This shock led to an adjustment process that finally implied lower equilibrium prices on the reader's side, as the share of advertising revenues in the financing mix of the newspapers increased considerably. It is conceivable that Rupert Murdoch, being first to spot this change in the market structure, was also first to react.

2.A Appendix

2.A.1 Conditions for the SOC (Monopoly and Oligopoly)

In order for the SOC to be satisfied, the Hessian matrix for each firm i should be negative definite.

$$H = \begin{bmatrix} N(f)\frac{\partial^2 R(a,m)}{\partial a^2} & \left(\frac{\partial R(a,m)}{\partial a} - \gamma\right)\frac{\partial N(f)}{\partial f} \\ \\ \left(\frac{\partial R(a,m)}{\partial a} - \gamma\right)\frac{\partial N(f)}{\partial f} & 2\frac{\partial N(f)}{\partial f} + [R(a,m) + f - \gamma a - c]\frac{\partial^2 N(f)}{\partial f^2} \end{bmatrix}$$

Since $\frac{\partial^2 R(a,m)}{\partial a^2} < 0$ by assumption and $\left(\frac{\partial R(a,m)}{\partial a} - \gamma\right) = 0$ by the FOC, the SOC is satisfied if and only if

$$2\frac{\partial N(f)}{\partial f} + [R(a,m) + f - \gamma a - c]\frac{\partial^2 N(f)}{\partial f^2} < 0,$$

or, equivalently, if and only if

$$\frac{\partial F_p}{\partial p} = 2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m) + p - c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2} < 0.$$

2.A.2 Proof of Lemma 2.2

$$\begin{aligned}
\frac{dp^*}{dm} &= -\frac{\frac{\partial F_p}{\partial m}}{\frac{\partial F_p}{\partial p^*}} = -\frac{\frac{\partial F_p}{\partial a}\frac{\partial a^*}{\partial m} + \frac{\partial F_p}{\partial m}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}} \\
&= -\frac{\left(\frac{\partial N(p+\gamma a)}{\partial a} + \frac{\partial R(a,m)}{\partial a}\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p \partial a}\right)\frac{\partial a^*}{\partial m} + \frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}} \\
&= -\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}} \\
&\quad - \left(\frac{\frac{\partial N(p+\gamma a)}{\partial a} + \frac{\partial R(a,m)}{\partial a}\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p \partial a}}{2\frac{\partial N(p+\gamma a)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}}\right)\frac{\partial a^*}{\partial m}
\end{aligned} \tag{A-1}$$

The first term is negative. Let consider the numerator of the term in brackets:

$$\begin{aligned}
&\frac{\partial N(p+\gamma a)}{\partial a}+\frac{\partial R(a,m)}{\partial a}\frac{\partial N(p+\gamma a)}{\partial p}+[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p \partial a}\\
&=\gamma\frac{\partial N(p+\gamma a)}{\partial p}+\frac{\partial R(a,m)}{\partial a}\frac{\partial N(p+\gamma a)}{\partial p}+\gamma[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}\\
&=\gamma\frac{\partial N(p+\gamma a)}{\partial p}+\gamma\frac{\partial N(p+\gamma a)}{\partial p}+\frac{\partial R(a,m)}{\partial a}\frac{\partial N(p+\gamma a)}{\partial p}-\gamma\frac{\partial N(p+\gamma a)}{\partial p}\\
&\quad+\gamma[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}\\
&=\gamma\Big(2\frac{\partial N(p+\gamma a)}{\partial p}+[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}\Big)\\
&\quad+\Big(\frac{\partial R(a,m)}{\partial a}-\gamma\Big)\frac{\partial N(p+\gamma a)}{\partial p}
\end{aligned} \tag{A-2}$$

The first term in brackets corresponds to the second order condition, while the second term in brackets is equal to zero by the FOC.

Hence,

$$\begin{aligned}
\frac{dp^*}{dm}&=-\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}+\Big(\frac{\partial R(a,m)}{\partial a}-\gamma\Big)\frac{\partial N(p+\gamma a)}{\partial p}\frac{\partial a^*}{\partial m}}{2\frac{\partial N(p+\gamma a)}{\partial p}+[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}}\\
&\quad-\gamma\frac{\partial a^*}{\partial m}\frac{\Big(2\frac{\partial N(p+\gamma a)}{\partial p}+[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}\Big)}{\Big(2\frac{\partial N(p+\gamma a)}{\partial p}+[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}\Big)}\\
&=-\left(\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N(p+\gamma a)}{\partial p}}{2\frac{\partial N(p+\gamma a)}{\partial p}+[R(a,m)+p-c]\frac{\partial^2 N(p+\gamma a)}{\partial p^2}}+\gamma\frac{\partial a^*}{\partial m}\right)
\end{aligned} \tag{A-3}$$

2.A.3 Proof of Lemma 2.4

Proof. By the implicit function theorem

$$\frac{d\mathbf{p}}{dm} = \begin{bmatrix} \frac{dp_1}{dm} \\ \frac{dp_2}{dm} \end{bmatrix} = - \begin{bmatrix} \frac{\partial F_{p_1}}{\partial p_1} & \frac{\partial F_{p_1}}{\partial p_2} \\ \frac{\partial F_{p_2}}{\partial p_1} & \frac{\partial F_{p_2}}{\partial p_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dF_{p_1}}{dm} \\ \frac{dF_{p_2}}{dm} \end{bmatrix}$$

$$= - \frac{1}{\left(\frac{\partial F_{p_1}}{\partial p_1} \frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial F_{p_1}}{\partial p_2} \frac{\partial F_{p_2}}{\partial p_1} \right)} \begin{bmatrix} \frac{\partial F_{p_2}}{\partial p_2} & -\frac{\partial F_{p_1}}{\partial p_2} \\ -\frac{\partial F_{p_2}}{\partial p_1} & \frac{\partial F_{p_1}}{\partial p_1} \end{bmatrix} \begin{bmatrix} \frac{dF_{p_1}}{dm} \\ \frac{dF_{p_2}}{dm} \end{bmatrix} \quad \text{(A-4)}$$

Since firms are assumed to be symmetric, $\frac{dF_{p_1}}{dm} = \frac{dF_{p_2}}{dm} = \frac{dF_p}{dm}$. Hence, the vector above can be rewritten as follows:

$$\frac{d\mathbf{p}}{dm} = -\frac{1}{\left(\frac{\partial F_p}{\partial p}\right)^2 - \left(\frac{\partial F_{p_1}}{\partial p_2}\right)^2} \begin{bmatrix} \left(\frac{\partial F_p}{\partial p} - \frac{\partial F_{p_1}}{\partial p_2} \right) \frac{dF_p}{dm} \\ \left(\frac{\partial F_p}{\partial p} - \frac{\partial F_{p_1}}{\partial p_2} \right) \frac{dF_p}{dm} \end{bmatrix}$$

It is reasonable to assume that the direct effect on marginal profit of a price change exceeds the cross effect (i.e., that $(\frac{\partial F_{p_1}}{\partial p_1} - \frac{\partial F_{p_1}}{\partial p_2}) < 0$). This implies also that $\frac{1}{\left(\frac{\partial F_p}{\partial p}\right)^2 - \left(\frac{\partial F_{p_1}}{\partial p_2}\right)^2} > 0$. Hence, $sign \frac{d\mathbf{p}}{dm} = sign \frac{d\mathbf{F_p}}{dm}$.

$$\frac{d\mathbf{F}_p}{dm} = \begin{bmatrix} \frac{dF_p}{dm} \\ \frac{dF_p}{dm} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_p}{\partial a} & \frac{\partial F_{p_1}}{\partial a_2} \\ \frac{\partial F_{p_2}}{\partial a_1} & \frac{\partial F_p}{\partial a} \end{bmatrix} \begin{bmatrix} \frac{\partial a}{\partial m} \\ \frac{\partial a}{\partial m} \end{bmatrix} + \begin{bmatrix} \frac{\partial F_p}{\partial m} \\ \frac{\partial F_p}{\partial m} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_p}{\partial a} \frac{\partial a}{\partial m} + \frac{\partial F_{p_1}}{\partial a_2} \frac{\partial a}{\partial m} + \frac{\partial F_p}{\partial m} \\ \frac{\partial F_{p_2}}{\partial a_1} \frac{\partial a}{\partial m} + \frac{\partial F_p}{\partial a} \frac{\partial a}{\partial m} + \frac{\partial F_p}{\partial m} \end{bmatrix}$$

Consider the first term of the vector.

$$\frac{dF_p}{dm} = \frac{\partial N(\cdot)}{\partial a} \frac{\partial a}{\partial m} + \frac{\partial R(a,m)}{\partial a} \frac{\partial N(\cdot)}{\partial p} \frac{\partial a}{\partial m} + [R(a,m) + p - c] \frac{\partial^2 N(\cdot)}{\partial p \partial a} \frac{\partial a}{\partial m}$$
$$+ \frac{\partial N_1(\cdot)}{\partial a_2} \frac{\partial a}{\partial m} + [R(a_1,m) + p - c] \frac{\partial^2 N_1(\cdot)}{\partial p_1 \partial a_2} \frac{\partial a}{\partial m} + \frac{\partial R(a_1,m)}{\partial m} \frac{\partial N(\cdot)}{\partial p}$$

By rearranging terms, we obtain:

$$\frac{dF_p}{dm} = 2\gamma\frac{\partial N(\cdot)}{\partial p}\frac{\partial a}{\partial m} + \gamma[R(a,m)+p-c]\frac{\partial^2 N(\cdot)}{\partial p^2}\frac{\partial a}{\partial m} - \gamma\frac{\partial N(\cdot)}{\partial p}\frac{\partial a}{\partial m} + \frac{\partial R(a,m)}{\partial a}\frac{\partial N(\cdot)}{\partial p}\frac{\partial a}{\partial m}$$

$$+ \gamma\frac{\partial N_1(\cdot)}{\partial p_2}\frac{\partial a}{\partial m} + \gamma[R(a,m)+p-c]\frac{\partial^2 N_1(\cdot)}{\partial p_1 \partial p_2}\frac{\partial a}{\partial m} + \frac{\partial R(a,m)}{\partial m}\frac{\partial N(\cdot)}{\partial p}$$

$$= \gamma\frac{\partial a}{\partial m}\left(\frac{2\partial N(\cdot)}{\partial p} + [R(a,m)+p-c]\frac{\partial^2 N(\cdot)}{\partial p^2}\right) + \frac{\partial N(\cdot)}{\partial p}\frac{\partial a}{\partial m}\left(\frac{\partial R(a,m)}{\partial a} - \gamma\right)$$

$$+ \gamma\left(\frac{\partial N_1(\cdot)}{\partial p_2}\frac{\partial a}{\partial m} + [R(a,m)+p-c]\frac{\partial^2 N_1(\cdot)}{\partial p_1 \partial p_2}\frac{\partial a}{\partial m}\right) + \frac{\partial R(a,m)}{\partial m}\frac{\partial N(\cdot)}{\partial p}$$

$$= \gamma\frac{\partial a}{\partial m}\frac{\partial F_p}{\partial p} + \gamma\frac{\partial a}{\partial m}\frac{\partial F_{p_1}}{\partial p_2} + \frac{\partial R(a,m)}{\partial m}\frac{\partial N(\cdot)}{\partial p}$$

F_p is decreasing in m (i.e., $\frac{\partial F_p}{\partial m} < 0$), and hence p_m is decreasing in m, if and only if $\gamma > \bar{\gamma}_o = -\frac{\frac{\partial R(a,m)}{\partial m}\frac{\partial N}{\partial p}}{\frac{\partial a}{\partial m}\left(\frac{\partial F_P}{\partial p} + \frac{\partial F_{P_1}}{\partial p_2}\right)}$.

□

2.A.4 Proof of Lemma 2.5

Proof.

$$\frac{d\mathbf{p}}{dm} = \begin{bmatrix} \frac{dp_1}{dm} \\ \frac{dp_2}{dm} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_{p_1}}{\partial p_1} & \frac{\partial F_{p_1}}{\partial p_2} \\ \frac{\partial F_{p_2}}{\partial p_1} & \frac{\partial F_{p_2}}{\partial p_2} \end{bmatrix}^{-1} \begin{bmatrix} \frac{dF_{p_1}}{dm} \\ \frac{dF_{p_2}}{dm} \end{bmatrix}$$

$$= -\frac{1}{\left(\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial F_{p_1}}{\partial p_2}\frac{\partial F_{p_2}}{\partial p_1}\right)} \begin{bmatrix} \frac{\partial F_{p_2}}{\partial p_2} & -\frac{\partial F_{p_1}}{\partial p_2} \\ -\frac{\partial F_{p_2}}{\partial p_1} & \frac{\partial F_{p_1}}{\partial p_1} \end{bmatrix} \begin{bmatrix} \frac{dF_{p_1}}{dm} \\ \frac{dF_{p_2}}{dm} \end{bmatrix}$$

$$= -\frac{1}{\left(\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial F_{p_1}}{\partial p_2}\frac{\partial F_{p_2}}{\partial p_1}\right)} \begin{bmatrix} \frac{\partial F_{p_2}}{\partial p_2}\frac{dF_{p_1}}{dm} - \frac{\partial F_{p_1}}{\partial p_2}\frac{dF_{p_2}}{dm} \\ -\frac{\partial F_{p_2}}{\partial p_1}\frac{dF_{p_1}}{dm} + \frac{\partial F_{p_1}}{\partial p_1}\frac{dF_{p_2}}{dm} \end{bmatrix} \quad \text{(A-5)}$$

where

$$\begin{bmatrix} \frac{dF_{p_1}}{dm} \\ \\ \frac{dF_{p_2}}{dm} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_{p_1}}{\partial a_1} & \frac{\partial F_{p_1}}{\partial a_2} \\ \\ \frac{\partial F_{p_2}}{\partial a_1} & \frac{\partial F_{p_2}}{\partial a_2} \end{bmatrix} \begin{bmatrix} \frac{\partial a_1}{\partial m} \\ \\ \frac{\partial a_2}{\partial m} \end{bmatrix} + \begin{bmatrix} \frac{\partial F_{p_1}}{\partial m} \\ \\ \frac{\partial F_{p_2}}{\partial m} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_{p_1}}{\partial a_1}\frac{\partial a_1}{\partial m} + \frac{\partial F_{p_1}}{\partial a_2}\frac{\partial a_2}{\partial m} + \frac{\partial F_{p_1}}{\partial m} \\ \frac{\partial F_{p_2}}{\partial a_1}\frac{\partial a_1}{\partial m} + \frac{\partial F_{p_2}}{\partial a_2}\frac{\partial a_2}{\partial m} + \frac{\partial F_{p_2}}{\partial m} \end{bmatrix} \tag{A-6}$$

$$\begin{aligned} \frac{dF_{p_1}}{dm} &= \gamma\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial a_1}{\partial m} + \frac{\partial R_1(a_1,m)}{\partial a_1}\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial a_1}{\partial m} + \gamma[R_1(a_1,m)+p_1-c_1]\frac{\partial^2 N_1(\cdot)}{\partial p_1^2}\frac{\partial a_1}{\partial m} \\ &\quad + \gamma\frac{\partial N_1(\cdot)}{\partial p_2}\frac{\partial a_2}{\partial m} + \gamma[R_1(a_1,m)+p_1-c_1]\frac{\partial^2 N_1(\cdot)}{\partial p_1 \partial p_2}\frac{\partial a_2}{\partial m} + \frac{\partial R_1(a_1,m)}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} \end{aligned} \tag{A-7}$$

$$\begin{aligned} \frac{dF_{p_2}}{dm} &= \gamma\frac{\partial N_2(\cdot)}{\partial p_1}\frac{\partial a_1}{\partial m} + \gamma[R_2(a_2,m)+p_2-c_2]\frac{\partial^2 N_2(\cdot)}{\partial p_2 \partial p_1}\frac{\partial a_1}{\partial m} + \gamma\frac{\partial N_2(\cdot)}{\partial p_2}\frac{\partial a_2}{\partial m} \\ &\quad + \frac{\partial R_2(a_2,m)}{\partial a_2}\frac{\partial N_2(\cdot)}{\partial p_2}\frac{\partial a_2}{\partial m} + \gamma[R_2(a_2,m)+p_2-c_2]\frac{\partial^2 N_2(\cdot)}{\partial p_2^2}\frac{\partial a_2}{\partial m} + \frac{\partial R_2(a_2,m)}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2} \end{aligned} \tag{A-8}$$

From which follows that

$$\begin{bmatrix} \frac{dF_{p_1}}{dm} \\ \\ \frac{dF_{p_2}}{dm} \end{bmatrix} = \begin{bmatrix} \gamma\frac{\partial a_1}{\partial m}SOC_1 - \gamma\frac{\partial a_1}{\partial m}FOC_1 + \gamma\frac{\partial a_2}{\partial m}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} + \frac{\partial R_1(a_1,m)}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} \\ \gamma\frac{\partial a_2}{\partial m}SOC_2 - \gamma\frac{\partial a_2}{\partial m}FOC_2 + \gamma\frac{\partial a_1}{\partial m}\frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2} + \frac{\partial R_2(a_2,m)}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2} \end{bmatrix} \tag{A-9}$$

Consider the vector $\frac{d\mathbf{p}}{dm}$ in (A-5). The denominator $\left(\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial F_{p_1}}{\partial p_2}\frac{\partial F_{p_2}}{\partial p_1}\right)$ is positive because we have assumed that the direct effect on marginal profit of a price change exceeds the cross effect (i.e., $\left|\frac{\partial F_{p_i}}{\partial p_i}\right| > \left|\frac{\partial F_{p_i}}{\partial p_j}\right|$). Hence, $sign\frac{d\mathbf{p}}{dm} = sign\frac{d\mathbf{F_p}}{dm}$. Let therefore study the first term of the column-vector in (A-5):

$$
\begin{aligned}
\frac{\partial F_{p_2}}{\partial p_2}\frac{dF_{p_1}}{dm} - \frac{\partial F_{p_1}}{\partial p_2}\frac{dF_{p_2}}{dm} &= \gamma\frac{\partial a_1}{\partial m}\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} + \gamma\frac{\partial a_2}{\partial m}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}\frac{\partial F_{p_2}}{\partial p_2} \\
&\quad + \frac{\partial R_1(a_1, m)}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \gamma\frac{\partial a_2}{\partial m}\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \\
&\quad - \gamma\frac{\partial a_1}{\partial m}\frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} - \frac{\partial R_2(a_2, m)}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} \\
&= \gamma\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}\right] \\
&\quad + \frac{\partial R_1(a_1, m)}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial R_2(a_2, m)}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2}\frac{\partial \Pi_1}{\partial p_1 \partial p_2}
\end{aligned}
\tag{A-10}
$$

In order for p_1 to be a decreasing function of m, the expression in (A-10) should be positive.

$\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{dF_{p_1}}{dm} - \frac{\partial F_{p_1}}{\partial p_2}\frac{dF_{p_2}}{dm}\right)$ is positive if and only if

$\gamma > \bar{\gamma}_{AO} = \frac{\frac{\partial R_2(a_2,m)}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2}\frac{\partial \Pi_1}{\partial p_1 \partial p_2} - \frac{\partial R_1(a_1,m)}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2}}{\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \frac{\partial^2 \Pi_2}{\partial p_1 \partial p_2}\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2}\right]}$.

Let consider each term in (A-10).

$\frac{\partial a_1}{\partial m}$ is positive by Lemma 2.3. The term in squared brackets is positive by assumption.

$\frac{\partial R_1(a_1,m)}{\partial m}$ and $\frac{\partial R_2(a_2,m)}{\partial m}$ are positive by Corollary 2, whereas $\frac{\partial N_1(\cdot)}{\partial p_1}$ and $\frac{\partial N_2(\cdot)}{\partial p_2}$ are negative, because demand are decreasing in prices.

$\frac{\partial F_{p_2}}{\partial p_2}$ is negative by the second order condition.

Hence, if $\gamma > 0$ and prices are strategic complements (i.e.,$\frac{\partial^2 \Pi_1}{\partial p_1 \partial p_2} > 0$), equation in (A-10) is positive. Therefore, equilibrium prices are decreasing in m.

□

2.A.5 Conditions for the SOC (Collusion)

In order for the SOC to be satisfied, the Hessian matrix should be negative definite, i.e., its k-th order leading principal minors should be negative when k is odd, and positive when k is even.

$$H = \begin{bmatrix} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f_2} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_2} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial f_2} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2^2} \end{bmatrix} = \begin{bmatrix} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} & 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f_2} \\ 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_2} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial f_2} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2^2} \end{bmatrix}$$

Let study the sign of each determinant:

a. $\left| \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} \right| = \frac{\partial^2 R_1}{\partial a_1^2} N_1(\mathbf{f}) < 0$ by assumption.

b. $\begin{vmatrix} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} & 0 \\ 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} \end{vmatrix} = \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} = \frac{\partial^2 R_1}{\partial a_1^2} \frac{\partial^2 R_1}{\partial a_1^2} N_1(\mathbf{f}) N_2(\mathbf{f}) > 0$ by assumption.

c.

$$\begin{vmatrix} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} & 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f1} \\ 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_1} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} & 0 & 0 \\ 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} & 0 \\ 0 & 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} \end{vmatrix} = \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} < 0 \text{ if and only if}$$

$$\frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} < 0$$

In fact, note that, by FOC:

$$\frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f_1} = \left(\frac{\partial R_1}{\partial a_1} - \gamma \right) \frac{\partial N_1}{\partial f_1} = 0;$$

$$\frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_1} = \left(\frac{\partial R_2}{\partial a_2} - \gamma \right) \frac{\partial N_2}{\partial f_1} = 0.$$

d. $$\begin{vmatrix} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} & 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1 \partial f_2} \\ 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2 \partial f_2} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial f_2} \\ \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial a_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial a_2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2^2} \end{vmatrix} = \begin{vmatrix} \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_1^2} & 0 & 0 & 0 \\ 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial a_2^2} & 0 & 0 \\ 0 & 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1^2} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_1 \partial f_2} \\ 0 & 0 & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2 \partial f_1} & \frac{\partial^2 \widetilde{\Pi}_m}{\partial f_2^2} \end{vmatrix} =$$

$$\frac{\partial^2\widetilde{\Pi}_m}{\partial a_1^2}\frac{\partial^2\widetilde{\Pi}_m}{\partial a_2^2}\left[\frac{\partial^2\widetilde{\Pi}_m}{\partial f_1^2}\frac{\partial^2\widetilde{\Pi}_m}{\partial f_2^2}-\left(\frac{\partial^2\widetilde{\Pi}_m}{\partial f_1\partial f_2}\right)^2\right]>0 \iff \frac{\partial^2\widetilde{\Pi}_m}{\partial f_1^2}\frac{\partial^2\widetilde{\Pi}_m}{\partial f_2^2}-\left(\frac{\partial^2\widetilde{\Pi}_m}{\partial f_1\partial f_2}\right)^2=$$

$$\frac{\partial F_{f_1}}{\partial f_1}\frac{\partial F_{f_2}}{\partial f_2}-\frac{\partial F_{f_1}}{\partial f_2}\frac{\partial F_{f_2}}{\partial f_1}>0$$

The SOC is satisfied if and only if profits are concave in prices and the direct effect on marginal profit of a price change exceeds the cross effect.

2.A.6 Proof of Lemma 2.7

Proof. As shown in A.2, in order to prove that prices are decreasing in m, it is sufficient to prove that $\frac{dF_{p_1}}{dm}<0$.

$$\begin{aligned}\frac{dF_{p_1}}{dm}&=\left(\frac{\partial R_1}{\partial a_1}-\gamma\right)\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial a}{\partial m}+\gamma\left[2\frac{\partial N_1(\cdot)}{\partial p_1}+[R_1(a_1,m)+p_1-c_1]\frac{\partial^2 N_1}{\partial p_1^2}+[R_2(a_2,m)+p_2-c_2]\frac{\partial^2 N_2}{\partial p_1^2}\right]\frac{\partial a}{\partial m}\\&+\gamma\left[\frac{\partial N_1(\cdot)}{\partial p_2}+\frac{\partial N_2(\cdot)}{\partial p_1}+[R_1(a_1,m)+p_1-c_1]\frac{\partial^2 N_1}{\partial p_1\partial p_2}+[R_2(a_2,m)+p_2-c_2]\frac{\partial^2 N_2}{\partial p_1\partial p_2}\right]\frac{\partial a}{\partial m}\\&+\left(\frac{\partial R_2}{\partial a_2}-\gamma\right)\frac{\partial N_2(\cdot)}{\partial p_1}\frac{\partial a}{\partial m}+\frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1}+\frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1}\\&=F_{a_1}\frac{\partial N_1(\cdot)}{\partial p_1}\frac{\partial a}{\partial m}+\gamma\frac{\partial a}{\partial m}\frac{\partial F_{p_1}}{\partial p_1}+F_{a_2}\frac{\partial N_2(\cdot)}{\partial p_1}\frac{\partial a}{\partial m}+\gamma\frac{\partial a}{\partial m}\frac{\partial F_{p_1}}{\partial p_2}+\frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1}+\frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1}\end{aligned}$$

The first and the third term are equal to zero by the FOC, whereas the second term is negative by the SOC.

$\frac{\partial F_p}{\partial m}<0$ if and only if $\gamma>\gamma_c=-\frac{\frac{\partial R(a,m)}{\partial m}\left(\frac{\partial N}{\partial p}+\frac{\partial N_2}{\partial p_1}\right)}{\frac{\partial a}{\partial m}\left(\frac{\partial F_p}{\partial p}+\frac{\partial F_{P_1}}{\partial p_2}\right)}$ □

2.A.7 Proof of Lemma 2.8

Proof.

$$\frac{d\mathbf{p}}{dm}=\begin{bmatrix}\frac{dp_1}{dm}\\ \\ \frac{dp_2}{dm}\end{bmatrix}=-\frac{1}{\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2}-\frac{\partial F_{p_1}}{\partial p_2}\frac{\partial F_{p_2}}{\partial p_1}}\begin{bmatrix}\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial F_{p_1}}{\partial m}-\frac{\partial F_{p_1}}{\partial p_2}\frac{\partial F_{p_2}}{\partial m}\\ \\ -\frac{\partial F_{p_2}}{\partial p_1}\frac{\partial F_{p_1}}{\partial m}+\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial m}\end{bmatrix} \tag{A-11}$$

where

$$\begin{bmatrix} \frac{\partial F_{p_1}}{\partial m} \\ \\ \frac{\partial F_{p_2}}{\partial m} \end{bmatrix} = \begin{bmatrix} \frac{\partial F_{p_1}}{\partial a_1} & \frac{\partial F_{p_1}}{\partial a_2} \\ \\ \frac{\partial F_{p_2}}{\partial a_1} & \frac{\partial F_{p_2}}{\partial a_2} \end{bmatrix} \begin{bmatrix} \frac{\partial a_1}{\partial m} \\ \\ \frac{\partial a_2}{\partial m} \end{bmatrix} + \begin{bmatrix} \frac{dF_{p_1}}{dm} \\ \\ \frac{dF_{p_2}}{dm} \end{bmatrix}$$

Moreover,

$$\frac{\partial F_{p_1}}{\partial a_1} = \frac{\partial N_1(\cdot)}{\partial a_1} + \frac{\partial R_1}{\partial a_1}\frac{\partial N_1(\cdot)}{\partial p_1} + [R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1 \partial a_1} + [R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1 \partial a_1}$$

$$\frac{\partial F_{p_1}}{\partial a_2} = \frac{\partial N_1(\cdot)}{\partial a_2} + \frac{\partial R_2}{\partial a_2}\frac{\partial N_2(\cdot)}{\partial p_1} + [R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1 \partial a_2} + [R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1 \partial a_2}$$

$$\frac{\partial F_{p_1}}{\partial m} = \frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1}$$

Let consider $\frac{\partial F_{p_1}}{\partial m}$

$$\begin{aligned}
\frac{\partial F_{p_1}}{\partial m} =& \left[\frac{\partial N_1(\cdot)}{\partial a_1} + \frac{\partial R_1}{\partial a_1}\frac{\partial N_1(\cdot)}{\partial p_1} + [R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1 \partial a_1} + [R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1 \partial a_1}\right]\frac{\partial a_1}{\partial m} \\
&+ \left[\frac{\partial N_1(\cdot)}{\partial a_2} + \frac{\partial R_2}{\partial a_2}\frac{\partial N_2(\cdot)}{\partial p_1} + [R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1 \partial a_2} + [R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1 \partial a_2}\right]\frac{\partial a_2}{\partial m} \\
&+ \frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1} \\
=& \left[\gamma\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_1}{\partial a_1}\frac{\partial N_1(\cdot)}{\partial p_1} + \gamma[R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1^2} + \gamma[R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1^2}\right]\frac{\partial a_1}{\partial m} \\
&+ \left[\gamma\frac{\partial N_1(\cdot)}{\partial p_2} + \frac{\partial R_2}{\partial a_2}\frac{\partial N_2(\cdot)}{\partial p_1} + \gamma[R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1 \partial p_2} + \gamma[R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1 \partial p_2}\right]\frac{\partial a_2}{\partial m} + \\
&+ \frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1} \\
=& \left[2\gamma\frac{\partial N_1(\cdot)}{\partial p_1} - \gamma\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_1}{\partial a_1}\frac{\partial N_1(\cdot)}{\partial p_1} + \gamma[R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1^2} + \gamma[R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1^2}\right]\frac{\partial a_1}{\partial m} \\
&+ \left[\gamma\frac{\partial N_1(\cdot)}{\partial p_2} + \gamma\frac{\partial N_2(\cdot)}{\partial p_1} - \gamma\frac{\partial N_2(\cdot)}{\partial p_1} + \frac{\partial R_2}{\partial a_2}\frac{\partial N_2(\cdot)}{\partial p_1} + \gamma[R_1(a_1, m) + p_1 - c_1]\frac{\partial^2 N_1}{\partial p_1 \partial p_2}\right]\frac{\partial a_2}{\partial m} \\
&+ \left[\gamma[R_2(a_2, m) + p_2 - c_2]\frac{\partial^2 N_2}{\partial p_1 \partial p_2}\right]\frac{\partial a_2}{\partial m} + \frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1} \\
=& \gamma\left(FOC_1 + SOC_1\right)\frac{\partial a_1}{\partial m} + \gamma\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial a_2}{\partial m} + \frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1}
\end{aligned} \tag{A-12}$$

Hence,

$$\frac{\partial F_{p_1}}{\partial m} = \gamma\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial a_1}{\partial m} + \gamma\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial a_2}{\partial m} + \frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} + \frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1} \tag{A-13}$$

$$\frac{\partial F_{p_2}}{\partial m} = \gamma\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial a_2}{\partial m} + \gamma\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial a_1}{\partial m} + \frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2} + \frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_2} \tag{A-14}$$

As showed above, $sign\frac{dp_1}{dm} = -sign\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial F_{p_1}}{\partial m} - \frac{\partial F_{p_1}}{\partial p_2}\frac{\partial F_{p_2}}{\partial m}\right)$.

$$
\begin{aligned}
\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial F_{p_1}}{\partial m} - \frac{\partial F_{p_1}}{\partial p_2}\frac{\partial F_{p_2}}{\partial m} =& \frac{\partial F_{p_2}}{\partial p_2}\gamma\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial a_1}{\partial m} + \frac{\partial F_{p_2}}{\partial p_2}\gamma\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial a_2}{\partial m} + \frac{\partial F_{p_2}}{\partial p_2}\frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_1} \\
&+ \frac{\partial F_{p_2}}{\partial p_2}\frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\gamma\frac{\partial a_2}{\partial m}\frac{\partial F_{p_2}}{\partial p_2} - \gamma\frac{\partial a_1}{\partial m}\left(\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\right)^2 \\
&- \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial R_2}{\partial m}\frac{\partial N_2(\cdot)}{\partial p_2} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial R_1}{\partial m}\frac{\partial N_1(\cdot)}{\partial p_2} \\
=& \gamma\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \left(\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\right)^2\right] + \frac{\partial R_1}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_1(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_1(\cdot)}{\partial p_2}\right) \\
&+ \frac{\partial R_2}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_2(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_2(\cdot)}{\partial p_2}\right)
\end{aligned}
\tag{A-15}
$$

In order for p_1 to be a decreasing function of m, the expression in (A-15) should be positive.

$\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{dF_{p_1}}{dm} - \frac{\partial F_{p_1}}{\partial p_2}\frac{dF_{p_2}}{dm}\right)$ is positive if and only if $\gamma > \bar{\gamma}_{AC}$,

with $\bar{\gamma}_{AC} = -\frac{\frac{\partial R_1}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_1(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_1(\cdot)}{\partial p_2}\right) + \frac{\partial R_2}{\partial m}\left(\frac{\partial F_{p_2}}{\partial p_2}\frac{\partial N_2(\cdot)}{\partial p_1} - \frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\frac{\partial N_2(\cdot)}{\partial p_2}\right)}{\frac{\partial a_1}{\partial m}\left[\frac{\partial F_{p_1}}{\partial p_1}\frac{\partial F_{p_2}}{\partial p_2} - \left(\frac{\partial^2 \Pi_C}{\partial p_1 \partial p_2}\right)^2\right]}$. □

2.B Appendix

2.B.1 Data

The dataset on the reader's side contains market level data on circulation, cover prices, and content characteristics of the four daily quality national newspapers in the UK (Guardian, The Times, Independent, and Daily Telegraph), with monthly observations from 1990 to 2000. Data on circulation come from the Audit Bureau of Circulation (ABC). Data on prices were collected from newspaper publishers themselves.

Data on the results of the political elections and on the political position of the newspapers were collected from the British Election Surveys (BES) in 1992 and 1997 and from the British Panel Election Survey (BPES) for the years 1992-1997 and 1997-2001. In particular, the relative political position of the newspaper was calculated as the percentage of readers of a given newspaper who a) voted for the conservative (or alternatively the labor) party b) felt closer to the conservative (or alternatively to the labor) party c) thought their newspaper favored the conservative (or alternatively the labor) party.

On the advertising side of the market we acquired market level data on advertising quantity and revenues of the same newspapers with monthly observations from 1991 to 2000 from Nielsen Media Research UK. The latter directly collects data on quantities and applies list prices in order to calculate advertising revenues. In doing so, however, Nielsen also applies an estimate of the discounts with respect to the posted list prices. We recovered nominal advertising tariffs dividing revenues by quantity. Finally, we deflated cover prices and advertising tariffs by the Consumer Price Index.

CHAPTER 3

MFN Clauses and Quality Disclosure on Online Platforms

3.1 Introduction

Over the past decade online platforms have proliferated and emerged as core business models.[1] They facilitate the interaction of supply and demand of goods and services sold through the Internet. From an antitrust perspective, online platforms raise many anti-competitive concerns. In particular, the Most Favored Nation clause (MFN) has come under the scrutiny of many antitrust authorities in recent years. For example, MFN clauses (also known as *price parity* clauses) included in the agreements between Apple and five major e-book publishers were prohibited both in the EU and in the USA. More recently, several antitrust authorities have investigated the use of MFN in the online hotel bookings market. MFN clauses are vertical agreements which prevent downstream firms from offering better deals to customers elsewhere. There are two types of price parity clauses, namely wide and narrow. A *wide* MFN clause prevents a firm from setting a lower price on any competing platform, or any other sales channel (e.g. its own website). With

[1] See, e.g., *Online Platforms and the Digital Single Market Opportunities and Challenges for Europe*, Communication, COM(2016) 288 final.

a *narrow* MFN clause a firm cannot charge a lower price through its own direct online sales channel, but it is free to offer consumers a lower price through its own direct (offline) sales channels and through any alternative platform. MFN agreements are adopted to address hold-up problems in vertical relations, minimize externalities and facilitate investment and efficiencies. However, they might also have anticompetitive effects. Indeed, MFNs might raise barriers to entry, as they prevent new platforms from offering better terms, and might reduce competition among existing platforms (e.g., by facilitating collusive behaviors).

In this chapter I draw from the recent antitrust case involving Online Travel Agencies (OTAs) which used MFN clauses in their agreements with hotels.

OTAs are two-sided Internet platforms, which facilitate interactions between hotels and travelers. On the one hand, consumers can easily search for, compare, and book rooms from a worldwide hotel portfolio. Moreover, they are offered multilingual websites and after-sales services. On the other hand, hotels are more visible and can reach a greater number of customers. Generally, the two groups of end-users have free access to the platform. The hotel pays a per-transaction fee only when the transaction is completed on the platform, with a commission typically ranging from 15% to 30% of room price.

In the case of OTAs, multi-homing is very common: consumers can search and compare hotels over several OTAs, and hotels can be listed on more than one OTA.

OTAs may use MFN clauses, which prevent hotels from offering better deals to customers elsewhere. Under a *wide* MFN clause, the hotel cannot charge a lower price through any alternative OTA, and through any other sales channel (e.g. hotel's own website). Under a *narrow* MFN clause, the hotel cannot undercut the platform on its own website, but it can set a lower price on its own direct (offline) sales channels and on other OTAs.

As mentioned before, some National Competition Authorities (NCAs), coordinated by the European Commission, between 2013 and 2014 have investigated the use of MFN clauses by the largest online booking platforms, namely Booking.com, Expedia and Hotel Reservation Service (HRS).

The main concern of Antitrust Authorities is that such clauses might restrict competition among platforms, and thus induce higher fees and prices. In particular, in presence of such clauses, a platform has no incentive to set low commissions, because they will not result in lower room prices (and hence in a higher demand). Moreover, as hotels are obliged to set equal room prices on all OTAs, MFNs might also discourage hotels from reducing their prices. Finally, such clauses might foreclose entry of new platforms. In fact, when hotels are bound by MFNs of incumbents, new entrants cannot attract consumers—and hence overcome indirect network effects—by charging low commissions.

OTAs, from their side, claim that MFN clauses are necessary both to prevent free-riding by hotels, which otherwise might set lower prices on their own direct channels (website, phone, email), and to protect platform's investments. In fact, without an MFN clause, customers might use the platform only to search and compare. A platform would have no incentive to invest in quality (better algorithms and rankings, customer feedbacks, purchase advice...) if consumers could buy more cheaply on other channels[2].

Interestingly, the various national authorities had divergent views. For example, the Italian, French and Swedish NCAs accepted commitments from Booking.com and Expedia to adopt narrow MFN clause, arguing that narrow MFN clauses enhance competition among OTAs on commission rates, while preventing free-riding by hotels[3]. By contrast, the German Bundeskartellamt wholly banned MFN clauses, stating that also the narrow one might violate competition law.

Using a theoretical model, I investigate the effects of MFN clauses on the quality of the hotel. Specifically, I consider the case in which a hotel can sell either only through its own sale channels or also through a platform. In the latter case,

[2] Notice that switching costs online are generally very low.

[3] However, more recently, some national laws made also narrow MFNs illegal. For example, the *Loi Macron* in France (July 2015) allows hotels to offer the prices they want through any channel, including on their own website, regardless of the prices offered on any online reservation platforms. Moreover, the Italian *Legge annuale per il mercato e la concorrenza* (August 2017) states that any contract between hotels and OTAs which contains a parity rate is invalid.

the hotel pays a fee to the platform, which certifies the quality of the hotel via customer feedbacks. I consider two possible scenarios, depending on whether an MFN clause is present or not. I show that a platform will always adopt an MFN clause to prevent hotel's free riding behaviors and make positive profits. My work points out three key findings. Firstly, with MFN clause in place, the hotel chooses the maximum quality when it sells through the platform, whereas it chooses the minimum quality when it sells only through its own channel. Secondly, a platform prefers to set a percentage fee rather than a non-percentage fee. Nonetheless, once MFN clause is prohibited, the platform might decide to change its business model and set a listing fee.

The chapter is organized as follows. In Section 2, I present recent literature dealing with the use of MFN clauses on platforms. In Section 3, I present the model. In Section 4, I discuss some policy implications. Section 5 concludes.

3.2 Related Literature

Recent theoretical papers exploring the effect of price parity clauses on platforms prices and fees show that MFN clauses can raise fees and prices (Boik and Corts, 2016; Johnson, 2017; Wang and Wright, 2016b).

Wang and Wright (2016b) study the effect of price parity clauses and of what they called "showrooming", a form of free riding in which consumers use the platform to search, and then buy elsewhere if price is lower. They first show that, when both showrooming and price parity clauses are not allowed, competition among firms increases, and consumers buy on the monopoly platform where search costs are lower. However, in equilibrium, prices on the platform are higher because of the fee it charges to hotels. When instead showrooming is allowed, consumers switch from the platform to hotels where prices are lower. Hence, showrooming restricts monopoly platform fees, and increases consumer surplus. Price parity clauses remove consumers' incentive to switch channel, since prices on the platform are never higher. Even though the platform extracts all consumer surplus, firms join the platform to be found by consumers. Wang and Wright (2016b) also consider a setting with two platforms and distinguish between narrow and wide MFN clause. Whereas the latter always harms consumers, narrow MFN can benefit consumers if platform competition is strong enough to restrict fees. Boik and Corts (2016) present a model with a monopolist seller which can sell through one or two platforms. They show that MFN clauses, which are adopted in equilibrium, always lead to higher fees and prices. The effects on platform's profit depend instead on the elasticity of aggregate demand, which in this linear model is inversely related to product substitutability: the closer substitutes the platforms (i.e., the less inelastic the aggregate demand), the more profitable the MFN agreements. Boik and Corts (2016) also explore the effect of MFN clauses on entry, finding that when the potential entrant and the incumbent have similar business model, parity clauses incentivize entry. By contrast, they discourage entry of a platform which is sufficiently differentiated. As already mentioned in Chapter 1, Johnson (2017) analyzes how the MFN clause affects equilibrium outcomes under a wholesale model and an agency model both in a bilateral monopoly and in a bilateral duopoly. He finds that consumers and retailers are better off without MFNs, whereas suppliers benefit from such clauses. Wang and Wright (2016a) analyze the effect of MFN on platforms' investment incentives. They consider three types of investment: investments in search technology, investments in advertising, and investments in benefits for consumers buying through the platform. For each type of investment, they analyze three possible scenarios: switching between selling channels and price parity clauses are not allowed, switching is allowed and price parity clauses are not imposed, and switching is allowed and price parity clauses are imposed. They show that investments to reduce search costs and to increase platform's visibility are insufficient without wide MFN clauses, because the platform cannot protect the investment, and thus cannot fully extract consumers' surplus. By contrast, the amount of investment in per-transaction benefits is efficient without price parity clauses, because the additional benefit can be compensated by a monopoly platform with a higher fee. They find that price parity clauses always lead platform to over-invest, since they allow it to fully extract consumers' surplus. Moreover, they find that wide parity clauses always lower consumer surplus.

My work is also related to the literature on two-sided market. Rochet and Tirole (2003); Armstrong (2006) present models of platform competition. Belleflamme and Peitz (2010) explore the role of for-profit platforms on seller investment incentives. Nocke et al. (2007) study and compare the effects of different platform ownership structures. They show that when network effects are strong, a monopoly platform is socially preferable. Baye and Morgan (2001) present a model of an Internet monopoly gatekeeper which charges fee to both sides of the market.

To the best of my knowledge, there is no work exploring the possible effects of parity clauses on quality investments by sellers. This work aims at giving a contribution in this respect.

3.3 The Model

3.3.1 Model Setup

There are three types of agents: a hotel, a platform and a mass of consumers.

The platform The hotel pays a per transaction fee to the platform, whereas consumers can access the platform for free. I assume that feedbacks on the platform perfectly reveal quality, i.e., platform is a certifier. I also assume that platform's cost are zero. The platform can impose a MFN clause or not. When the platform adopts an MFN clause, prices are the same on all channels, otherwise the hotel is free to set a lower price on its direct channels.

The hotel The hotel offers rooms of quality s uniformly distributed in $[0, 1]$. It can sell either only through its own sale channels or also through the platform. The constant marginal cost of producing a good of quality s is $c \geq 0$.

Consumers When quality is known, the utility function of a generic consumer is $v = (r + s)\theta - p_k$. Parameter $\theta \in [0, 1]$ captures consumers' willingness to pay for quality. Consumers with a large θ value quality more strongly, i.e., they are more sensitive to quality. p_k is the price of a room, and r a reservation value. Consumers buy if they have positive utility. Demand function for a room is then given by $Pr(v > 0) = Pr[(r + s)\theta - p_k > 0] = 1 - Pr(\theta < \frac{p_k}{s+r}) = 1 - \frac{p_k}{s+r}$.

When the hotel is not on the platform, quality is unknown and consumers assume $s = 0$. The utility function of a generic consumer is then $v = r\theta - p_u$. Hence, demand for rooms when the hotel is not on the platform is $q_u = 1 - \frac{p_u}{r}$.

I assume that consumers search first on the platform. They know when there is MFN because it is advertised on the platform. In this case, they buy directly on the platform since they know they pay the lowest price. Otherwise, they visit also the hotel's website and then decide where to buy based on relative prices.[4]

I assume that the fee the platform charges to the hotel is private information of the two parties.

Timing The timing of the game is as follows.

1. The platform decides whether to use or not the MFN clause.
2. The platform sets a per transaction fee.
3. The hotel chooses quality.
4. The hotel chooses where to sell (only through its own direct channels or also through the platform).
5. The hotel sets prices.
6. Consumers buy.

[4]In this setting, I assume switching costs for consumers to be zero.

Payoffs When the hotel is on the platform and consumers buy there, platform's profit is $\Pi_p = q_k f$; otherwise the platform will make no profit. The profit for the hotel selling through the platform is given by $\Pi_h = q_k(p_k - f) - cs$, where p_k is the final price, f is the fee and q_k is the demanded quantity on the platform. The profit for the hotel when it does not sell through the platform is given by $\widetilde{\Pi}_h = q_u p_u$, where p_u is the final price, and q_u is the demanded quantity "offline".

I first consider the case in which the platforms charges a percentage fee, a. Then I analyze the case with a non-percentage fee. I solve the problem by backward induction. It is straightforward to see that, when allowed, the platform will always adopt MFN. In fact, without MFN, platform's profits will be zero, regardless of hotel's choice. When the hotel does not join the platform, platform's profits are zero. When the hotel joins the platform, it will set a price on the platform slightly higher than the price offline, so as to divert demand from the platform to its own sale channels. In this case, consumers use the platform only to discover quality, but buy cheaply on hotel's direct channels. The hotel will be indifferent between selling offline and selling through the platform—and hence will set the same price on all channels—when $f = 0$. Also in this case platform's profits will be zero. For this reason, I will consider only the case with MFN.

3.3.2 Percentage Fee

At stage 5, the hotel sets the price. If it does not sell through the platform, the maximization problem will be:

$$\max_{p_u} \widetilde{\Pi}_h = \left(1 - \frac{p_u}{r}\right) p_u - cs, \tag{3.1}$$

which implies $p_u^* = \frac{r}{2}$, and $\widetilde{\Pi}_h^* = \frac{r}{4} - cs$.

If instead the hotel sells through the platform, the maximization problem will be:

$$\max_{p_k} \Pi_h = \left(1 - \frac{p_k}{s+r}\right) p_k(1-a) - cs, \tag{3.2}$$

which implies $p_h^* = \frac{(s+r)}{2}$ and $\Pi_h^* = \frac{(s+r)(1-a)}{4} - cs$.

At stage 4 the hotel chooses the channel. It sells also on the platform if it derives a higher profit, i.e., if and only if

$$\frac{(s+r)(1-a)}{4} - cs \geq \frac{r}{4} - cs. \tag{3.3}$$

The inequality holds for $s \geq \tilde{s}$, with $\tilde{s} = \frac{ar}{1-a}$. Hence, for $s < \tilde{s}$, the hotel sells only offline, whereas for $s \geq \tilde{s}$ it will sell also through the platform.

Note that $\frac{d\tilde{s}}{da} = \frac{r}{(1-a)^2} > 0$. Indeed, as one might expect, the higher the fee, the higher $\tilde{s}$ and, hence, the less likely the hotel will sell through the platform.

At stage 3 the hotel chooses the quality s:

$$\max_{s \in [0, \tilde{s})} \widetilde{\Pi}_h = \frac{r}{4} - cs \tag{3.4}$$

and

$$\max_{s \in [\tilde{s}, 1]} \Pi_h = \frac{(s+r)}{4}(1-a) - cs. \tag{3.5}$$

Consider first the maximization problem in (3.4). It is easily verified that $\frac{\partial \widetilde{\Pi}_h}{\partial s} = -c$. Since hotel's profit is decreasing in s, it will choose the lowest quality, $s^* = 0$.

Therefore, when the hotel does not sell through the platform, $p_u^* = \frac{r}{2}$, and $\widetilde{\Pi}_h^* = \frac{r}{4}$. Platform's profits are zero.

By contrast, when the hotel sells also through the platform, the first order condition for (3.5) is $\frac{1-a}{4} - c = 0$. Hence, when the cost of quality is sufficiently low—specifically, when $c < \frac{1-a}{4}$—hotel's profit is increasing in s. Hence, it will choose $s^* = 1$. The higher the fee a, the less likely the hotel will choose the maximum s. Indeed, the higher a, the lower the threshold which ensures that hotel's profit is increasing in s.

By contrast, when $c > \frac{1-a}{4}$, hotel's profit is decreasing in s. Hence, a hotel facing high costs for quality would choose $s^* = \tilde{s} = \frac{ar}{1-a}$. However, in such a case, the hotel would prefer to sell only offline. In fact, the profit of the hotel selling on the platform would be $\Pi_h^* = \frac{r}{4} - \frac{ar}{1-a}c$, which, since $a \in [0,1]$, is always less than the profit outside the platform, $\widetilde{\Pi}_h^* = \frac{r}{4}$.

At stage 2 the platform chooses the fee a:

$$\max_a \Pi_p = \frac{(1+r)a}{4}$$
$$\text{s.t.} \quad \frac{(1+r)(1-a)}{4} - c \geq \frac{r}{4} \tag{3.6}$$

Since its profit is increasing in a, the platform will set the highest fee satisfying the hotel's participation constraint (eq. (3.6)).

Hence, the optimal fee is $a^* = \frac{1-4c}{1+r}$[5].

The higher c, the lower should be the optimal fee.

To sum up, when quality costs are sufficiently low, i.e., when $c < \frac{1}{4}$, the hotel sells also through the platform and chooses the highest quality $s^* = 1$. It sets a room price $p_k^* = \frac{1+r}{2}$ and makes profits $\Pi_h^* = \frac{r}{4}$. Room prices and hotel's profits are increasing in the reservation value r. The platform sets a percentage fee $a^* = \frac{1-4c}{1+r}$ and makes profits $\Pi_p^* = \frac{1}{4} - c$. Fees and platform profits are decreasing in the hotel's marginal cost c. In fact, the higher the cost, the lower should be the fee. Otherwise, the hotel will choose not to sell on the platform.

When quality costs are high, i.e., when $c > \frac{1}{4}$, the hotel sells only through its own channels and chooses the minimum quality $s^* = 0$. It sets a room price $p_u^* = \frac{r}{2}$, and makes profits $\widetilde{\Pi}_h^* = \frac{r}{4}$. Platform's profits are zero.

3.3.3 Non-percentage Fee

I here consider the case of the platform charging a non-percentage fee, f.

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[5]When $a^* = \frac{1-4c}{1+r}$, hotel's profits are the same offline and on the platform. I assume that, when indifferent, the hotel sells also through the platform.

At stage 5 the hotel sets the price. As above, if the hotel does not sell through the platform, the maximization problem will be:

$$\max_{p_u} \widetilde{\Pi}_h = \left(1 - \frac{p_u}{r}\right) p_u - cs, \tag{3.7}$$

which implies $p_u^* = \frac{r}{2}$, and $\widetilde{\Pi}_h^* = \frac{r}{4} - cs$.

If the hotel sells through the platform, the maximization problem is:

$$\max_{p_k} \Pi_h = \left(1 - \frac{p_k}{s+r}\right) (p_k - f) - cs, \tag{3.8}$$

which implies $p_h^* = \frac{(s+r+f)}{2}$, and $\Pi_h^* = \frac{(s+r-f)^2}{4(s+r)} - cs$.

At stage 4, the hotel chooses where to sell. It sells also through the platform if and only if $\Pi_h^* \geq \widetilde{\Pi}_h^*$:

$$\frac{(s+r-f)^2}{4(s+r)} - cs \geq \frac{r}{4} - cs \tag{3.9}$$

The inequality holds for $s \geq s_2$, with $s_2 = \frac{2f-r+\sqrt{r(r+4f)}}{2}$.

(The proof is left in Appendix 3.A.)

At stage 3, the hotel chooses s:

$$\max_{s\in[0,s_2)} \widetilde{\Pi}_h = \frac{r}{4} - cs \tag{3.10}$$

and

$$\max_{s\in[s_2,1]} \Pi_h = \frac{(s+r-f)^2}{4(s+r)} - cs. \tag{3.11}$$

As shown in the previous section, when $s < s_2$, hotel's profit is decreasing in s and, hence, it chooses the minimum quality $s = 0$, and sells only offline.

The FOC for equation (3.11) is given by:

$$\begin{aligned}\frac{\partial \Pi_h}{\partial s} &= \frac{2(s+r-f)(s+r) - (s+r-f)^2}{4(s+r)^2} - c \\ &= \frac{(s+r-f)(s+r+f)}{4(s+r)^2} - c \\ &= \frac{1}{4} - c - \frac{f^2}{4(s+r)^2}\end{aligned}$$

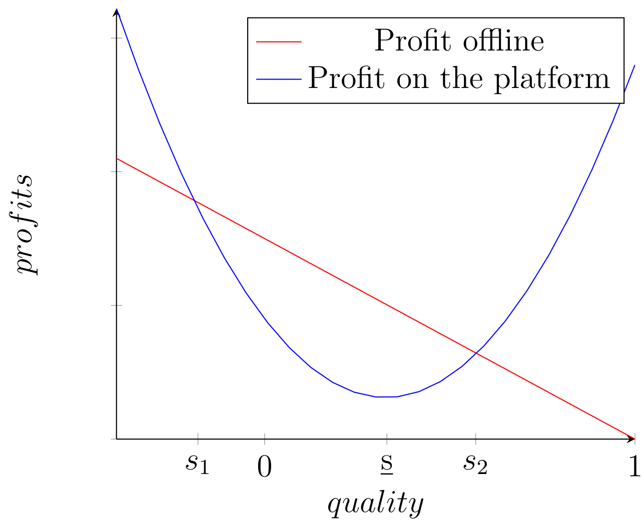

Figure 3.1: s_2 is to the right of $\underline{s}$

If $c > \frac{1}{4} - \frac{f^2}{4(s+r)}$, the first derivative is always negative. Hence, the profit is decreasing in s. In such a case, the hotel chooses the minimum quality $s = 0$ and does not sell on the platform.

If $c < \frac{1}{4} - \frac{f^2}{4(s+r)}$, the profit is increasing in s. Since the second derivative w.r.t. s is positive, $\frac{\partial^2 \Pi_h}{\partial s^2} = \frac{f^2}{2(s+r)^3}$, the stationary point $\underline{s} = (\frac{f}{\sqrt{1-4c}} - r)$ is a minimum.

The minimum, $\underline{s}$, can be either greater or less than s_2, depending on the marginal cost c. If $c < \bar{c} = \frac{r^2+4rf+(r+2f)\sqrt{r^2+4rf}}{4\left[r^2+4rf+2f^2+(r+2f)\sqrt{r^2+4rf}\right]}$, then $\underline{s} < s_2$: in such a case, hotel's profit is increasing in $s \in [s_2, 1]$. Therefore, the hotel chooses the maximum quality, $s = 1$, and sells also on the platform.

By contrast, when $c > \bar{c}$, $\underline{s} > s_2$. The profit function is not monotonic in $s \in [s_2, 1]$, hence the hotel chooses either $s = s_2$ or $s = 1$, depending on which one yields the greater profit. (See Appendix 3.A)

Let consider the case in which $c < \bar{c}$ and, therefore, the hotel chooses the maximum quality.

At stage 2 the platform sets the fee. The maximization problem is:

$$\max_f \Pi_p = \frac{(1+r-f)}{2(1+r)} f$$
$$\text{s.t.} \frac{(1+r-f)^2}{4(1+r)} - c \geq \frac{r}{4} \tag{3.12}$$

In order for platform's profits to be positive, the optimal fee should be included in $[0, 1+r]$. The platform sets the highest $f \in [0, 1+r]$ which satisfies the hotel's participation constraint in (3.12).

The optimal fee is then $f^* = r + 1 - \sqrt{(1+r)(r+4c)}$.

Hence, equilibrium outcomes are the following:

$$p_k^* = r + 1 - \frac{\sqrt{(1+r)(r+4c)}}{2}$$
$$f^* = r + 1 - \sqrt{(1+r)(r+4c)}$$
$$\Pi_h^* = \frac{r}{4} \tag{3.13}$$
$$\Pi_p^* = \frac{\sqrt{(1+r)(r+4c)}}{2} - \frac{r+4c}{2} \tag{3.14}$$

Proposition 3.8. *A hotel facing high costs for quality chooses the lowest quality, $s = 0$, and does not sell through the platform, whereas a hotel facing low quality costs chooses the highest quality, $s = 1$, and sells through the platform.*

Proposition 3.9. *A monopoly platform always prefers a percentage fee to a non-percentage fee.*

Proof. The platform prefers to set a percentage fee if and only if it yields higher profits:

$$\frac{1}{4} - c > \frac{\sqrt{(1+r)(r+4c)}}{2} - \frac{r+4c}{2} \tag{3.15}$$

$$(4c-1)^2 > 0$$

The inequality holds for any value of $c \neq \frac{1}{4}$. Hence, the platform always prefers a percentage fee. □

3.4 Policy Implications

An MFN clause, by ensuring platforms make positive profits and do not exit the market, can have positive effects on hotels' quality. Nonetheless, when the MFN clause is prohibited, the platform does not make any profit if it charges a transaction-based commission. In this case, it might decide to change business model and set a listing fee F. The hotel pays a fee, regardless of whether the transaction is concluded ot not through the platform.

Hotel's profit is given by $\Pi_h = q_k p_k - F - cs = \left(1 - \frac{p_k}{s+r}\right) p_k - F - cs$, whereas platform's profit is $\Pi_p = F$.

Note that in this case the hotel has no incentive to divert demand from the platform. In fact, when the platform charges a listing fee, the hotel is indifferent between selling through the platform and through its direct channels. Hence, it will set the same price on all channels, even in absence of MFN clause.

At stage 5, hotel sets p:

$$\max_{p_k} \Pi_h = \left(1 - \frac{p_k}{s+r}\right) p_k - cs, \tag{3.16}$$

which implies $p_k^* = \frac{(s+r)}{2}$ and $\Pi_h^* = \frac{(s+r)}{4} - F - cs$.

At stage 4, the hotel chooses the channel. It will sell also through the platform if and only if it gains a higher profit:

$$\frac{(s+r)}{4} - F - cs \geq \frac{r}{4} - cs. \tag{3.17}$$

The inequality holds for $s \geq \bar{s}$, with $\bar{s} = 4F$.

At stage 3, the hotel chooses s:

$$\max_{s \in [0,\bar{s})} \Pi_h = \frac{r}{4} - cs, \tag{3.18}$$

and

$$\max_{s \in [\bar{s},1]} \Pi_h = \frac{s+r}{4} - F - cs. \tag{3.19}$$

As shown in the previous section, a hotel selling only offline chooses the minimum quality.

If $c < \frac{1}{4}$, hotel's profit is increasing in s; therefore, the hotel will sell also on the platform and will choose $s^* = 1$.

Hence, $p_k^* = \frac{(1+r)}{2}$ and $\Pi_h^* = \frac{(1+r)}{4} - F - c$.

At stage 2 the platform sets the fee. Since its profit is increasing in F, the platform chooses the highest fee which satisfies the hotel's participation constraint,

$$\frac{(1+r)}{4} - F - c \geq \frac{r}{4}.$$

Hence, $0 \leq F^* \leq \frac{1}{4} - c$.

Equilibrium outcomes are the following:

$$p_k^* = \frac{1+r}{2}$$

$$F^* = \frac{1}{4} - c$$

$$\Pi_h^* = \frac{r}{4} \tag{3.20}$$

$$\Pi_p^* = \frac{1}{4} - c \tag{3.21}$$

Note that platform's profits are the same with a percentage fee and a listing fee. This result could depend on the model setting. In fact, I assumed that the amount of feedbacks is given and platform's costs are null. However, taking into account that the amount of feedbacks depends on the amount of transactions might give different results. Indeed, one might plausibly assume that the cost of certifying is decreasing in the number of feedbacks, which, in turn, increases with the amount of transactions. In such a case, the platform might prefer the type of fee which ensures more transactions through it.

3.5 Conclusions and Future Research

Research literature has shed light on the anticompetitive and harmful effects of MFN clauses in platforms. I show that the MFN clause, by ensuring platforms make positive profits and do not exit the market, can have positive effects on hotels' quality. Indeed, with MFN clause in place, the hotel chooses the maximum quality when it sells through the platform, whereas it chooses the minimum quality when it sells only through its own channel. I also find that a platform prefers a percentage fee to a non-percentage fee. I show that a platform always adopts an MFN clause, if allowed. However, once MFN is prohibited, the platform might decide to change its business model and set a listing fee.

Yet, the model should be further developed since it presents some drawbacks. In particular, further work should be devoted to a deeper analysis of the role of feedbacks. Indeed, I assume that search on the platform perfectly reveals quality, but I take the amount of feedbacks as given, without considering that it depends on the number of transactions completed on the platform. In fact, the more transactions on the platform, the more feedbacks, the more reliable and accurate is the information about quality in the platform. In such a case, also the hotel might prefer that transactions are completed on the platform. Moreover, as already pointed out, in this setting the platform is indifferent between imposing a percentage fee with MFN and a listing fee without MFN. However, the amount of feedbacks is crucial.

Furthermore, in this setting, I do not take into account that when a hotel is on the platform, demand for room might increase. Also in this case, the hotel might have an incentive to sell also through the platform, even if the profit for each transaction is lower.

3.A Appendix

3.A.1 Choice of Channel

Proof. At stage 4, the hotel chooses where to sell. It will sell also on the platform if and only if

$$\frac{(s+r-f)^2}{4(s+r)} - cs \geq \frac{r}{4} - cs. \tag{A-1}$$

$$s^2 + r^2 + f^2 + 2sr - 2sf - 2rf \geq sr + r^2$$

$$s^2 + f^2 + sr - 2sf - 2rf - 2sr \geq 0$$

$$s^2 - s(2f - r) + f^2 - 2rf \geq 0$$

The inequality holds for

$$s \leq s_1 = \frac{2f - r - \sqrt{r(r+4f)}}{2} \tag{A-2}$$

and

$$s \geq s_2 = \frac{2f - r + \sqrt{r(r+4f)}}{2} \tag{A-3}$$

Since $s \in [0,1]$, there are two constraints:

$$\begin{cases} 0 \leq s_1 \leq 1 \\ 0 \leq s_2 \leq 1 \end{cases} \tag{A-4}$$

Let consider the constraint $s_1 \geq 0$.

$$\frac{2f - r - \sqrt{r(r+4f)}}{2} \geq 0$$

$$\begin{cases} 2f - r \geq 0 \\ 2f - r \geq \sqrt{r(r+4f)} \end{cases} \tag{A-5}$$

$$\begin{cases} f \geq \frac{r}{2} \\ 4f^2 + r^2 - 4rf \geq r^2 + 4rf \end{cases} \tag{A-6}$$

$$\begin{cases} f \geq \frac{r}{2} \\ f(f - 2r) \geq 0 \end{cases} \tag{A-7}$$

Hence, $s_1 \geq 0$ if and only if $f \geq f_1 = 2r$.

Let now consider the constraint $s_1 \leq 1$.

$$\frac{2f - r - \sqrt{r(r+4f)}}{2} \leq 1$$

$$2f - r - \sqrt{r(r+4f)} \leq 2$$

$$2f - r - 2 \leq \sqrt{r(r+4f)}$$

The inequality is equivalent to the set of systems:

$$\begin{cases} 2f - r - 2 \geq 0 \\ r(r+4f) \geq 0 \\ (2f - r - 2)^2 \leq r(r+4f) \end{cases} \quad \bigcup \quad \begin{cases} 2f - r - 2 < 0 \\ r(r+4f) \geq 0 \end{cases}$$

$$\begin{cases} f \geq \frac{r+2}{2} \\ 4f^2 + r^2 + 4 - 4fr - 8f + 4r \leq r^2 + 4fr \end{cases} \quad \bigcup \quad f < \frac{r+2}{2}$$

$$\begin{cases} f \geq \frac{r+2}{2} \\ f^2 - 2f(r+1) + r + 1 \leq 0 \end{cases} \quad \bigcup \quad f < \frac{r+2}{2}$$

$$\begin{cases} f \geq \frac{r+2}{2} \\ \widetilde{f_1} \leq f \leq \widetilde{f_2} \end{cases} \quad \bigcup \quad f < \frac{r+2}{2}$$

where $\widetilde{f_1} = r + 1 - \sqrt{r(r+1)}$ and $\widetilde{f_2} = r + 1 + \sqrt{r(r+1)}$.

Let check that both solutions are acceptable, that is both $\widetilde{f_1}$ and $\widetilde{f_2}$ are positive. Whereas it is straightforward to see that $\widetilde{f_2}$ is positive, I have to study the sign of $\widetilde{f_1}$:

$$r + 1 - \sqrt{r(r+1)} \geq 0$$

$$(r+1)^2 > r(r+1)$$

$$r + 1 > r$$

The inequality is always true.

$s_1 \leq 1$ if and only if $f \leq \widetilde{f_2} = r + 1 + \sqrt{r(r+1)}$.

Hence, $0 \leq s_1 \leq 1$ if and only if $f_1 = 2r \leq f \leq \widetilde{f_2} = r + 1 + \sqrt{r(r+1)}$.

Let now consider the constraint $s_2 \geq 0$.

$$\frac{2f - r + \sqrt{r(r+4f)}}{2} \geq 0$$

$$\sqrt{r(r+4f)} \geq r - 2f$$

The inequality is equivalent to the set of systems:

$$\begin{cases} r - 2f \geq 0 \\ r(r+4f) \geq 0 \\ r(r+4f) \geq (r-2f)^2 \end{cases} \quad \bigcup \quad \begin{cases} r - 2f < 0 \\ r(r+4f) \geq 0 \end{cases}$$

$$\begin{cases} f < \frac{r}{2} \\ r^2 + 4fr \geq 4f^2 + r^2 - 4fr \end{cases} \qquad \bigcup \qquad f \geq \frac{r}{2}$$

$$\begin{cases} f < \frac{r}{2} \\ f(f - 2r) \leq 0 \end{cases} \qquad \bigcup \qquad f \geq \frac{r}{2}$$

$$\begin{cases} f < \frac{r}{2} \\ f \leq 2r \end{cases} \qquad \bigcup \qquad f \geq \frac{r}{2}$$

s_2 is always positive.

Let now find the conditions under which $s_2 < 1$.

$$\frac{2f - r + \sqrt{r(r + 4f)}}{2} \leq 1$$

$$\sqrt{r^2 + 4rf} \leq 2 + r - 2f \tag{A-8}$$

$$\begin{cases} 2 + r - 2f \geq 0 \\ r^2 + 4rf \leq 4 + r^2 + 4f^2 + 4r - 8f - 4rf \end{cases}$$

$$\begin{cases} f \leq 1 + \frac{r}{2} \\ f^2 - 2f(r + 1) + (r + 1) \geq 0 \end{cases}$$

$$\begin{cases} f \leq 1 + \frac{r}{2} \\ f \leq \widetilde{f}_1 \vee f \geq \widetilde{f}_2 \end{cases}$$

Hence, $s_2 \leq 1$ if and only if $f \leq \widetilde{f}_1 = (r + 1) - \sqrt{r(r + 1)}$.

For $r > \frac{1}{3}$, we have:

		0	$\widetilde{f}_1$	f_1	$\widetilde{f}_2$
(I)	$0 \leq s_1 \leq 1$	–	–		–
(II)	$0 \leq s_2 \leq 1$		–	–	–

Conditions (I) and (II) are never satisfied simultaneously. Hence, the hotel would either choose $s \leq s_1$ or $s \geq s_2$.

Let consider the case in which $s \leq s_1$ and show that it is not feasible. Since hotel profit is decreasing in $s \in [0, s_1]$, the hotel would choose $s = 0$. However, at stage 2, the platform would face the following maximization problem:

$$\max_f \Pi_p = \frac{(r-f)}{2r} f \tag{A-9}$$

$$\text{s.t.} \frac{(r-f)^2}{4(1+r)} \geq \frac{r}{4}$$

The platform would never choose a fee f greater than r, otherwise it would make negative profits.

Nonetheless, condition (I) is satisfied only when $2r \leq f \leq r + 1 + \sqrt{r(r+1)}$.

Hence, the hotel will sell also through the platform if and only if

$$s \geq s_2 = \frac{2f - r + \sqrt{r(r+4f)}}{2} \tag{A-10}$$

□

3.A.2 Choice of s

Proof. At stage 3, the hotel chooses the level of quality, depending on its costs.

If the minimum point $\underline{s}$ is to the right of s_2 (the minimum quality on the platform) (see Figure 3.2), the hotel will choose either $s = s_2$ or $s = 1$, depending on which one yields the greater profit. Hence, I have to compare $\Pi_{h|_{s=s_2}}$ and $\Pi_{h|_{s=1}}$.

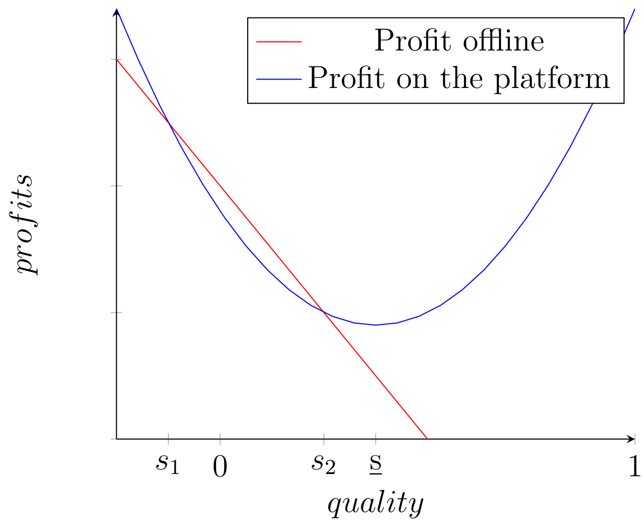

Figure 3.2: s_2 is to the left of $\underline{s}$

$$\Pi_{h|_{s=1}} = \frac{(1+r-f)^2}{4(1+r)} - c \geq \frac{r}{4} - cs_2 = \Pi_{h|_{s=s_2}} \tag{A-11}$$

$$(1+r-f)^2 - r(1+r) > 4c(1+r)(1-s_2)$$

$$4c(1-s_2)(1+r) < 1 + r^2 + f^2 + 2r - 2rf - 2f - r - r^2$$

$$4c(1-s_2)(1+r) < f^2 - 2f(1+r) + 1 + r$$

$$c < \frac{f^2 - 2f(1+r) + 1 + r}{4(1-s_2)(1+r)},$$

where $s_2 = \frac{2f - r + \sqrt{r(r+4f)}}{2}$.

Hence,

$$c^* \leq \frac{f^2 - 2f(1+r) + 1 + r}{2(1+r)(2 - 2f + r - \sqrt{r(r+4f)})} \tag{A-12}$$

Cost	Channel	Quality
(1) $c > \frac{1}{4} - \frac{f^2}{4(s+r)} \rightarrow$ profit is decreasing in s	Offline	$s = 0$
(2) $\begin{cases} c < \frac{1}{4} - \frac{f^2}{4(s+r)} \rightarrow \text{profit increasing in s} \\ c < \frac{r^2+4rf+(r+2f)\sqrt{r^2+4rf}}{4\left[r^2+4rf+2f^2+(r+2f)\sqrt{r^2+4rf}\right]} \rightarrow \underline{s} < s_2 \end{cases}$	Platform	$s = 1$
(3) $\begin{cases} c < \frac{1}{4} - \frac{f^2}{4(s+r)} \rightarrow \text{profit increasing in s} \\ c > \frac{r^2+4rf+(r+2f)\sqrt{r^2+4rf}}{4\left[r^2+4rf+2f^2+(r+2f)\sqrt{r^2+4rf}\right]} \rightarrow \underline{s} > s_2 \\ c \leq \frac{f^2-2f(1+r)+1+r}{2(1+r)\left(2-2f+r-\sqrt{r^2+4rf}\right)} \rightarrow \Pi_{h\|_{s=s_2}} < \Pi_{h\|_{s=1}} \end{cases}$	Platform	$s = 1$
(4) $\begin{cases} c < \frac{1}{4} - \frac{f^2}{4(s+r)} \rightarrow \text{profit increasing in s} \\ c > \frac{r^2+4rf+(r+2f)\sqrt{r^2+4rf}}{4\left[r^2+4rf+2f^2+(r+2f)\sqrt{r^2+4rf}\right]} \rightarrow \underline{s} > s_2 \\ c > \frac{f^2-2f(1+r)+1+r}{2(1+r)\left(2-2f+r-\sqrt{r^2+4rf}\right)} \rightarrow \Pi_{h\|_{s=s_2}} > \Pi_{h\|_{s=1}} \end{cases}$	Offline	$s = 0$

Let now consider case (4): chose the hotel to sell also on the platform, it would set $s = s_2$ which yields greater profits than $s = 1$.

However, selling only offline and setting a quality $s = 0$ is more profitable than selling also on the platform and setting a quality $s = s_2$. Indeed, at $s = s_2$ profits on the platform are equal to profits offline, which, in turn, are lower than profits offline with $s = 0$. Hence, when $c > \frac{f^2-2f(1+r)+1+r}{2(1+r)(2-2f+r-\sqrt{r(r+4f))}}$, the hotel will sell only offline and will set the lowest quality, $s = 0$.

□

I here substitute $f^* = 1 + r - \sqrt{(1+r)(r+4c)}$ into $c < \frac{1}{4} - \frac{f^2}{4(s+r)}$ to find the cost above which the hotel will always sell only offline.

$$c \leq \frac{1}{4} - \frac{(1+r)^2 - 2(1+r)\sqrt{(1+r)(r+4c)} + (1+r)(r+4c)}{4(1+r)}$$

$$4c \leq 1 - 1 - r + 2\sqrt{(1+r)(r+4c)} - r - 4c$$

$$8c + 2r \leq 2\sqrt{(1+r)(r+4c)}$$

$$(4c+r)^2 \leq (1+r)(r+4c)$$

$$16c^2 + r^2 + 8rc \leq r + 4c + r^2 + 4rc$$

$$16c^2 + 4c(r-1) - r \leq 0$$

$$c = \frac{-2(r-1) \pm 2(r+1)}{16}$$

$$c^* \leq \frac{1}{4}$$

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